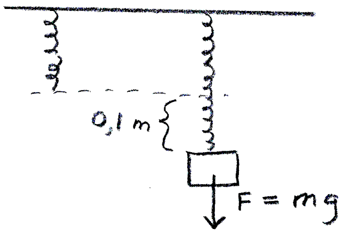
Lösungen für Schwingungen Aufgaben

Aufgabe 1



a. Since $100\,grams$ stretches the spring $10\,cm$, we can determine the spring constant c:

$$\begin{aligned} F &= c\Delta x \\ (0,1\,kg)(9,8\,\frac{m}{s^2}) &= c(0,1\,m) \\ c &= 9,8\,\frac{N}{m} \end{aligned}$$

The angular frequency $\omega_0 = \sqrt{c/m} = \sqrt{9,8/0,1} \approx 9,9\,rad/s$. The general expression $x(t)$ for undamped oscillations is

$$\begin{aligned} x(t) &= A\,cos(\omega_0 t - \phi) \\ x(t) &= A\,cos(9,9\,t - \phi) \end{aligned}$$

Since $x(0) = -6\,cm$ and $v(0) = 0$, we have:

$$\begin{aligned} -6 &= A\,cos(-\phi) \\ 0 &= -A\omega_0\,sin(-\phi) \end{aligned}$$

which yields $\phi = 0$ and $A = -6\,cm$:

$$x(t) = -6\,cos(9,9\,t)\,cm \qquad (1)$$

Aufgabe 2

Since the period is $T = 2\,s$, $\omega_0 = 2\pi/T = \pi\,s^{-1}$

a. A general expression for $x(t)$ is $x(t) = A\,cos(\omega_0 t - \phi)$. Since the amplitude equals $15\,cm$, $A = 15$ cm, we have

$$x(t) = 15\,cos(\pi\,t - \phi)\,cm \tag{2}$$

At $t = 0$ the position is $x = 0$, we have

$$\begin{aligned} 0 &= 15\,cos(-\phi) \\ \phi &= \pm\frac{\pi}{2} \end{aligned}$$

To determine which sign ϕ has, we use the condition that at $t = 0$, $v_0 > 0$. Since $v(t) = \dot{x} = -15\pi sin(\pi t - \phi)$,we have

$$\begin{aligned} v_0 &= -15\pi\,sin(-\phi) \\ v_0 &= +15\pi\,sin(\phi) \\ \rightarrow sin(\phi) &> 0 \end{aligned}$$

so $\phi = +\pi/2$. So, $x(t)$ is

$$\begin{aligned} x(t) &= 15\,cos(\pi\,t - \pi/2)\,cm \\ x(t) &= 15\,sin(\pi\,t)\,cm \end{aligned}$$

b. The velocity is the derivative of $x(t)$:

$$v(t) = \dot{x} = 15\pi\,cos(\pi\,t)\,\frac{cm}{s} \tag{3}$$

So v_{max} is $15\pi\,cm/s \approx 47,1\,\frac{cm}{s}$.

c. The acceleration is the derivative of $v(t)$:

$$a(t) = \dot{v} = -15\,\pi^2\,sin(\pi\,t)\,\frac{cm}{s^2} \tag{4}$$

So $a_{max} = 15\,\pi^2 cm/s^2 \approx 148\,\frac{cm}{s^2}$

d. The total energy is the maximum kinetic energy:

$$E_{tot} = \frac{m}{2}v_{max}^2 = \frac{0,1\,kg}{2}(0.471\frac{m}{s})^2 \approx 0,0111\,J \tag{5}$$

Aufgabe 3

a. The amplitude is $6\,m$.

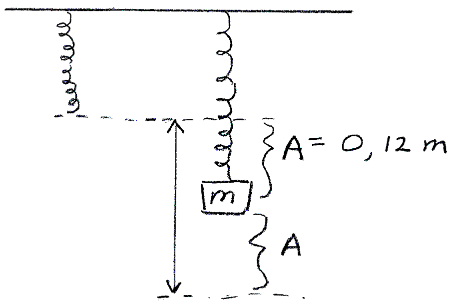
b. The angular frequency is $\omega_0 = 3\pi\,rad/s$.

$$T = \frac{2\pi}{\omega_0} = \frac{2\pi}{3\pi} = \frac{2}{3}s \tag{6}$$

c. The frequency $f = 1/T$:

$$f = \frac{1}{T} = \frac{1}{2/3\,s} = 1.5\,Hz \tag{7}$$

Aufgabe 4



a. The mass was let go when the spring was unstretched. Since the amplitude is $0,12\,m$, the mass will fall $0,24\,m$ below the unstretched length when it oscillates. Thus, the equilibrium position of the mass is $0,12\,m$ below the unstretched position. Therefore, a weight of mass $0,3\,kg$ will stretch the spring $\Delta x = 0,12\,m$:

$$\begin{aligned} F &= c\Delta x \\ (0,3)9,8\,N &= c(0,12\,m) \\ c &\approx 24,5\,\frac{N}{m} \end{aligned}$$

The period of oscillation is

$$\begin{aligned}
T &= \frac{2\pi}{\omega_0} \\
&= 2\pi\sqrt{\frac{m}{c}} \\
T &= 2\pi\sqrt{\frac{0,3}{24,5}} \approx 0,695\,s
\end{aligned}$$

Aufgabe 5

The relationship between the frequency and length of a simple pendulum in the small angle limit is

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{l}} \tag{8}$$

a. The above relationship holds for both pendulum 1 and 2:

$$\begin{aligned}
f_1 &= \frac{1}{2\pi}\sqrt{\frac{g}{l_1}} \\
f_2 &= \frac{1}{2\pi}\sqrt{\frac{g}{l_2}}
\end{aligned}$$

Dividing the two equations by each other,

$$\frac{f_1}{f_2} = \sqrt{\frac{l_2}{l_1}} \tag{9}$$

The above equation holds even if the amplitudes are not small, but friction is neglected. We have $l_2 = (0,9)l_1$ and $f_2 = f_1 + 0,1\,Hz$:

$$\begin{aligned}
\frac{f_1}{f_1 + 0,1} &= \sqrt{\frac{0.9 l_1}{l_1}} = \sqrt{0.9} \\
f_1 &\approx 1,85\,Hz
\end{aligned}$$

The length l_1 is found:

$$l_1 = \frac{g}{(2\pi f_1)^2} \approx 0,0725\,m \tag{10}$$

Aufgabe 6

Let T_A be the period of pendulum A, and T_B be the period of pendulum B.

a. We have the following two equations:

$$\begin{aligned} \frac{T_A}{T_B} &= \frac{19}{20} \\ \frac{15}{T_A} - \frac{15}{T_B} &= 3 \end{aligned}$$

From these two equations, with two unknowns, we can solve for T_A and T_B.

$$\begin{aligned} T_B &= T_A(\frac{20}{19}) \\ \rightarrow \frac{15}{T_A} - \frac{15}{T_A(20/19)} &= 3 \\ \frac{1}{T_A} - \frac{19}{20T_A} &= \frac{1}{5} \\ T_A &= \frac{1}{4}\,s \end{aligned}$$

Solving for the frequencies:

$$\begin{aligned} f_A &= \frac{1}{T_A} = 4\,Hz \\ f_B &= (\frac{19}{20})f_A = 3,8\,Hz \end{aligned}$$

Aufgabe 7

a. The angular frequency is $\omega_0 = 90s^{-1}$. The period T is

$$T = \frac{2\pi}{\omega_0} = \frac{2\pi}{90} = 0.0698\,s \tag{11}$$

b. The position function, and velocity function are

$$\begin{aligned} x(t) &= A\,cos(90t - \phi_0) \\ v(t) &= \dot{x} = -90A\,sin(90t - \phi_0) \end{aligned}$$

Substituting in the initial conditions $x(0) = 0,02\,m$ and $v(0) = -3\,m/s$

$$\begin{aligned} 0,02 &= A\,cos(-\phi_0) \\ -3 &= \dot{x} = -90A\,sin(-\phi_0) \end{aligned}$$

By dividing these two equations, the amplitude A cancels:

$$\begin{aligned} \frac{0,02}{-3} &= \frac{1}{-90} cotan(-\phi_0) \\ tan(-\phi_0) &= \frac{5}{3} \\ tan(\phi_0) &= -\frac{5}{3} \end{aligned}$$

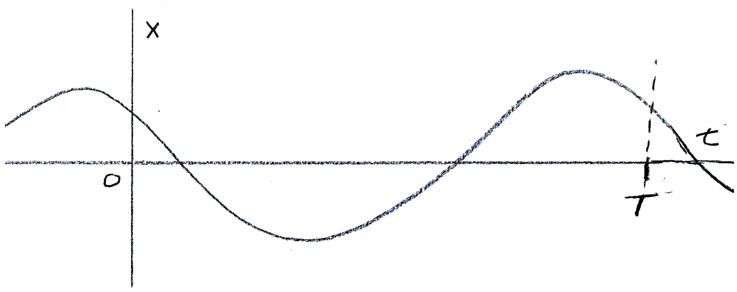
There are two solutions: $\phi_0 = 121°$ and $\phi_0 = -59°$. Since $x(0) > 0$ and $v(0) < 0$, the phase angle must be such that $-\phi_0$ is in the first quadrant. Hence, ϕ_0 is in the fourth quadrant, and the correct phase is:

$$\phi_0 \approx -59° \tag{12}$$

To find the amplitude A:

$$\begin{aligned} 0,02 &= A\,cos(-\phi_0) \\ A &= \frac{0,02}{cos(59°)} \\ A &\approx 0,0389\,m = 3,89\,cm \end{aligned}$$

c. So the solution is $x(t) = 3,89\,cos(90t + 59°)\,cm$ as shown in the sketch.



Aufgabe 8

The angular frequency $\omega = \frac{210}{60} 2\pi \approx 21,99\, rad/s$.

a.

$$\begin{aligned} y(t) &= r\, sin(\omega t) \\ v_y(t) &= \dot{y} = \omega r\, cos(\omega t) \\ v_{y_1} &= \omega r\, cos(15°) \\ v_{y_1} &= (21,99)(0,18)\, cos(15°) \approx 3,82\, \frac{m}{s} \end{aligned}$$

b. Similarly

$$v_{y_2} = (21,99)(0,18)\, cos(125°) \approx -2,27\, \frac{m}{s} \tag{13}$$

c. The maximum value for $v_y(t)$ is when $cos(\omega t) = 1$.

$$v_{y-max} = (21,99)(0,18) = 3,958\, \frac{m}{s} \tag{14}$$

d. The acceleration is the derivative of velocity

$$\begin{aligned} a_y &= \dot{v} = -\omega^2 r\, sin(\omega t) \\ |a_{y-max}| &= \omega^2 r \\ &= (21,99)^2(0,18) \approx 87,04\, \frac{m}{s^2} \end{aligned}$$

Aufgabe 9

a. The undamped angular frequency $\omega_0 = \sqrt{c/m}$.

$$\begin{aligned} \omega_0 &= \sqrt{\frac{c}{m}} \\ c &= m\omega_0^2 \\ c &= 1,0(10)^2 = 100\, \frac{N}{m} \end{aligned}$$

b. First we find δ:

$$\begin{aligned} \omega^2 &= \omega_0^2 - \delta^2 \\ \delta^2 &= \omega_0^2 - \omega^2 \\ &= 10^2 - 8^2 \\ \delta &= 6\,\frac{1}{s} \end{aligned}$$

Now we solve for d:

$$\begin{aligned} \delta &= \frac{d}{2m} \\ d &= 2m\delta \\ &= 2(1)(6) = 12\,\frac{N}{m/s} \end{aligned}$$

c. The amplitude decreases in time as $A = A_0 e^{-\delta t}$. The period $T = 2\pi/\omega = 2\pi/8\,s$.

$$\begin{aligned} p &= \frac{A}{A_0} = e^{-\delta t} \\ p_1 &= e^{-\delta T} \\ &= e^{-6(2\pi/8)} \\ &= 0,00898 = 0,898\,\% \end{aligned}$$

After 2 periods:

$$\begin{aligned} p_2 &= e^{-\delta\, 2T} \\ &= e^{-6(2)(2\pi/8)} \\ &= 0,0000807 = 0,00807\,\% \end{aligned}$$

Aufgabe 10

a. To determine the spring constant c, we use the fact that a force of $(100\,kg)(9,8\,m/s)\,N$ causes the spring length to change $0,03\,m$:

$$\begin{aligned} F &= c(\Delta x) \\ c &= \frac{F}{\Delta x} \\ c &= \frac{(100\,kg)(9,8\,\frac{m}{s})}{0,03\,m} \\ c &\approx 3,27 \times 10^4\,\frac{N}{m} \end{aligned}$$

The frequency of oscillation $f = \frac{1}{2\pi}\sqrt{c/m}$, where m is the amount of mass that is oscillating.

$$
\begin{aligned}
f &= \frac{1}{2\pi}\sqrt{\frac{3,27 \times 10^4\, N/m}{2100\, kg}} \\
&\approx 0,628\, Hz
\end{aligned}
$$

Aufgabe 11

a. A total force of $(1800\, kg)(9,8\, m/s^2)$ causes the truck to be lowered $0,1\, m$. Since there are four tires, each spring recieves $1/4$ of the total force:

$$
\begin{aligned}
F &= c\Delta x \\
\frac{(1800\, kg)(9,8\, m/s^2)}{4} &= c(0,1\, m) \\
c &= 44100\, \frac{N}{m} = 44,1\, \frac{kN}{m}
\end{aligned}
$$

The oscillations are **aperiodischen Grenzfall**:

$$
\begin{aligned}
\delta &= \omega_0 \\
\frac{d}{2m_R} &= \sqrt{\frac{c}{m_R}} \\
d &= 2\sqrt{cm_R} \\
d &= 2\sqrt{44100(40)} \approx 2656\, \frac{N}{m/s}
\end{aligned}
$$

Aufgabe 12

The undamped angular frequency is $\omega_0 = \sqrt{c/m} = \sqrt{56/0.75} \approx 8,64\, rad/s$. The parameter $\delta = b/(2m) = 0,162/(2(0.75)) \approx 0,108\, s^{-1}$.

a. The angular frequency ω is

$$\omega = \sqrt{\omega_0^2 - \delta^2} = \sqrt{8,64^2 - 0,108^2} \approx 8,64\, rad/s \tag{15}$$

The period T is thus

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{8,64} \approx 0,727\, s \tag{16}$$

b. The logarithmische Dekrement $\Lambda = \delta T = (0,108)(0,727) = 0,0785$

c. A general expression for $x(t)$ is

$$x(t) = Ae^{-\delta t} cos(\omega t - \phi) \tag{17}$$

The initial condition $x(0) = 0$ allows one to determine ϕ:

$$\begin{aligned} 0 &= Ae^0\, cos(-\phi) \\ \phi &= \pm\frac{\pi}{2} \end{aligned}$$

Without loss of generality, we can take ϕ to be $\pi/2$, since A can be positive or negative. So

$$\begin{aligned} x(t) &= Ae^{-\delta t} cos(\omega t - \frac{\pi}{2}) \\ x(t) &= Ae^{-\delta t} sin(\omega t) \end{aligned}$$

The other condition $x(1) = 0,12\,m$ allows one to determine A:

$$\begin{aligned} x(t) &= Ae^{-\delta t} sin(\omega t) \\ 0,12 &= Ae^{-(0,108)(1)} sin(8,64(1)) \\ A &\approx 0,189\,m \end{aligned}$$

The function $x(t)$ is

$$x(t) = 0,189\, e^{-0,108\,t} sin(8,64\,t)\, m \tag{18}$$

Aufgabe 13

a. We have sinusoidal forced oscillations, with a driving angular frequency of $\Omega = 6\,rad/s$. The resulting amplitude A is found in the formula sheet:

$$\begin{aligned} A &= (\frac{F_0}{m}) \frac{1}{\sqrt{(\omega_0^2 - \Omega^2)^2 + 4\delta^2\Omega^2}} \\ &= (\frac{0,1\,N}{0,05\,kg}) \frac{1}{\sqrt{(10^2 - 6^2)^2 + 4(2^2)6^2}} \\ A &\approx 0,029\,m = 2,9\,cm \end{aligned}$$

The phase ϕ between the driving force and the position function $x(t)$ is also found in the formula sheet:

$$\begin{aligned} tan(\phi) &= \frac{2\delta\Omega}{\omega_0^2 - \Omega^2} \\ &= \frac{2(2)6}{10^2 - 6^2} \\ \phi &\approx 20,6^\circ = 0,359\, rad \end{aligned}$$

The position function $x(t)$ is therefore:

$$x(t) = 0,029\, cos(6t - 0,359)\, m = 2,9\, cos(6t - 0,359)\, cm \tag{19}$$

b. The resonance frequency Ω_R for the system is

$$\Omega_R = \sqrt{\omega_0^2 - 2\delta^2} = \sqrt{10^2 - 2(2)^2} \approx 9,59\, \frac{rad}{s} \tag{20}$$

c. The amplitude A_{res} is

$$\begin{aligned} A_{res} &= f_0 \frac{1}{2\delta\sqrt{\omega_0^2 - \delta^2}} \\ &= (\frac{0,1\, N}{0,05\, kg}) \frac{1}{2(2)\sqrt{10^2 - 2^2}} \\ A_{res} &\approx 0,051\, m = 5,1\, cm \end{aligned}$$

The phase angle between the driving force and $x(t)$ is

$$\begin{aligned} tan(\phi) &= \frac{2\delta\Omega_R}{\omega_0^2 - \Omega_R^2} = \frac{\Omega_R}{\delta} \\ &= \frac{9,59}{2} \\ \phi &\approx 78,2^\circ = 1,365\, rad \end{aligned}$$

The position function $x_R(t)$ is therefore

$$x_R(t) = 0,051\, cos(9,59t - 1,365)\, m = 5,1\, cos(9,59t - 1,365)\, cm \tag{21}$$

d. The half-life t_H is the time it takes the amplitude to decrease to one half its value. The amplitude as a function of time is $A = A_0 e^{-\delta t}$.

$$\begin{aligned} A_0 e^{-\delta(t+t_H)} &= \frac{A_0}{2} e^{-\delta t} \\ e^{-\delta t_H} &= \frac{1}{2} \\ -\delta t_H &= ln(\frac{1}{2}) \\ \delta t_H &= ln(2) \\ t_H &= \frac{ln(2)}{\delta} \\ &= \frac{ln(2)}{2}\, s \end{aligned}$$

Aufgabe 14

a. The beat frequency $f_s = 1/T_s$. So, $f_s = 1/0,5 = 2\,Hz$:

$$f_2 - f_1 = 2\,Hz \tag{22}$$

The average frequency is $(f_1 + f_2)/2$:

$$\frac{f_1 + f_2}{2} = 441 \tag{23}$$

We have two equations with two unknowns:

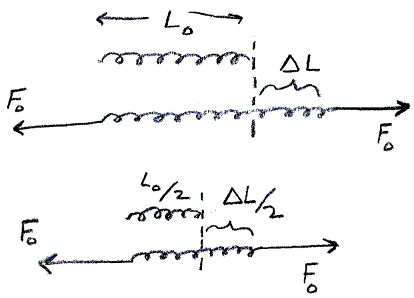
$$\begin{aligned} f_2 - f_1 &= 2 \\ f_2 + f_1 &= 882 \end{aligned}$$

We can solve for f_2 by adding the two equations:

$$\begin{aligned} 2f_2 &= 884 \\ f_2 &= 442\,Hz \end{aligned}$$

The other frequency, $f_1 = f_2 - 2 = 440\,Hz$.

Aufgabe 15



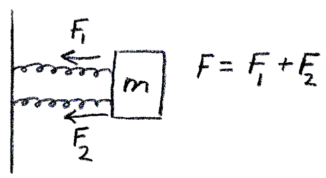
a. Suppose a force of magnitude F_0 acting on each end of a spring of length L_0 causes the spring to stretch a length of ΔL. Then,

$$c = \frac{F_0}{\Delta L} \tag{24}$$

If the same force of magnitude F_0 acts on each side of a similar spring that is half as long, i.e. of length $L_0/2$, then the spring will stretch half as much, $(\Delta L)/2$. In this case, the spring constant c' is

$$c' = \frac{F_0}{(\Delta L)/2} = 2\frac{F_0}{\Delta L} = 2c \tag{25}$$

Aufgabe 16



a. Parallel Case: When the block is moved a distance Δx from equilibrium, spring 1 will exert a restoring force of $c_1\,\Delta x$ and spring 2 will exert a restoring force of $c_2\,\Delta x$. The forces are in the same direction, and will add:

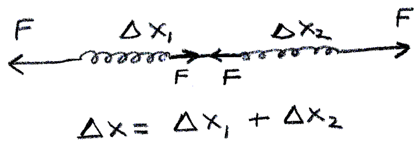
$$\begin{aligned} F &= -c_1\Delta x - c_2\Delta x \\ F &= -(c_1 + c_2)\Delta x \end{aligned}$$

The spring constant for the parallel case is thus: $c = c_1 + c_2$, and the frequency of oscillation is

$$\begin{aligned} f &= \frac{1}{2\pi}\sqrt{\frac{c_1+c_2}{m}} \\ f &= \frac{1}{2\pi}\sqrt{\frac{c_1}{m}+\frac{c_2}{m}} \end{aligned}$$

However, $\frac{c_1}{m} = (2\pi f_1)^2$ and $\frac{c_2}{m} = (2\pi f_2)^2$, so

$$\begin{aligned} f &= \frac{1}{2\pi}\sqrt{\frac{c_1}{m}+\frac{c_2}{m}} \\ &= \frac{1}{2\pi}\sqrt{(2\pi f_1)^2+(2\pi f_2)^2} \\ f &= \sqrt{f_1^2+f_2^2} \end{aligned}$$



b. Series case: Suppose a force F is applied to each end of the springs in series. Then, each spring will experience the force F at its ends. Spring 1 will stretch a distance Δx_1 and spring 2 will stretch a distance Δx_2. The total stretch will be $\Delta x = \Delta x_1 + \Delta x_2$.

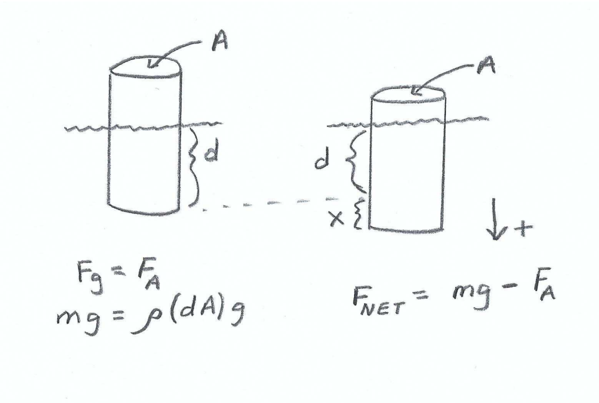
$$\begin{aligned} \Delta x &= \Delta x_1 + \Delta x_2 \\ \frac{F}{c} &= \frac{F}{c_1}+\frac{F}{c_2} \\ \frac{1}{c} &= \frac{1}{c_1}+\frac{1}{c_2} \end{aligned}$$

Since the spring constants are equal to $m(2\pi f)^2$, we have

$$\frac{1}{m(2\pi f)^2} = \frac{1}{m(2\pi f_1)^2}+\frac{1}{m(2\pi f_2)^2}$$

$$
\begin{aligned}
\frac{1}{f^2} &= \frac{1}{f_1^2} + \frac{1}{f_2^2} \\
f &= \frac{f_1 f_2}{\sqrt{f_1^2 + f_2^2}}
\end{aligned}
$$

Aufgabe 17



a. We need to find an expression for the restoring force on the block when it is pushed a distance x from its floating equilibrium position. Let d be the distance the block sinks in water when it floats (see the figure). Since the block floats, we have $F_g = F_A$:

$$
\begin{aligned}
mg &= \rho V g \\
mg &= \rho(dA)g \\
mg &= \rho d A g
\end{aligned}
$$

where A is the area of the block. Now, if the block is pushed a distance x down from its floating equilibrium position, the net force on the block is

$$
\begin{aligned}
F_{net} &= F_g - F_A \\
&= mg - \rho(d+x)Ag \\
&= mg - \rho d A g - \rho x A g \\
&= mg - mg - \rho x A g \\
&= -\rho x A g \\
&= -x A \rho g
\end{aligned}
$$

Note that this "restoring" force will be the additional weight of the fluid that is displaced when the block is moved the distance x. This force will be directed in the opposite direction to the displacement x. Using Newton's second law of motion:

$$\begin{aligned} F_{net} &= -xA\rho g \\ m\ddot{x} &= -(A\rho g)x \\ \ddot{x} &= -(\frac{A\rho g}{m})x \end{aligned}$$

Since the acceleration $\ddot{x}$ equals minus times a constant times x, the resulting motion is simple harmonic, i.e. sinusoidal.

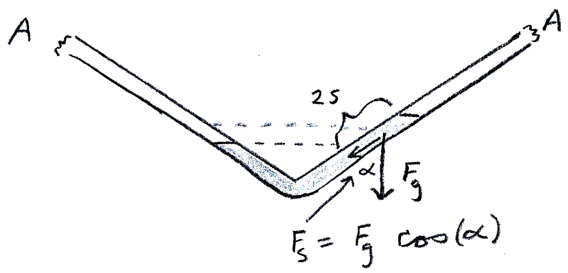
b. The constant in the above equation equals ω_0^2:

$$\begin{aligned} \omega_0^2 &= \frac{A\rho g}{m} \\ \omega_0 &= \sqrt{\frac{A\rho g}{m}} \end{aligned}$$

and hence the period T is

$$T = 2\pi/\omega_0 = 2\pi\sqrt{\frac{m}{A\rho g}} \tag{26}$$

Aufgabe 18



a. When the mercury moves a distance s from equilibrium, the net force on the mercury is the amount of weight in a length $2s$ and is directed downward:

$$\begin{aligned} \vec{F}_{net} &= \vec{F}_g \\ &= (2sA\rho)\vec{g} \end{aligned}$$

The component of the force along the direction of the glass tube equals $F_g\,cos(\alpha)$ and is directed opposite to s:

$$\begin{aligned} F_s &= -2sA\rho g\,cos(\alpha) \\ m\ddot{s} &= -(2A\rho g\,cos(\alpha))s \\ \ddot{s} &= -(\frac{2A\rho g\,cos(\alpha)}{m})s \end{aligned}$$

b. Since $\ddot{s}$ equals minus times a constant times s, the motion is sinusoidal with angular frequency ω_0 equal to

$$\begin{aligned} \omega_0^2 &= \frac{2A\rho g\,cos(\alpha)}{m} \\ \omega_0 &= \sqrt{\frac{2A\rho g\,cos(\alpha)}{m}} \\ \omega_0 &= \sqrt{\frac{2(\pi(0,005)^2)13530(9,8)\,cos(30°)}{0,5}} \\ \omega_0 &\approx 6,01\,\frac{rad}{s} \end{aligned}$$

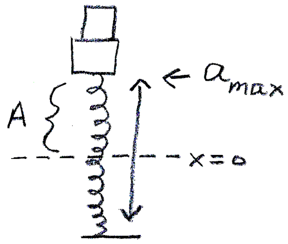
The frequency and period are

$$\begin{aligned} f_0 &= \frac{\omega_0}{2\pi} \approx 0,956\,Hz \\ T_0 &= \frac{1}{f_0} \approx 1,05\,s \end{aligned}$$

Aufgabe 19

a. If the two masses stay together, the angular frequency of the oscillation is

$$\omega_0 = \sqrt{c/m} = \sqrt{\frac{5\,N/m}{0,15\,kg}} \approx 5,77\,\frac{1}{s} \tag{27}$$



The maximum acceleration of the blocks equals $A\omega_0^2$, where A is the amplitude of oscillation. For the top block to stay on the lower block, this acceleration must be less than g:

$$\begin{aligned} a_{max} &= A\omega_0^2 = g \\ A &\leq \frac{g}{\omega_0^2} \\ A_{max} &= \frac{9,8\, m/s^2}{5,77^2\, \frac{1}{s^2}} \\ A_{max} &\approx 0,294\, m = 29,4\, cm \end{aligned}$$

Aufgabe 20

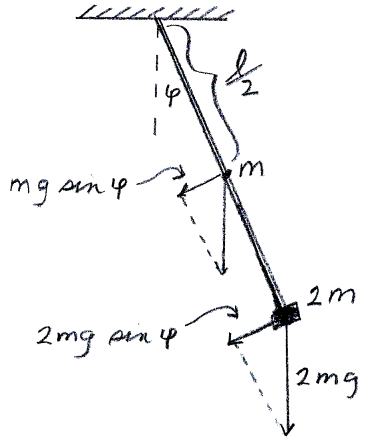
a.

$$\begin{aligned} J &= 3(2)^2 + 8(5)^2 \\ J &= 212\, kg - m^2 \end{aligned}$$

b.

$$\begin{aligned} J &= 4(3)^2 + 3(1)^2 + 8(2)^2 \\ J &= 71\, kg - m^2 \end{aligned}$$

Aufgabe 21



a.

$$
\begin{aligned}
J\ddot{\phi} &= M_{tot} \\
(\frac{ml^2}{3} + 2ml^2)\ddot{\phi} &= M_{tot} \\
(\frac{ml^2}{3} + 2ml^2)\ddot{\phi} &= -\frac{l}{2}mg\, sin(\phi) - 2mg\, sin(\phi) \\
\frac{7}{3}ml^2\,\ddot{\phi} &= -\frac{5}{2}mgl\, sin(\phi)
\end{aligned}
$$

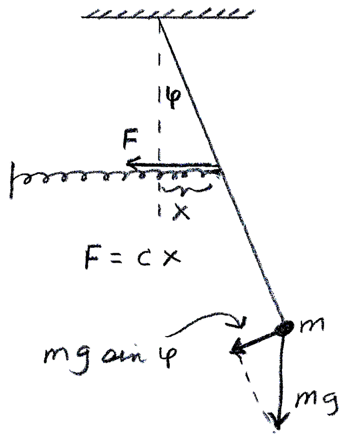
If $\phi << 1$ then $sin(\phi) \approx \phi$:

$$
\begin{aligned}
\frac{7}{3}l\,\ddot{\phi} &\approx -\frac{5}{2}g\phi \\
\ddot{\phi} &\approx -\frac{15g}{14l}\phi
\end{aligned}
$$

The angular frequency squared is the constant in front of ϕ on the left:

$$
\begin{aligned}
\omega^2 &= \frac{15g}{14l} \\
\omega &= \sqrt{\frac{15g}{14l}} \\
f &= \frac{1}{2\pi}\sqrt{\frac{15g}{14l}}
\end{aligned}
$$

Aufgabe 22



a.

$$
\begin{aligned}
J\ddot{\phi} &= M_{tot} \\
&= M_g + M_{feder} \\
ml_1^2\ddot{\phi} &= -mgl_1\, sin(\phi) - l_2F \\
ml_1^2\ddot{\phi} &= -mgl_1\, sin(\phi) - l_2cx
\end{aligned}
$$

If $\phi << 1$, $sin(\phi) \approx \phi$ and $x \approx l_2\phi$:

$$
\begin{aligned}
ml_1^2\ddot{\phi} &\approx -mgl_1\,\phi - l_2^2c\,\phi \\
\ddot{\phi} &\approx -(\frac{mgl_1 + cl_2^2}{ml_1^2})\phi
\end{aligned}
$$

The expression in parenthesis is the angular frequency squared:

$$
\begin{aligned}
\omega^2 &\approx \frac{mgl_1 + cl_2^2}{ml_1^2} \\
&\approx \frac{1(9,8)(0,3) + 100(0,15)^2}{1(0,3)^3} \\
\omega &\approx 7,594\,\frac{rad}{s}
\end{aligned}
$$

The frequency f is therefore,

$$f = \frac{\omega}{2\pi} = \frac{7,594}{2\pi} \approx 1,209\,Hz \tag{28}$$