Physik Übung Lösungen
Kinematik von Punktmassen

Aufgabe 1

a. Since the velocity is constant, the speed is just the distance divided by time:

$$v = \frac{s}{t} \to t = \frac{s}{v} = \frac{11m}{\frac{120m}{3{,}6s}} \approx 0{,}333s \tag{1}$$

Aufgabe 2

a. Der Basketballspieler benötigt die gleiche zeit vom Boden bis zum höchsten Punkt wie vom höchsten Punkt wieder zum Boden. Wir haben also einen freien Fall aus 1,5m Höhe.

$$\begin{aligned} y &= \frac{1}{2} * gt^2 \\ t &= \sqrt{\frac{2y}{g}} = \sqrt{\frac{2 * 1{,}5m}{9{,}81\frac{m}{s^2}}} = 0{,}553\,s \end{aligned}$$

The total time is twice the time to fall $1{,}5\,m$:

$$t_{ges} = 2t \approx 1{,}11\,s \tag{2}$$

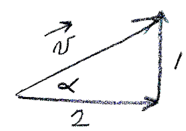
Aufgabe 3

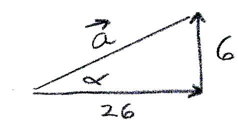
a. Die Geschwindigkeit ist die erste Zeitableitung des Ortes. Since the motion is in two dimensions, we need to differentiate both $x(t)$ and $y(t)$:

$$\begin{aligned} x(t) &= 2t^3 + t^2 + 2t + 5 \\ y(t) &= 3t^2 - t + 7 \\ v_x(t) &= 6t^2 + 2t + 2 \to v_x(0) = 2 \\ v_y(t) &= 6t - 1 \to v_y(0) = -1 \end{aligned}$$

The magnitude and direction of $\vec{v}(0)$ is

$$\begin{aligned} |v| &= \sqrt{2^2 + 1^2} \approx 2{,}24\frac{m}{s} \\ \alpha &= \arctan(\frac{1}{2}) \approx 26{,}6^\circ \end{aligned}$$





b. Die Beschleunigung ist die erst Zeitableitung der Geschwindigkeit. Since the motion is in two dimensions, we need to differentiate both $v_x(t)$ and $v_y(t)$:

$$\begin{aligned} a_x(t) &= 12t + 2 \rightarrow a_x(2) = 26 \\ a_y(t) &= 6 \rightarrow a_y(2) = 6 \\ |a| &= \sqrt{26^2 + 6^2} \approx 26,7\frac{m}{s^2} \\ \alpha &= \arctan(\frac{6}{26}) \approx 13^\circ \end{aligned}$$

Aufgabe 4

a. Man benutzt die Formel für konstante Beschleunigung:

$$\begin{aligned} a &= \frac{\Delta v}{\Delta t} \\ \Delta t &= \frac{\Delta v}{a} = \frac{\frac{50m}{3,6s} - \frac{80m}{3,6s}}{-6\frac{m}{s^2}} \approx 1,39s \end{aligned}$$

b. Der Bremsweg lässt sich folgendermaen berechnen:

$$\begin{aligned} v^2 &= v_0^2 + 2ax \\ x &= \frac{v^2 - v_0^2}{2a} = \frac{(\frac{50m}{3,6s})^2 - (\frac{80m}{3,6s})^2}{2 * (-6\frac{m}{s^2})} \approx 25,1\,m \end{aligned}$$

Aufgabe 5

a. Um von der Beschleunigung auf die Geschwindigkeit zu kommen muss man integrieren.

$$\begin{aligned} v(t) &= \int_0^t a(t)dt = \int_0^t 4tdt = 2t^2 \\ v(3) &= 2 * 3^2 = 18\frac{m}{s} \end{aligned}$$

b. Durch erneutes Integrieren kommt man auf die Positionsfunktion.

$$\begin{aligned} x(t) &= \int_0^t v(t)dt = \frac{2}{3}t^3 \\ x(3) &= \frac{2}{3}3^3 = 18\,m \end{aligned}$$

c. Man muss zuerst die Zeit berechnen zu der das Auto die Geschwindigkeit $8\frac{m}{s}$ hat und kann diese Zeit dann in die Positionsfunktion einsetzen.

$$\begin{aligned} v &= 2t^2 \\ t &= \sqrt{\frac{v}{2}} = \sqrt{\frac{8}{2}} = 2 \\ x(2) &= \frac{2}{3}2^3 = \frac{16}{3}m \end{aligned}$$

Aufgabe 6

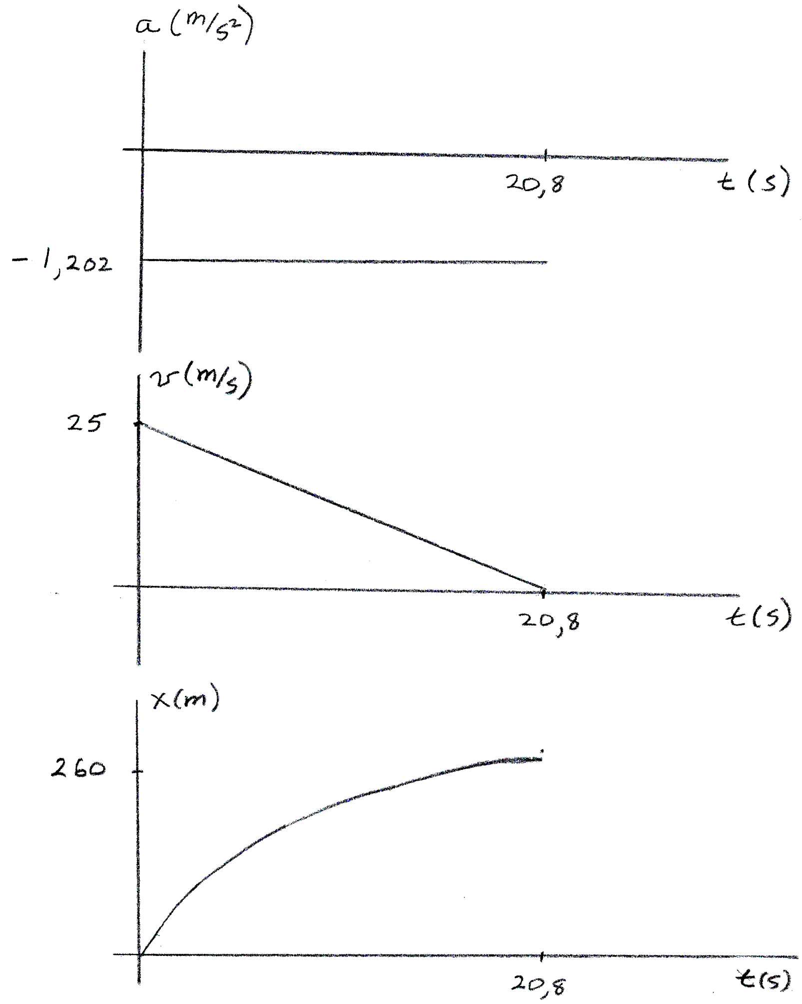
a. Formel für konstante Beschleunigung mit $v = 0$. The initial speed is $v_0 = 90/3,6 = 25\,m/s$, and the final velocity v is zero.

$$\begin{aligned} v^2 &= v_0^2 + 2ax \\ a &= \frac{0^2 - v_0^2}{2x} = \frac{-(\frac{90m}{3,6s})^2}{2 * 260m} \approx -1,202\frac{m}{s^2} \end{aligned}$$

b.

$$t = \frac{v - v_0}{a} = \frac{-\frac{90m}{3,6s}}{-1,202\frac{m}{s^2}} \approx 20,8\,s \tag{3}$$

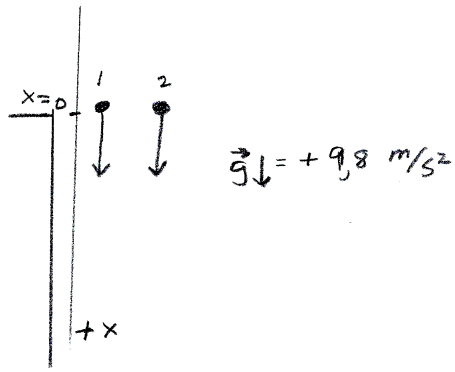
c. $a(t) = -1,202\,m/s^2$, $v(t) = 25 - 1,202t$, and $x(t) = -0,601t^2 + 25t$. Die Graphen von a(t), v(t) und x(t) sehen folgendermaen aus:



a (m/s²)
20,8
t (s)
- 1,202
v (m/s)
25
20,8
t (s)
x (m)
260
20,8
t (s)

Aufgabe 7

Let $x_1(t)$ be the position of Stein 1, and $x_2(t)$ be the position of Stein 2.



a. $x_1 = \frac{1}{2}gt^2$

b. Der zweite Stein hat eine Sekunde weniger Zeit am Boden der Klippe anzukommen also muss man t durch (t-1) ersetzen.

$$\begin{aligned}
x_2 &= \frac{1}{2}g(t-1)^2 + v_0(t-1) \\
x_1 &= x_2 \\
\frac{1}{2}gt^2 &= \frac{1}{2}g(t-1)^2 + v_0(t-1) \\
0 &= -2t + 1 + \frac{2v_0 t}{g} - \frac{2v_0}{g} \\
-1 + \frac{2v_0}{g} &= t(-2 + \frac{2v_0}{g}) \\
t &= \frac{-1 + \frac{2v_0}{g}}{-2 + \frac{2v_0}{g}} \approx 1,48\,s \\
h &= \frac{1}{2}gt^2 \approx 10,7m
\end{aligned}$$

Aufgabe 8

a. Man muss den Zeitpunkt herausfinden, zu dem beide Fahrzeuge die gleiche Strecke zurückgelegt haben.

$$\begin{aligned} x_1 &= \frac{1}{2}at^2 \\ x_2 &= v_0 t \\ x_1 &= x_2 \\ \frac{1}{2}at^2 &= v_0 t \\ t &= \frac{2v_0}{a} \approx 29,6\,s \end{aligned}$$

Aufgabe 9

a. Man setzt wieder die Strecken, die die Fahrzeuge zurücklegen gleich und erhält mithilfe der Mitternachtsformel die Zeit.

$$\begin{aligned} x_1 &= \frac{1}{2}at^2 \\ x_2 &= = v_0 t + x_0 \\ \frac{1}{2}at^2 &= v_0 t + x_0 \\ 0 &= \frac{1}{2}at^2 - v_0 t - x_0 \end{aligned}$$

Mithelfe der Mitternachtsformel de Zeit: $\alpha = \frac{1}{2}a \quad \beta = -v_0 \quad \gamma = -x_0$

$$t_1 = \frac{-\beta + \sqrt{\beta^2 - 4\alpha\gamma}}{2\alpha} = 23,7m \tag{4}$$

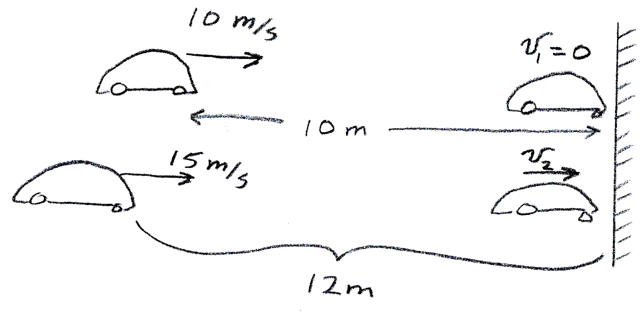
b. Man setzt die Zeit in die Positionsfunktion ein und erhält die Strecke.

$$x_1 = \frac{1}{2}at^2 = \frac{1}{2}1,2\frac{m}{s^2}(23,7s)^2 \approx 337m \tag{5}$$

Aufgabe 10

a. Zuerst muss man die Beschleunigung der beiden Autos ausrechnen. Anschließend kann man mit der gleichen Formel die Aufprallgeschwindigkeit berechnen.

$$\begin{aligned} v_1^2 &= v_0^2 + 2ax \\ 0^2 &= v_0^2 + 2ax \\ a &= \frac{-v_0^2}{2x} = \frac{-(10\frac{m}{s})^2}{2 * 10m} = -5\frac{m}{s^2} \end{aligned}$$



Der gleichen Formel für auto 2 mit $a = -5\,m/s^2$:

$$v_2 = \sqrt{v_0^2 + 2ax} = \sqrt{(15\frac{m}{s})^2 + 2(-5\frac{m}{s^2})12m} \approx 10,25\frac{m}{s} \tag{6}$$

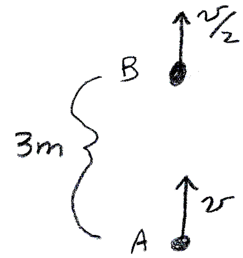
Aufgabe 11

a. Man setzt die Näherung des TÜV mit der realen Formel gleich und kann somit auf die Reaktionszeit schließen.

$$\begin{aligned} x_{naeherung} &= \frac{v}{10} * 3m = \frac{3}{10} * v[m] \\ x_{real} &= \frac{v}{3,6}\frac{m}{s} * t = \frac{1}{3,6} * t * v[m] \\ x_{real} &= x_{naeherung} \\ t &= \frac{\frac{3}{10} * v[m]}{\frac{1}{3,6} * v[\frac{m}{s}]} \\ t &= 1,08[s] \end{aligned}$$

b. Man muss den genäherten Bremsweg berechnen und diesen in die zeitunabhängige Formel für konstante Beschleunigung einsetzen. Dies geht am einfachsten mithilfe eines Zahlenwertes, z.B. $36\frac{km}{h} = 10\,m/s$.

$$\begin{aligned} x &= (\frac{36}{10})^2 \approx 12,96\,m \\ v^2 &= v_0^2 + 2ax \\ a &= \frac{(0^2 - v_0)^2}{2x} = \frac{(-10\frac{m}{s})^2}{2 * 12,96m} \approx -3,86\frac{m}{s^2} \end{aligned}$$



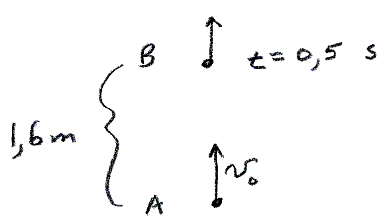
Aufgabe 12

a. $v = v_0/2$. Formel für constante Beschleunigung:

$$\begin{aligned} v^2 &= v_0^2 + 2as \\ (\frac{v_0}{2})^2 &= v_0^2 - 2(9,8)3 \\ \frac{3}{4}v_0^2 &= 2(9,8)3 \\ v_0 &= \sqrt{8(9,8)} \approx 8,85\,s \end{aligned}$$

Aufgabe 13

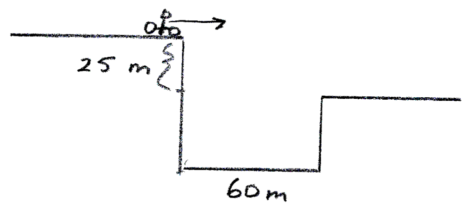
As the ball travels upward, it has a constant acceleration of g downward.



a.

$$\begin{aligned} x &= \frac{1}{2}gt^2 + v_0 t \\ 1.6 &= \frac{1}{2}(-9,8)(0,5)^2 + v_0(0,5) \\ v_0 &\approx 5,65\frac{m}{s} \end{aligned}$$

Aufgabe 14



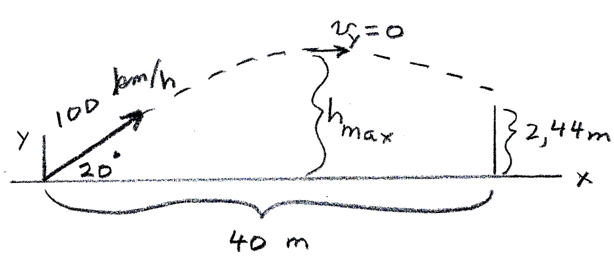
a. We first calculate the time it takes for the Motorrad to fall 25 meters:

$$\begin{aligned} \Delta y &= \frac{1}{2} g t^2 \\ t &= \sqrt{\frac{2 * \Delta y}{g}} = \sqrt{\frac{2 * 25m}{9,81 \frac{m}{s^2}}} = 2,26\, s \end{aligned}$$

b. The Motorrad will travel horizontally a distance $60\,m$ for a time of $2,26\,s$:

$$\begin{aligned} \Delta x &= v_0 * t \\ v_0 &= \frac{\Delta x}{t} = \frac{60\,m}{2,26\,s} = 26,5 \frac{m}{s} \end{aligned}$$

Aufgabe 15
The best approach is to analyze the horizontal, $x(t)$, and the vertical, $y(t)$, directions independently.



a. First, for the horizontal direction the velocity v_x equals $v_x = v\,cos(20°)$. The horizontal acceleration is zero: $a_x = 0$.

$$\begin{aligned} \Delta x &= v_{0x} * t \\ t &= \frac{\Delta x}{v_{0x}} = \frac{40\,m}{\frac{100\,m}{3{,}6s} * \cos 20°} = 1{,}53\,s \end{aligned}$$

b. For the vertical direction, the initial $v_y = v_{0y}$ equals $v_{0y} = v\,sin(20°)$. The vertical acceleration is $-g$, $a_y = -g$.

$$\begin{aligned} y(t) &= v_{0y}t - \frac{1}{2}gt^2 \\ y(1{,}53s) &= \frac{100m}{3{,}6s} * sin20° * 1{,}53s - \frac{1}{2} * 9{,}81\frac{m}{s^2} * (1{,}53s)^2 = 3{,}05\,m \end{aligned}$$

c. Der Ball hat die maximale Höhe bei $v_y = 0$

$$\begin{aligned} v_y(t) &= v_{0y} - gt \\ 0 &= v_{0y} - gt \\ t &= \frac{v_{0y}}{g} = \frac{\frac{100m}{3{,}6s} * sin20°}{9{,}81\frac{m}{s^2}} = 0{,}968s \end{aligned}$$

d.

$$\begin{aligned} y(t) &= v_{0y}t - \frac{1}{2}gt^2 \\ y(0{,}968s) &= \frac{100m}{3{,}6s} * sin20° * 0{,}968s - \frac{1}{2} * 9{,}81\frac{m}{s^2} * (0{,}968s)^2 = 4{,}6\,m \end{aligned}$$

e. Der Fuballspieler schiet ber das Tor hinaus.

Aufgabe 16

As with the previous problem, the best approach is to analyze the horizontal, $x(t)$, and the vertical, $y(t)$, directions independently.

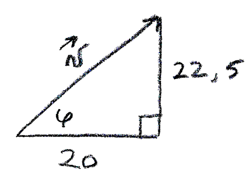
a. First, the horizontal direction where v_x is constant.

$$\begin{aligned} \Delta x &= v_{0x} * t \\ v_{0x} &= \frac{\Delta x}{t} = \frac{100m}{5s} = 20\frac{m}{s} \end{aligned}$$

Now, for the vertical direction where the acceleration is $-g$.

$$
\begin{aligned}
y(t) &= y_0 + v_{0y}t - \frac{1}{2}gt^2 \\
y(5s) &= 0 \\
0 &= 10 + v_{0y}(5) - \frac{1}{2}(9,8)(5)^2 \\
v_{0y} &= \frac{-10m + \frac{1}{2} * 9,81\frac{m}{s^2} * (5s)^2}{5s} \approx 22,5\frac{m}{s}
\end{aligned}
$$

We know the two components of the initial velocity, so we can find the magnitude and direction of $\vec{v}_0 = v_{0x}\hat{i} + v_{0y}\hat{j}$:

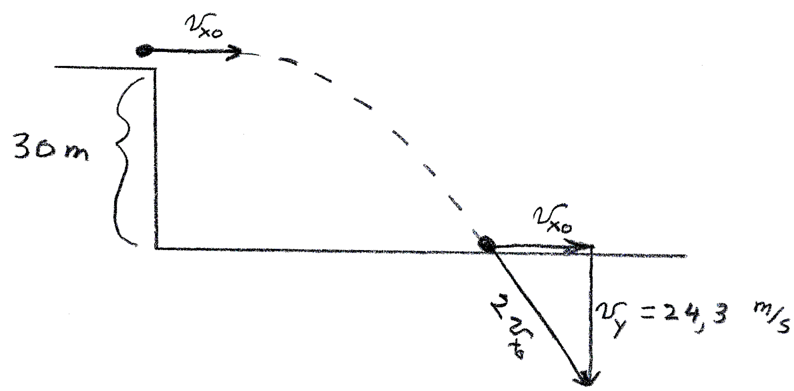


$$
\begin{aligned}
|v| &= \sqrt{v_{0x}^2 + v_{0y}^2} \\
&= = \sqrt{(20\frac{m}{s})^2 + (22,5\frac{m}{s})^2} \approx 30,1\frac{m}{s} \\
\varphi &= \arctan\frac{v_{y0}}{v_{x0}} \\
&= \arctan\frac{22,5\frac{m}{s}}{20\frac{m}{s}} \approx 48,4^\circ
\end{aligned}
$$

Aufgabe 17

a. Since we know how tall the cliff is, and that $v_{y0} = 0$, we can find the vertical component v_y of the ball when it hits the ground.

$$
\begin{aligned}
v_y^2 &= v_{y0}^2 + 2g\Delta y \\
v_y^2 &= 2g\Delta y \\
v_y &= \sqrt{2g\Delta y} \\
&= \sqrt{2 * 9,81\frac{m}{s^2} * 30m} \approx 24,3\frac{m}{s}
\end{aligned}
$$



We also know that $|\vec{v}| = 2v_x$ when the ball hits the ground:

$$\begin{aligned} |v| &= \sqrt{v_x^2 + v_y^2} = 2 * v_x \\ 4v_x^2 &= v_x^2 + v_y^2 \\ v_x &= \sqrt{\frac{v_y^2}{3}} = \sqrt{\frac{(24,3\frac{m}{s})^2}{3}} \approx 14\frac{m}{s} \end{aligned}$$

Aufgabe 18

a. For circular motion, $v = \omega R$:

$$\begin{aligned} v &= \omega * R \\ &= \frac{2\pi}{T} * R = \frac{2\pi}{24 * 3600s} * 6371000m \approx 463\frac{m}{s} \end{aligned}$$

For circular motion, the acceleration towards the center is $\omega^2 R = v^2/R$:

$$\begin{aligned} a_z &= \omega^2 * R \\ &= (\frac{2\pi}{T})^2 * R = (\frac{2\pi}{24 * 3600s})^2 * 6371000m \approx 0,0337\frac{m}{s^2} \end{aligned}$$

b.

$$\begin{aligned} v &= \omega * R \cos\varphi \\ &= \frac{2\pi}{T} * Rcos\varphi = \frac{2\pi}{24 * 3600s} * 6371000m * cos(48,5°) \approx 307\frac{m}{s} \\ a_z &= \omega^2 * Rcos\varphi \\ &= (\frac{2\pi}{T})^2 * Rcos\varphi = (\frac{2\pi}{24 * 3600s})^2 * 6371000m * cos(48,5°) \approx 0,0223\frac{m}{s^2} \end{aligned}$$

Aufgabe 19

a. Since the motion is in a circle of radius R with a constant speed v, the acceleration is toward the center with a value of a_z:

$$a_z = \frac{v^2}{R} = \frac{(\frac{1800m}{3,6s})^2}{3500m} \approx 71,43\frac{m}{s^2} = 7,281g \tag{7}$$

Aufgabe 20

a. The motion has a constant angular acceleration of ψ, and starts with $\omega_0 = 0$:

$$\begin{aligned}\omega &= \psi * t \\ &= 1\frac{rad}{s^2} * 4s = 4\frac{rad}{s}\end{aligned}$$

b. The acceleration has two components, a_z towards the center, and a_r tangential to the circle. $a^2 = |\vec{a}|^2 = a_z^2 + a_r^2$:

$$\begin{aligned}a &= \sqrt{(a_z)^2 + (a_r)^2} \\ a_z &= \omega^2 R = (4\frac{rad}{s})^2(2m) = 32\,m/s \\ a_r &= \psi R = (1\,rad/s^2)(2m) = 2\,m/s \\ a &= \sqrt{32^2 + 2^2} \approx 32,06\,m/s\end{aligned}$$

c. After $6\,s$, $\omega = 6\,rad/s$:

$$\begin{aligned}v &= \omega * R \\ &= (1\,rad/s^2)(6s) * 2m = 12\frac{m}{s}\end{aligned}$$

d.

$$\varphi(t) = \frac{1}{2}\psi t^2$$

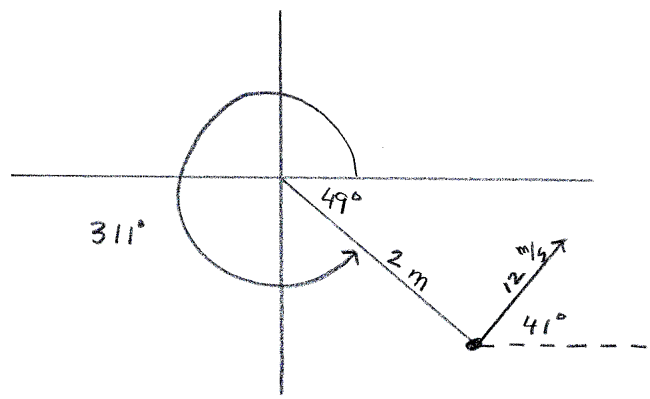
$$\varphi(6s) = \frac{1}{2} * 1\frac{rad}{s^2} * (6s)^2 = 18\,rad$$

$18\,rad$ ist $\frac{18}{2\pi} = 2,865\,rev.$

$$\varphi(6s) = 0,865 * 360° \approx 311° \tag{8}$$

$$
\begin{aligned}
x &= (2m) * cos(311^\circ) \approx 1,32m \\
y &= (2m) * sin(311^\circ) \approx -1,5m
\end{aligned}
$$

$\vec{v}$ is perpendicular to R, and at an angle of $311 - 270 = 41^\circ$.



$$
\begin{aligned}
v_x &= 12\frac{m}{s} * sin(41,3^\circ) \approx 7,92\frac{m}{s} \\
v_y &= 12\frac{m}{s} * cos(41,3^\circ) \approx 9,01\frac{m}{s}
\end{aligned}
$$

Aufgabe 21

a. Since $v(t)$ is not constant, we need to integrate to find x.

$$
\begin{aligned}
\frac{dx}{dt} &= v = v_0 e^{-\alpha t} \\
dx &= v_0 e^{-\alpha t} * dt \\
\int_0^x dx' &= \int_0^t v_0 e^{-\alpha t'} * dt' \\
x &= \frac{v_0}{\alpha}(1 - e^{-\alpha t}) \\
\rightarrow x_{max} &= \frac{v_0}{\alpha}
\end{aligned}
$$

Aufgabe 22

a. The acceleration is the first derivative of $v(t)$:

$$\begin{aligned} a(t) &= \frac{dv}{dt} \\ &= 11 * e^{-t} \\ a(10s) &= 11 * e^{-10} \approx 5 * 10^{-4} \frac{m}{s^2} \end{aligned}$$

b.

$$v_{max} = 11 \frac{m}{s} \tag{9}$$

c. Since v is not constant, we need to integrate to find $x(t)$:

$$\begin{aligned} x(t) &= \int_0^t 11 - 11e^{-t} \\ x(t) &= 11t - 11 * (1 - e^{-t}) \end{aligned}$$

d. $x(10s) = 99m$