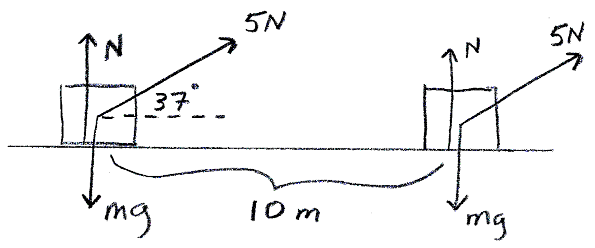
Lösungen für Arbeit, Leistung und Energie

Aufgabe 1



a. For a constant force acting through a straight distance, $W = \vec{F} \cdot \vec{s}$:

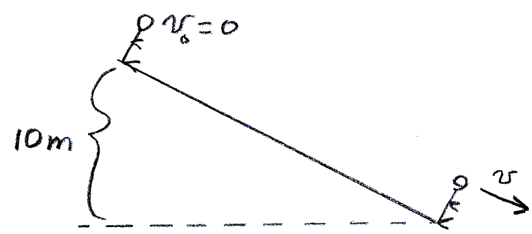
$$W = (5\,cos(37^\circ\,N))(10\,m) = 40\,J \tag{1}$$

b. Since the weight is perpendicular to the path, $W_{mg} = 0$

c. Since the normal force is perpendicular to the pathe, $W_N = 0$

d. $W_{net} = 40 + 0 + 0 = 40\,J$

Aufgabe 2



a. Using the energy conservation equation:

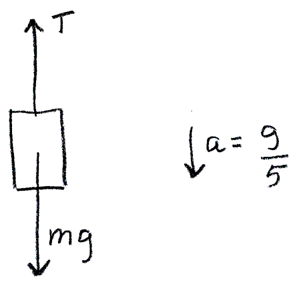
$$
\begin{aligned}
K^i + E^i_{pot} + E^i_{sp} + W_{ext} &= K^f + E^f_{pot} + E^f_{sp} \\
0 + mg(10\,m) + 0 + 0 &= \frac{m}{2}v^2 + 0 + 0 \\
mg(10\,m) &= \frac{m}{2}v^2 \\
v^2 &= 2g(10) \\
v &= \sqrt{2(9.8)10} \approx 14\,m/s
\end{aligned}
$$

Aufgabe 3

a. Using the work-energy theorum:

$$
\begin{aligned}
W_{net} &= \Delta(K.E.) \\
W_F + W_{GR} &= \frac{m}{2}v^2 - 0 \\
(3\,N)(5\,m) - (1\,N)(5\,m) &= \frac{0,002\,kg}{2}v^2 \\
v &= \sqrt{20/(0,002)} = 100\,m/s
\end{aligned}
$$

Aufgabe 4



a.

b. The work done by gravity is just the weight times the distance d: $W_{grav} = +mgd$

c. First we find the tension T using Newton's second law:

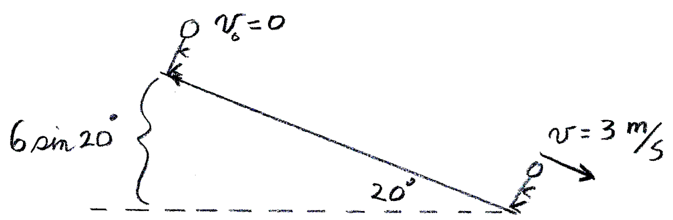
$$\begin{aligned} mg - T &= ma \\ mg - T &= m\frac{g}{5} \\ T &= \frac{4}{5}mg \end{aligned}$$

Now, we find the work done by the tension T.

$$W_T = (-\frac{4}{5}mg)(d) = -\frac{4}{5}mgd \qquad (2)$$

The work is negative since the tension acts in the opposite direction as the distance moved.

Aufgabe 5



a. The work done by gravity is just mg times the change in height:

$$\begin{aligned} W_{grav} &= mg(6\,sin(20°)) \\ &= 40(9,8)(6)sin(20°) \approx 804,4\,J \end{aligned}$$

b. One can use the energy conservation equation to solve for the work done by kinetic friction, W_{GR}:

$$\begin{aligned} K^i + E^i_{pot} + E^i_{sp} + W_{ext} &= K^f + E^f_{pot} + E^f_{sp} \\ 0 + mg(6\,sin(20°)) + 0 + W_N + W_{GR} &= \frac{m}{2}v^2 + 0 + 0 \\ 804,4\,J + W_{GR} &= \frac{40}{2}(3)^2 = 180\,J \\ W_{GR} &= 180\,J - 804,4\,J \\ W_{GR} &\approx -624,4\,J \end{aligned}$$

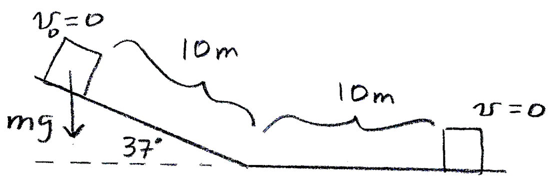
Since the work done by the normal force, W_N, equals zero.

Aufgabe 6.

a. We can use the work-energy theorum. The work done by gravity and the normal force is zero.

$$\begin{aligned} W_{net} &= \Delta(K.E.) \\ W_{GR} + W_{grav} + W_N &= \frac{m}{2}v^2 - \frac{m}{2}v_0^2 \\ -\mu_{GR}mgs + 0 + 0 = 0 - \frac{m}{2}v_0^2 \\ s &= \frac{v_0^2}{2\mu_{GR}\,g} \\ s &= \frac{2^2}{2(0,1)(9,8)} \approx 2,04\,m \end{aligned}$$

Aufgabe 7



a. We can use the energy conservation equation to find the work done by friction, W_{GR}. The K.E. at the start and end are zero,

$$\begin{aligned} K^i + E^i_{pot} + E^i_{sp} + W_{ext} &= K^f + E^f_{pot} + E^f_{sp} \\ 0 + mg(10\,sin(37°)) + 0 + W_{GR} &= 0 + 0 + 0 \\ W_{GR} &= -mg(10\,sin(37°)) \approx -6mg \end{aligned}$$

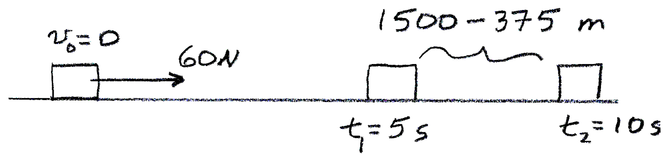
We can also find the work done by friction from the frictional force and distance traveled:

$$\begin{aligned} W_{GR} &= -mg\,cos(37°)\,\mu_{GR}\,10 - mg\mu\,10 \\ &= -mg(10\,cos(37°) + 10)\mu_{GR} \end{aligned}$$

By equating the two expressions, we can solve for μ_{GR}:

$$\begin{aligned}-mg(10\,sin(37°)) &= -mg(10\,cos(37°)+10)\mu_{GR}\\ \mu_{GR} &= \frac{sin(37°)}{cos(37°)+1} \approx 0,333\end{aligned}$$

Aufgabe 8



a. We first find the acceleration of the object:

$$a = \frac{F}{m} = \frac{60N}{2kg} = 30\frac{m}{s^2} \tag{3}$$

Then, we determine the distance the object moved from $5\,s$ to $10\,s$:

$$\begin{aligned}s_1 &= \frac{a}{2}t_1^2 = \frac{30}{2}(5)^2 = 375\,m\\ s_2 &= \frac{a}{2}t_2^2 = \frac{30}{2}(10)^2 = 1500\,m\\ \Delta s &= 1500 - 375 = 1125\,m\end{aligned}$$

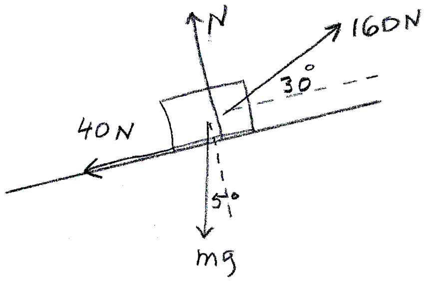
Finally, the work is just force times distance since F is constant:

$$W_F = F(\Delta s) = 60N(1125\,m) = 67500\,J \tag{4}$$

Aufgabe 9

a. The distance traveled in $125\,s$ at a constant velocity is

$$s = vt = (1,1\,m/s)(125\,s) = 137,5\,m \tag{5}$$



The work done by the constant force F is therefore:

$$W_F = (160\,cos(30°))(137,5) = 19052\,J \tag{6}$$

The power is work divided by time:

$$P = \frac{W_F}{t} = \frac{19052\,J}{125\,s} \approx 152\,W \tag{7}$$

b. The change in vertical distance, h, during the $125\,s$ is:

$$h = 137,5\,m(sin(5°)) \approx 11,98\,m \tag{8}$$

c. Since the K.E. does not change, $W_{net} = 0$:

$$\begin{aligned} W_{net} &= 0 \\ W_{grav} + W_F + W_{RR} + W_N &= 0 \\ -mgh + 19052\,J - 40(137,5)\,J + 0 &= 0 \\ m &= \frac{19052 - 40(137,5)}{9,8(11,98)} \approx 115,4\,kg \end{aligned}$$

d. For the forces perpendicular to the street:

$$\begin{aligned} N + 160\,sin(30°) - mg\,cos(5°) &= 0 \\ N &= mg\,cos(5°) - 160\,sin(30°) \\ &= 115,4(9,8)\,cos(5°) - 160\,sin(30°) \\ N &\approx 1046,9\,N \end{aligned}$$

The coefficient of rolling friction, μ_{RR} is therefore:

$$\mu_{RR} = \frac{40}{1046,9} \approx 0,0382 \tag{9}$$

Aufgabe 10

a. The force of the Triebwerke, F_T, is constant, so the work done by F_T is just the force times the distance:

$$W_T = F_T s = (130 \times 10^5\, N)(100\, m) = 1,3 \times 10^7\, J = 13\, MJ \qquad (10)$$

b. The force of the catapult is not constant, so we need to integrate F_K over the distance:

$$\begin{aligned} W_K &= \int_0^{100} F_K\, dx \\ &= (\frac{1000+100}{2} kN)(100\, m) \\ W_K &= 5,5 \times 10^7\, J = 55\, MJ \end{aligned}$$

Aufgabe 11 The final speed of the car in units of m/s is $v = \frac{108}{3,6} = 30\, m/s$

a. Since the power is constant in time, the net work is just power times time:

$$\begin{aligned} W_{net} &= \Delta(K.E.) \\ Pt &= \frac{m_{tot}}{2} v^2 - 0 \\ (50000 \frac{J}{s})(10\, s) &= \frac{m_{tot}}{2} (30\, \frac{m}{s})^2 \\ m_{tot} &\approx 1111\, kg \end{aligned}$$

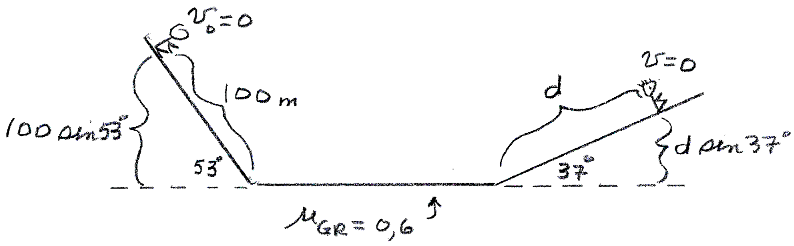
Subtracting the mass of the driver, we have for the mass of the PKW:

$$m_{PKW} = 1111 - 80 \approx 1031\, kg \qquad (11)$$

b. Using the same formula, but with a larger mass:

$$\begin{aligned} Pt_5 &= \frac{m}{2} v^2 - 0 \\ (50000\, \frac{J}{s}) t_5 &= \frac{1031+400}{2} (30\, \frac{m}{s})^2 \\ t_5 &\approx 12,88\, s \end{aligned}$$

Aufgabe 12



a. We can use the energy conservaton equation. The initial and final K.E. are both zero.

$$\begin{aligned}
K^i + E^i_{pot} + E^i_{sp} + W_{ext} &= K^f + E^f_{pot} + E^f_{sp} \\
0 + mg(\,100\,sin(53°) + 0 + W_{GR} &= 0 + mgd\,sin(37°)) + 0 \\
0 + mg(\,100\,sin(53°) + 0 - mg\mu_{GR}(50\,m) &= 0 + mgd\,sin(37°)) + 0 \\
100\,sin(53°) - \mu_{GR}(50\,m) &= d\,sin(37°) \\
d &= \frac{100\,sin(53°) - 0,6(50)}{sin(37°)} \\
d &\approx 83,3\,m
\end{aligned}$$

b. We can use the energy conservation equation, and the initial and final K.E. are both zero.

$$\begin{aligned}
K^i + E^i_{pot} + E^i_{sp} + W_{ext} &= K^f + E^f_{pot} + E^f_{sp} \\
0 + mg(\,100\,sin(53°) + 0 + W_{GR} &= 0 + 0 + 0 \\
mg(100\,sin(53°)) - mg\mu_{GR}\,s &= 0 \\
s &= \frac{100\,sin(53°)}{0,6} \\
s &\approx 133\,m
\end{aligned}$$

Each pass of the parking lot is $50\,m$, so she ends up $133 - 50 - 50 = 33\,m$ from the bottom of the left hill after crossing the parking lot 2 times.

Aufgabe 13

a. To find the speed of the satelite we use Newton's second law and his law for the gravitational **force**, $F_{grav} = \gamma Mm/r^2$:

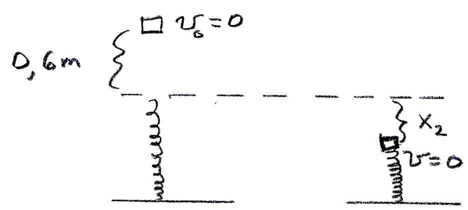
$$\vec{F}_{net} = m\vec{a}$$

$$\begin{aligned}
\frac{\gamma M m}{r^2} &= m\frac{v^2}{r} \\
v &= \sqrt{\frac{\gamma M}{r}} \\
v &= \sqrt{\frac{(6,673 \times 10^{-11})(5,975 \times 10^{24})}{(6370 + 1000) \times 10^3}} \\
v &\approx 7355\,\frac{m}{s}
\end{aligned}$$

b. The gravitational potential energy function, $U_{grav} = \int \frac{\gamma M m}{r^2} dr = -\frac{\gamma M m}{r}$. We can use the energy conservation equation:

$$\begin{aligned}
K^i + E^i_{pot} + E^i_{sp} + W_{ext} &= K^f + E^f_{pot} + E^f_{sp} \\
0 + (-\frac{\gamma M m}{R}) + 0 + W_{ext} &= \frac{m}{2}v^2 + (-\frac{\gamma M m}{r}) + 0 \\
-\frac{\gamma M m}{R} + W_{ext} &= (-\frac{\gamma M m}{r}) + \frac{m}{2}v^2 \\
W_{ext} &= \gamma M m(\frac{1}{R} - \frac{1}{r}) + \frac{m}{2}v^2 \\
&= (6,67 \times 10^{-11})(5,98 \times 10^{24})(200)(\frac{1}{6370} \\
&\quad -\frac{1}{7370}) \times 10^{-3} + \frac{200}{2}7355^2 \\
W_{ext} &\approx 7,14 \times 10^9\, J
\end{aligned}$$

Aufgabe 14



a. We can use the energy conservation equation:

$$\begin{aligned}
K^i + E^i_{pot} + E^i_{sp} + W_{ext} &= K^f + E^f_{pot} + E^f_{sp} \\
0 + mg(0,6) + 0 + 0 &= 0 + mg(-x_2) + \frac{c}{2}x_2^2 \\
mg(0,6 + x_2) - \frac{c}{2}x_2^2 &= 0 \\
10(9,8)(0,6 + x_2) - \frac{1960}{2}x_2^2 &= 0 \\
100x_2^2 - 10x_2 - 6 &\approx 0 \\
x_2 &\approx 0,3\,m
\end{aligned}$$

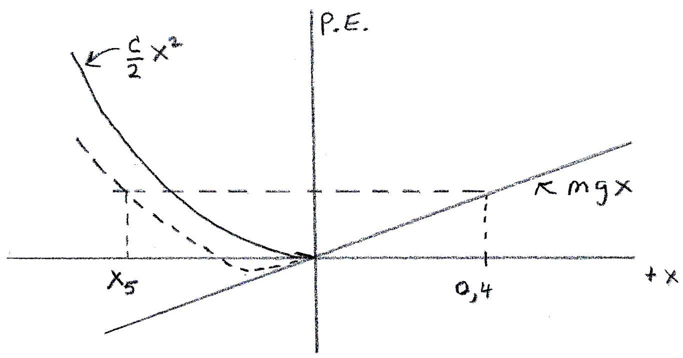
b. We can use the energy conservation equation for this part also:

$$\begin{aligned}
K^i + E^i_{pot} + E^i_{sp} + W_{ext} &= K^f + E^f_{pot} + E^f_{sp} \\
0 + mg(0,6) + 0 + 0 &= \frac{m}{2}v_3^2 + mg(-0,1) + \frac{c}{2}(0,1)^2 \\
mg(0,6 + 0,1) - \frac{c}{2}(0,1)^2 &= \frac{m}{2}v_3^2 \\
v_3^2 &= 2(9,8)(0,7) - \frac{1960}{10}(0,1)^2 \\
v_3 &\approx 3,43\,m/s
\end{aligned}$$

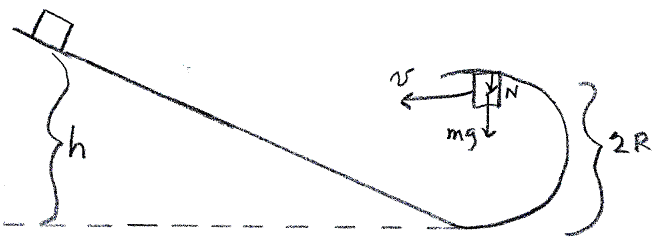
c. The power is just $\vec{F} \cdot \vec{v}$:

$$|P_3| = |F_3 \cdot v_3| = c|x_3|v_3 = 1960(0,1)(3,43) = 672\,W \tag{12}$$

d.



Aufgabe 15



a. When the block is at the top of the loop, the normal force N is such that $N + mg = m\frac{v^2}{R}$, and N must be positive. The minimum value for the speed v will be when $N = 0$:

$$\begin{aligned} mg &= m\frac{v_{min}^2}{R} \\ v_{min}^2 &= Rg \end{aligned}$$

We can use the energy conservation equation to find the relationship between h and v:

$$\begin{aligned} K^i + E_{pot}^i + E_{sp}^i + W_{ext} &= K^f + E_{pot}^f + E_{sp}^f \\ 0 + mgh + 0 + 0 &= \frac{m}{2}v^2 + mg(2R) + 0 \\ mg(h - 2R) &= \frac{m}{2}v^2 \end{aligned}$$

To find h_{min}, substitute v_{min} into the equation:

$$\begin{aligned}
mg(h_{min} - 2R) &= \frac{m}{2}v_{min}^2 \\
mg(h_{min} - 2R) &= \frac{m}{2}Rg \\
h_{min} - 2R &= \frac{R}{2} \\
h_{min} &= \frac{5}{2}R
\end{aligned}$$

Aufgabe 16

a. Since the force is not constant, we need to integrate to find the work done:

$$\begin{aligned}
W_F &= \int F_x\,dx \\
&= \int_0^2 6x\,dx \\
&= 12\,J
\end{aligned}$$

The amount of work W_F equals the change in kinetic energy.

$$\begin{aligned}
W_F &= \Delta(K.E.) \\
12\,J &= \frac{m}{2}v^2 - 0 \\
12\,J &= \frac{10}{2}v^2 \\
v &\approx 1,55\,\frac{m}{s}
\end{aligned}$$

Aufgabe 17

a. The work done by gravity equals the weight, mg, times the change in elevation:

$$\begin{aligned}
W_{grav} &= mg(\Delta h) \\
&= 80kg(9,8\frac{m}{s})(-12000\,sin(10°)\,m) \\
&\approx -1,63 \times 10^6\,J
\end{aligned}$$

The work is negative, since the force of gravity is downward and the motion is upward.

b. Since v is constant, $\Delta(K.E.) = 0$,

$$\begin{aligned} W_{net} &= \Delta(K.E.) = 0 \\ W_{grav} + W_{rider} &= 0 \\ -1,63 \times 10^6\,J + W_{rider} &= 0 \\ W_{rider} &= 1,63 \times 10^6\,J \end{aligned}$$

c.

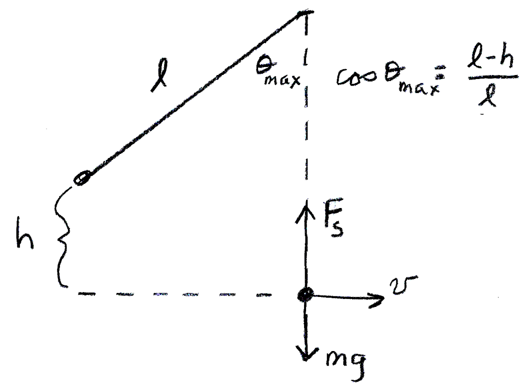
$$P = \frac{W}{t} = \frac{1,63 \times 10^6\,J}{3600\,s} \approx 454\,Watt \tag{13}$$

Aufgabe 18

a. We first find the speed of the father at the bottom of the swing using the energy conservation equation:

$$\begin{aligned} K^i + E^i_{pot} + E^i_{sp} + W_{ext} &= K^f + E^f_{pot} + E^f_{sp} \\ 0 + mgh + 0 + 0 &= \frac{m}{2}v^2 + 0 + 0 \\ mgh &= \frac{m}{2}v^2_{max} \\ v^2_{max} &= 2gh \end{aligned}$$

Since the work done by the string (seil) equals zero. At the bottom of the swing,



the forces satisfy the following equation:

$$\begin{aligned}
F_{S-max} - mg &= ma = m\frac{v_{max}^2}{l} \\
\frac{F_{S-max}}{mg} - 1 &= \frac{v_{max}^2}{gl} \\
\frac{F_{S-max}}{mg} - 1 &= \frac{2gh}{gl} \\
\frac{F_{S-max}}{mg} - 1 &= \frac{2h}{l}
\end{aligned}$$

From the figure, we see that $cos(\theta_{max}) = (l-h)/l = 1 - h/l$, so we have

$$\begin{aligned}
\frac{F_{S-max}}{mg} - 1 &= \frac{2h}{l} \\
\frac{F_{S-max}}{mg} - 1 &= 2(1 - cos(\theta_{max})) \\
\frac{F_{S-max}}{mg} &= 2 - 2\,cos(\theta_{max}) \\
\frac{2000\,N}{150(9,8)\,N} &= 2 - 2\,cos(\theta_{max}) \\
cos(\theta_{max}) &\approx 0,82 \rightarrow \theta_{max} \approx 34,9^\circ
\end{aligned}$$

Aufgabe 19

a. We can use the energy conservation equation. Let W_{LW} be the work done by air resistance. For the way up:

$$\begin{aligned}
K^i + E_{pot}^i + E_{sp}^i + W_{ext} &= K^f + E_{pot}^f + E_{sp}^f \\
\frac{m}{2}v_0^2 + 0 + 0 + W_{LW} &= 0 + mgh_{max} + 0 \\
\frac{m}{2}v_0^2 + 0 + 0 - (xmg)h_{max} &= 0 + mgh_{max} + 0 \\
-mgh_{max} - xmgh_{max} &= -\frac{m}{2}v_0^2 \\
h_{max} &= \frac{v_0^2}{2g(1+x)}
\end{aligned}$$

b. On the way down we have:

$$\begin{aligned}
K^i + E^i_{pot} + E^i_{sp} + W_{ext} &= K^f + E^f_{pot} + E^f_{sp} \\
0 + mgh_{max} + 0 + W_{LW} &= \frac{m}{2}v^2 + 0 + 0 \\
0 + mgh_{max} + 0 - (xmg)h_{max} &= \frac{m}{2}v^2 + 0 + 0 \\
+mgh_{max} - xmgh_{max} &= \frac{m}{2}v^2 \\
v^2 &= 2gh_{max}(1-x)
\end{aligned}$$

From part a) we have $2gh_{max} = v_0^2/(1+x)$, which we can substitute on the right side of the equation above:

$$\begin{aligned}
v^2 &= v_0^2(\frac{1-x}{1+x}) \\
v &= v_0\sqrt{\frac{1-x}{1+x}}
\end{aligned}$$

Aufgabe 20

a. Analyzing the forces at the top of the loop give the following equation:

$$T_T + mg = m\frac{v_T^2}{R} \tag{14}$$

At the bottom of the loop we have

$$T_B - mg = m\frac{v_B^2}{R} \tag{15}$$

Subtracting these two equations yields:

$$T_B - T_T = 2mg + \frac{m}{R}(v_B^2 - v_T^2) \tag{16}$$

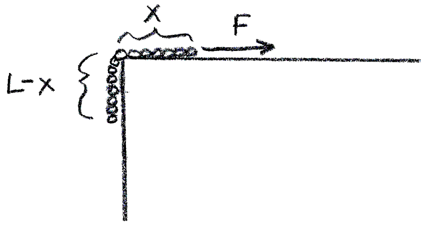
We can use the energy conservation equation. $W_{ext} = 0$ since the string tension force is perpendicular to the path:

$$\begin{aligned}
K^i + E^i_{pot} + E^i_{sp} + W_{ext} &= K^f + E^f_{pot} + E^f_{sp} \\
\frac{m}{2}v_T^2 + mg(2R) + 0 + 0 &= \frac{m}{2}v_B^2 + 0 + 0 \\
m(v_B^2 - v_T^2) &= 4mgR
\end{aligned}$$

Substituting into the first equation gives

$$
\begin{aligned}
T_B - T_T &= 2mg + \frac{m}{R}(v_B^2 - v_T^2) \\
T_B - T_T &= 2mg + \frac{4mgR}{R} \\
T_B - T_T &- 6mg
\end{aligned}
$$

Aufgabe 21



a. The force F_x required when the chain as a length x on the table equals the weight that is hanging over the end:

$$F_x = mg(\frac{L-x}{L}) = mg - mg\frac{x}{L} \tag{17}$$

The force is not constant, so we need to integrate:

$$
\begin{aligned}
W &= \int F_x \, dx \\
&= \int_{L/4}^{L} (mg - mg\frac{x}{L}) \, dx \\
&= (mgx - mg\frac{x^2}{2L})|_{L/4}^{L} \\
W &= \frac{9}{32} mgL
\end{aligned}
$$