

Exercises on Thermodynamics

Exercise 1.1

Tom wants to measure his temperature using a thermocouple as a thermometer. He defines temperature such that T is to be proportional to the thermocouple voltage. He places the thermocouple in ice water (0°), in boiling water (100°), and in his mouth. Below are the voltage readings he obtains:

Substance	Thermocouple Voltage (mV)
ice water	3.1
Tom's mouth	4.7
boiling water	7.1

What is temperature of Tom's mouth using the thermocouple as a thermometer?

If the numerical value of the temperature is such that the thermocouple's voltage is proportional to T , then we have

$$\frac{T}{4.7 - 3.1} = \frac{100}{7.1 - 3.1} \quad (1)$$

Solving for T using this equation gives $T = (1.6/4)100 = 40^\circ$. Since the normal temperature of a person is around 37° , should Tom worry that he has a fever? No! The thermocouple does not have the same properties as an ideal gas. If Tom used a constant volume ideal gas thermometer (or a standard mercury thermometer) he would have gotten $\sim 37^\circ\text{C}$ as the value for his temperature.

Exercise 1.2

The temperatures at the Golden Gate bridge can vary from -20°C to $+40^\circ\text{C}$. If the bridge is made out of steel ($\alpha = 1.1 \times 10^{-5} \text{ }^\circ\text{C}^{-1}$), how much will the length of the bridge change for this temperature change? The length of the Golden Gate Bridge is around 1250 meters.

Using the definition for α in linear expansion, we have

$$\begin{aligned} \Delta L &= L_0 \alpha \Delta T \\ \Delta L &= 1250(1.1 \times 10^{-5})60 \\ \Delta L &\approx 0.825 \text{ meters} \end{aligned}$$

since ΔT is 60°C . Expansion joints are needed to handle the temperature change.

Exercise 1.3

Dave uses an Aluminum ruler to measure a steel rod. The ruler is calibrated to be accurate at 20°C . Dave measures the steel when both the steel rod and the Aluminum ruler are at a temperature of 100°C . At this temperature, the rod measures 60 cm as read off the "hot" Aluminum ruler. What is the "true" length of the steel rod when it is at a temperature of 20°C ?

Let L_0 be the "true" length of the steel rod when it is 20°C . At 100°C , the length of the rod is L :

$$L = L_0(1 + \alpha_{steel}80^\circ) \quad (2)$$

The length L is also the length of 60 centimeter of Aluminum that is heated to 100°C . That is,

$$L = 60(1 + \alpha_{Al}80^\circ) \text{ cm} \quad (3)$$

Equating the two expressions for L , we have

$$L_0(1 + \alpha_{steel}80^\circ) = 60(1 + \alpha_{Al}80^\circ) \text{ cm} \quad (4)$$

Solving for L_0 gives

$$\begin{aligned} L_0 &= 60 \frac{1 + \alpha_{Al}80^\circ}{1 + \alpha_{steel}80^\circ} \\ &= 60 \frac{1 + (2.3 \times 10^{-5})80^\circ}{1 + (1.1 \times 10^{-5})80^\circ} \\ L_0 &= 60.058 \text{ cm} \end{aligned}$$

Exercise 1.4

Your task is to design an immersion heater that will bring a cup of 400 grams of water from a temperature of 20°C up to a temperature of 100°C . You want the heating time to be 2 minutes. What should the wattage be for your immersion heater.

We first need to determine the amount of energy needed to increase the temperature of 400 grams of water by 80°C :

$$\begin{aligned}
Q &= mc\Delta T \\
&= (400 \text{ grams})(1\frac{\text{cal}}{\text{gram } ^\circ\text{C}})80^\circ\text{C} \\
&= 32000 \text{ cal} \\
&= 133952 \text{ Joules}
\end{aligned}$$

If this much energy needs to be transferred in 2 minutes (120 seconds), the power of the heater must be

$$Power = \frac{133952 \text{ Joules}}{120 \text{ sec}} \approx 1116 \text{ Watts} \quad (5)$$

Exercise 1.5

Ronald McDonald wants to help with the energy problem by not wasting the ice in his drink. He wants to know how much ice at 0°C he should add to 500 grams of water at 20°C so that the water cools down to 0°C just as the last bit of ice melts. How many grams of ice should he add to his 500 gram drink?

The energy lost by the water when the temperature decreases from 20°C to 0°C equals the energy gained by the ice as it melts (staying at 0°C). The heat capacity of water is $1 \text{ cal}^\circ\text{C}/\text{gram}$, and the latent heat of ice is $80 \text{ cal}/\text{gram}$. Let x be the number of grams of ice. Then:

$$\begin{aligned}
\text{Energy gained by ice} &= \text{Energy lost by water} \\
x80 &= 500 \text{ g} (1\frac{\text{cal}}{\text{g}^\circ\text{C}})20^\circ\text{C} \\
x &= 125 \text{ grams}
\end{aligned}$$

After the 125 grams of ice melts to 0°C water, the initial 500 grams of water has cooled to 0°C . No ice is wasted. So, in order to not waste ice, your drink should start with 1/4 of it being ice. Remember this the next time you fill up your drink at the soda fountain.

Exercise 1.6

Raman is served 500 grams of very hot tea, which is at a temperature 100°C . He

wants to add some ice at -10°C to cool it to 60°C . How much ice should he add?

This problem is similar to the previous one, except the ice goes through a phase change. The physics is the same: the energy gained by the ice is equal to the energy lost by the tea. The energy lost by the tea is easy to calculate, $Q = mc\Delta T = 500(1)(40^{\circ}\text{C}) = 20000$ cal. Let x be the mass of ice needed. The ice gains energy in three stages: first the x grams of ice rises to 0°C , then the ice melts, and finally the x grams rises in temperature from 0° to 60°C . The heat capacity of ice is 0.5 cal/ $^{\circ}\text{C}/\text{gram}$, so we have

$$20000 = x(0.5)10 + 80x + x(1)(60) \quad (6)$$

The first term on the right is the energy gained by the ice as its temperature increases from -10° to 0° . The middle term, $80x$, is the energy gained when the x grams of ice melts without changing temperature. The last term on the right is the energy gained when x grams of water has a temperature increase of 60°C . Solving for x gives

$$x = \frac{20000}{145} \approx 138 \text{ grams} \quad (7)$$

Exercise 1.7

Rick has just installed solar collectors to heat water. His large water tank holds 400 liters of water. The solar collectors have an area of 8 m^2 (2 meters by 3 meters). If the water in his tank starts out with a temperature of 20°C in the morning, what is its temperature in the evening after the sun has set? Some useful information: solar energy intensity is around 800 W/m^2 , solar collectors have an efficiency of around 30%, and suppose the sun shines for 8 hrs/day.

First let's calculate the energy Q collected by the solar collectors. Since a Watt = 1 Joule/sec, the number of Joules of energy collected is

$$\begin{aligned} Q &= (800 \frac{\text{Joules}}{\text{sec m}^2})(3600 \text{ sec})(8 \text{ hours})(8 \text{ m}^2)(0.3) \\ &\approx 5.53 \times 10^7 \text{ Joules} \\ &\approx 1.32 \times 10^7 \text{ cal} \end{aligned}$$

where we have used the conversion $1 \text{ cal} = 4.186 \text{ Joules}$. To determine how much the temperature will change, we use the property of the specific heat: $Q = mc\Delta T$:

$$\begin{aligned}
\Delta T &= \frac{Q}{mc} \\
&= \frac{1.32 \times 10^7}{(400000)1} \\
\Delta T &\approx 33^\circ C
\end{aligned}$$

where we have used the conversion 1 *liter* = 1000 *grams* of water. So the final temperature of the water is around 53°C. 400 liters of water at 53°C is enough hot water to last a family for a day. Start installing solar collectors.

Exercise 1.8

Merton the civil engineer needs to determine the energy transfer through two materials that are next to each other. Each material has an area A . One material has a thickness L_1 and a thermal conductivity constant of k_1 . The other material has a thickness L_2 and a thermal conductivity constant of k_2 . If the higher temperature on one side is T_H , and lower temperature is T_C , what is the rate of energy transfer through the two materials? See the figure.

Once a steady energy flow has been reached, the energy transfer rate through material 1 equals the energy transfer rate through material 2. Let T be the temperature in the middle of the two materials. Then we have

$$k_1 \frac{A(T_H - T)}{L_1} = k_2 \frac{A(T - T_C)}{L_2} \quad (8)$$

Since the areas cancel, the critical factor is the ratio k/L . It is useful to define a new quantity $R \equiv L/k$, which is called the "Thermal Resistance" or R value. We can solve the above equation for T :

$$T = \frac{R_1 T_H + R_2 T_C}{R_1 + R_2} \quad (9)$$

where $R_1 = L_1/k_1$ and $R_2 = L_2/k_2$. Now that we know the temperature in the middle, we can solve for the energy transfer rate through the materials. Since the rate is the same through each one, we only need to calculate the rate through (say) material 1:

$$\begin{aligned}\frac{Q}{At} &= \frac{1}{R_1}(T - T_C) \\ &= \frac{T_H - T_C}{R_1 + R_2} \\ \frac{Q}{t} &= \frac{A}{R_1 + R_2}(T_H - T_C)\end{aligned}$$

From the last line, we see that the total thermal resistance R_{tot} of the two materials equals the sum of the thermal resistances of each one: $R_{tot} = R_1 + R_2$. This is an important relationship when civil engineers need to calculate the thermal conductivity of materials placed together.

Exercise 2.1

Lenny has a balloon filled with helium. He wonders: how fast are the helium atoms moving around in the balloon. What are you going to tell him? The temperature in the room is 20°C.

We can figure out how fast the helium molecules are moving. From the kinetic theory of gases we derived in class that the "root-mean-square" velocity is $v_{RMS} = \sqrt{3kT/m}$ where m is the mass of a molecule. Multiplying the top and bottom of the fraction under the square-root by Avagadro's number N_A , we get

$$v_{RMS} = \sqrt{\frac{3RT}{M}} \quad (10)$$

where $R = N_A k \approx 8.34 \text{ J}/(\text{mole K})$ is the gas constant and $M = N_A m$ is the molar mass. For helium the molar mass is 0.004 Kg, so we have

$$\begin{aligned}v_{RMS} &= \sqrt{\frac{3(8.34)(293)}{0.004}} \\ &\approx 1354 \text{ m/s}\end{aligned}$$

since $T = 273 + 20 = 293^\circ\text{K}$. It is interesting that we don't need to know the volume of the balloon, the pressure inside it, or the number of molecules in the balloon. The average speed of the molecules only depends on the temperature of the helium gas.

Exercise 2.2

While sitting in class one day, Nathan was wondering: how much energy does the air in the room have? Can we answer his question?

Yes we can, approximately. Air consists mainly of diatomic gases: N_2 and O_2 . The energy of a diatomic gas starts out like a monatomic gas for temperatures less than around 150°K : $U = (3/2)nRT$. For temperatures above 150° energy can be transferred into rotating the molecules, and the change in energy $\Delta U = (5/2)nR\Delta T$. Since room temperature is around 300°K , the energy of a diatomic gas at this temperature is:

$$\begin{aligned} U &\approx \frac{3}{2}nR150 + \frac{5}{2}nR(T - 150) \\ &\approx 4nR150 \end{aligned}$$

where we used $T = 300^\circ\text{K}$. It is easier to cast the right side of this equation in terms of P and V . At equilibrium the gas variables satisfy: $PV = nRT$, so $nR = PV/300$. Thus, the total energy can be written as

$$U \approx 4 \frac{PV}{300} 150 = 2PV \quad (11)$$

This is an easier form to work with. The pressure in the room is around one atmosphere: $P \approx 1.013 \times 10^5 \text{ N/m}^2$. The volume of the room is around $V \approx (10\text{m})(5\text{m})(2\text{m}) = 100 \text{ m}^3$. Using these values, we have

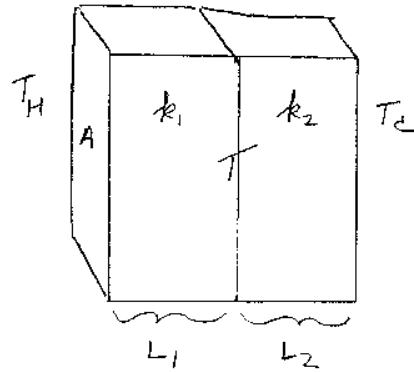
$$\begin{aligned} U &\approx 2(1.013 \times 10^5)(100) \\ &\approx 2.02 \times 10^7 \text{ Joules} \end{aligned}$$

Too bad we can't transfer some of the 20 million Joules of energy in the air of the room to help us solve exam problems.

Exercise 2.3

While sitting in her igloo, Lutaq realized that the air she breaths out is much warmer than the air she takes in. She wondered: how much energy am I transferring into the air with each breath? Can we answer her question?

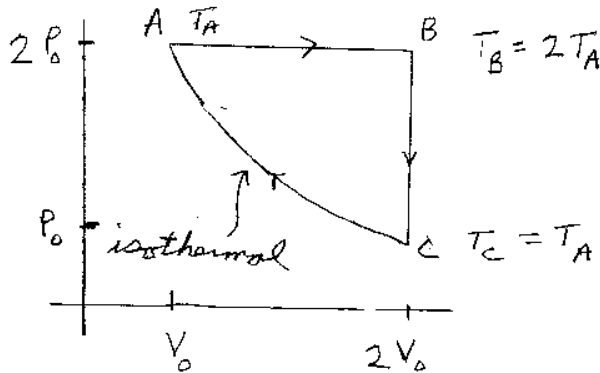
1.8



$T =$ temperature
at the interface
between the
two materials

$$k_1 \frac{A(T_H - T)}{L_1} = k_2 \frac{A(T - T_C)}{L_2}$$

2.4



$\Delta U(C \rightarrow A) = 0$
isothermal

$W(B \rightarrow C) = 0$

$$W(A \rightarrow B) = 2P_0 (2V_0 - V_0) = 2P_0 V_0$$

$$W(C \rightarrow A) = \int P dV = \int_{2V_0}^{V_0} \frac{nRT_A}{V} dV = nRT_A \ln\left(\frac{1}{2}\right) = -2V_0 P_0 \ln 2$$

Yes we can, at least approximately. Let's assume that the air she breaths in is around a temperature of 20°C , and that the air she breaths out is around her body temperature of 40°C . Since air is a diatomic gas, the energy of n moles of air **at room temperature** is $U = (5/2)nRT$. The change in the energy of the air she breaths in, after she breaths it out, is

$$\Delta U = \frac{5}{2}nR\Delta T \quad (12)$$

We just need to know approximately how many moles of air we breath in and out. This volume of air is called the tidal volume and is around 500 ml or 0.5 liters of air. Since one mole of air at room temperature and pressure is 22.6 liters, the number of moles we breath in and out is around $0.5/22.6 \approx 0.022$ moles. Using these numbers we obtain

$$\begin{aligned} \Delta U &\approx \frac{5}{2}(0.022)(8.34)(20^\circ) \\ &\approx 9.2 \text{ Joules} \end{aligned}$$

This is a small amount of energy, but 9.2 Joules gets transferred with each breath. The average breathing rate is 12 breaths/min, so in an hour, $(9.2)(12)(60) \approx 6600$ Joules of energy gets transferred to the air in the igloo. Energy also gets transferred through her skin, so the food Lutaag eats helps keep the igloo at a comfortable temperature.

Exercise 2.4

A gas consisting of n moles of a monatomic gas goes through the cyclic process shown in the figure. It starts off at point A in the $P - V$ plane. It first expands at constant pressure (isobaric) to the point B . Then the pressure is reduced, with constant volume, to the point C (isometric process). Finally, the gas is compressed with a constant temperature $T_C = T_A$ back to the point A (isothermal). The pressures and volumes are labeled in the figure. For each process, and the whole cycle, find ΔU , Q , W , and ΔS .

It is easiest to first find ΔU for each process, then W for each process. To find Q , we can use $Q = \Delta U + W$. Since there are n moles of a monatomic gas, $U = (3/2)nRT = (3/2)PV$. So

$$\Delta U(A \rightarrow B) = U_B - U_A$$

$$\begin{aligned}
&= \frac{3}{2}(2V_0)(2P_0) - \frac{3}{2}(V_0)(2P_0) \\
&= 3V_0P_0
\end{aligned}$$

Similarly, for the process from $B \rightarrow C$ we have

$$\begin{aligned}
\Delta U(B \rightarrow C) &= U_C - U_B \\
&= \frac{3}{2}(2V_0)(P_0) - \frac{3}{2}(2V_0)(2P_0) \\
&= -3V_0P_0
\end{aligned}$$

The process from C to A is isothermal, i.e. $\Delta T = 0$, so $\Delta U = 0$.

The work done is the next easiest to calculate, since it is the area under the curve describing the process. The work done **by** the gas from A to B is area under the line, or simply

$$W_{by}(A \rightarrow B) = (2P_0)(2V_0 - V_0) = 2V_0P_0 \quad (13)$$

because the pressure is constant during the process. The work from B to C is zero, since the volume doesn't change: $W_{by}(B \rightarrow C) = 0$. To calculate the work from C to A we need to integrate, since the pressure is not constant as the volume changes:

$$W_{by} = \int P dV \quad (14)$$

For an isothermal process, T is constant, so $P = nRT_C/V$. The integral becomes:

$$\begin{aligned}
W_{by}(C \rightarrow A) &= \int_{2V_0}^{V_0} \frac{nRT_C}{V} dV \\
&= nRT_C \ln\left(\frac{V_0}{2V_0}\right) \\
&= -nRT_C \ln(2) \\
&= -2V_0P_0 \ln(2)
\end{aligned}$$

Since $Q = \Delta U + W_{by}$, we can determine Q for all processes. The results are summarized in the chart below:

Process	Q	W_{by}	ΔU
$A \rightarrow B$	$5V_0P_0$	$2V_0P_0$	$3V_0P_0$
$B \rightarrow C$	$-3V_0P_0$	0	$-3V_0P_0$
$C \rightarrow A$	$-2V_0P_0 \ln(2)$	$-2V_0P_0 \ln(2)$	0
Whole Cycle	$2V_0P_0(1 - \ln(2))$	$2V_0P_0(1 - \ln(2))$	0

Note that the change in the internal energy ΔU for a complete cycle equals zero. It has to be, since we believe that U is a state function and doesn't depend on how the gas got to that state. Getting $\Delta U = 0$ is a good check that we haven't made any mistakes.

The change in entropy, ΔS , can be calculated for the processes by evaluating the heat transfer divided by T : $\Delta S = \Delta Q/T$. For the process A to B , the temperature is constantly changing so we need to integrate. The A to B process is isobaric, so we can use the heat capacity at constant pressure C_p to relate ΔQ to ΔT : $\Delta Q = nC_p\Delta T$. For a monatomic gas, $C_p = (5/2)R$. So we have

$$\begin{aligned}\Delta S(A \rightarrow B) &= \int_{T_A}^{2T_A} \frac{\Delta Q}{T} \\ &= \int_{T_A}^{2T_A} \frac{5}{2}nR \frac{dT}{T} \\ &= \frac{5}{2}nR \ln(2)\end{aligned}$$

For the process from B to C , the temperature is also continuously changing, so we have to integrate to find the change in entropy. However, in this case the process is isometric, so ΔQ is related to ΔT via the heat capacity for constant volume, C_v , which is $C_v = (3/2)R$ for a monatomic gas. So we have

$$\begin{aligned}\Delta S(B \rightarrow C) &= \int_{2T_C}^{T_C} \frac{\Delta Q}{T} \\ &= \int_{2T_C}^{T_C} \frac{3}{2}nR \frac{dT}{T} \\ &= -\frac{3}{2}nR \ln(2)\end{aligned}$$

For the isothermal process C to A , the temperature is not changing, so the change in entropy is simply $\Delta S = Q/T_C$, where Q is the heat transferred into the gas. From the table above, $Q = -2V_0P_0 \ln(2)$. So we have

$$\begin{aligned}\Delta S &= \frac{-2V_0P_0 \ln(2)}{T_C} \\ &= -nR \ln(2)\end{aligned}$$

since $P_0(2V_0) = nRT_C$. The entropy results can be summarized in the following chart:

Process	ΔS
$A \rightarrow B$	$(5/2)nR \ln(2)$
$B \rightarrow C$	$-(3/2)nR \ln(2)$
$C \rightarrow A$	$-nR \ln(2)$
Whole Cycle	0

Note that the change in entropy ΔS for a complete cycle equals zero. It has to, since we believe that Entropy is a state function and doesn't depend on how the gas got to that state. Getting $\Delta S = 0$ is a good check on our calculations.

Exercise 2.5

A monatomic gas is contained in a cylinder with a movable piston. Initially it has a volume V_0 , pressure P_0 and temperature T_0 . A weight is placed on the piston, and the gas is compressed adiabatically to a pressure $10P_0$. What is the final volume of the gas, and what is the final temperature? Express your answers in terms of V_0 and T_0 respectively.

Since the gas is monatomic, $PV^{5/3} = \text{constant}$ for adiabatic processes:

$$\begin{aligned}
 P_f V_f^{5/3} &= P_0 V_0^{5/3} \\
 V_f &= V_0 \left(\frac{P_0}{P_f}\right)^{3/5} \\
 V_f &= V_0 \left(\frac{1}{10^{3/5}}\right) \\
 V_f &\approx 0.25 V_0
 \end{aligned}$$

The final temperature can be determined using the ideal gas law:

$$\begin{aligned}
 nRT_f &= P_f V_f \\
 &= (10P_0)(0.25V_0) \\
 &= 2.5 P_0 V_0 \\
 &= 2.5 nRT_0 \\
 T_f &= 2.5 T_0
 \end{aligned}$$

The temperature in Kelvin increases by a factor of 2.5. The work done on the gas increases the internal energy of the gas.

Exercise 2.6

John pours himself a cup of tea. The tea contains 500 grams of water initially at a temperature of 100°C . While waiting for the tea to cool down, John falls asleep. When he wakes up, the tea is at the room temperature of 20°C . The temperature of the room has hardly changes. How much has the entropy of the universe increased?

When the tea cools down, its entropy decreases. However, the entropy of the room increases. Lets first calculate the decrease in the entropy of the tea. As the tea loses energy, its temperature is continuously changing. When we add up the changes in entropy, $\Delta S = \Delta Q/T$ we need to integrate since T is always changing:

$$\begin{aligned}\Delta S_{tea} &= \int_{T_i}^{T_f} \frac{\Delta Q}{T} \\ &= \int_{T_i}^{T_f} \frac{mcdt}{T} \\ &= mc \ln\left(\frac{T_f}{T_i}\right) \\ &= (0.5 \text{ Kg})(4186 \frac{\text{Joules}}{\text{Kg}^\circ\text{K}})\ln(293/373) \\ &\approx -505 \text{ J}/^\circ\text{K}\end{aligned}$$

We now need to calculate the increase in entropy of the room. Since the temperature of the room essentially did not change, the increase in entropy is just

$$\Delta S_{room} \approx \frac{Q}{T_{room}} \approx \frac{Q}{293^\circ\text{K}} \quad (15)$$

where Q is the total energy transferred to the air. This is the same amount of energy lost by the cooling tea. So $Q = mc\Delta T = 0.5(4186)(80^\circ) \approx 167440$ Joules. Using this value for Q , we have

$$\Delta S_{room} \approx \frac{167440}{293} \approx +571 \text{ J}/^\circ\text{K} \quad (16)$$

Adding the two changes together, we find that the net change of the entropy of the tea and the room as the tea cools down is around $571 - 505 = +67 \text{ J}/^\circ\text{K}$.

Exercise 2.7

John wants to try to make tea again. To heat the water, he puts 500 grams of water

into his microwave oven. The water is initially at 20°C. The oven can supply energy to the water at a rate of 1200 Watts. He turns the oven on and waits for the water to heat up to 100°C. However, he falls asleep again and the oven keeps heating up the water till it all evaporates, and turns to steam. When the pressure of the steam reaches 10 atmospheres the microwave explodes waking John up. How long did it take for the water to a) reach 100°C, b) completely evaporate, and c) reach a pressure of 10 atmospheres?

a) To increase the temperature of 500 grams of water from 20° to 100° requires an energy of $Q = mc\Delta T = 500(1 \text{ cal}/^\circ\text{g})(80) = 40000$ calories. In units of Joules, this is $Q = 40000(4.186) = 167440$ Joules. Since the oven can supply energy at a rate of 1200 Joules/sec, the time needed to heat the water to 100° is

$$t = \frac{167440}{1200} \approx 140 \text{ seconds} \quad (17)$$

So John should have set the timer for around 2.5 minutes.

b) The energy needed to change 500 grams of water (at 100°) to steam (at 100°) is $Q = mL = 500(540 \text{ cal}/\text{gram}) = 270000$ calories. Since the latent heat to change water to steam at 100° is 540 cal/gram. In units of Joules, this is $Q = 270000(4.186) = 1130220$ Joules. Since the oven can supply energy at a rate of 1200 Joules/sec, the time needed to heat evaporate the water is

$$t = \frac{1130220}{1200} \approx 941 \text{ seconds} \quad (18)$$

So around 15 minutes after the water has reached 100° the water has evaporated to become steam.

c) Now, the microwave contains 500 grams of water vapor at a temperature of $T = 373^\circ\text{K}$, and we will assume a pressure of one atmosphere. If the pressure increases to 10 atmospheres, while keeping the volume constant, the temperature must reach $T_f = 10(373) = 3730^\circ\text{K}$. That is, the change in temperature must be 3357°K . The heat capacity of water vapor at constant volume is around $(9/2)R$ per mole. Water molecules have rotational and vibrational modes of energy. The amount of energy needed to double the pressure is then $Q = nR\Delta T = (500/18)(9/2)(8.314)(3357) = 3488762$ Joules, since the molar mass of water is 18 grams. Since the oven can supply energy at a rate of 1200 Joules/sec, the time needed to bring the steam up to 10 atmospheres is

$$t = \frac{3488762}{1200} \approx 2907 \text{ seconds} \quad (19)$$

So around an hour after John puts the water in the microwave oven, it explodes and John wakes up. Do not try this experiment at home, use an alarm clock instead.

Exercise 2.7

Phil has a large cylinder filled with helium. It starts off with a volume of 2 liters, a pressure of 10^5 Pa, and a temperature of 300°K . He quickly decreases the volume to one liter adiabatically. Then, he waits for the gas to cool back down to 300°K at this one liter volume. Finally, he expands the piston slowly out to 2 liters such that the temperature of the gas remains constant at 300°K . The three processes, adiabatic-isometric-isothermal, form a cycle as shown in the figure. Calculate ΔU , Q , W_{by} , and ΔS for each leg of the cycle, and for the whole cycle.

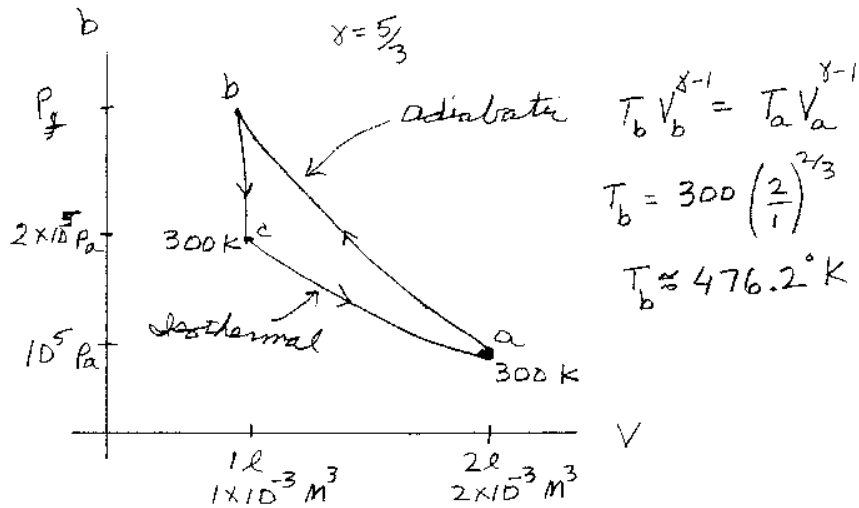
It is easiest to first determine which quantities do not change. For the complete cycle, ΔU and ΔS are zero: $\Delta U = 0$ and $\Delta S = 0$. This must be true, since the total internal energy and the entropy are state functions of the gas. Their values only depend on the state of the gas and not how the gas was produced. For the adiabatic process, $Q = 0$, thus $Q(a \rightarrow b) = 0$. Since there is no "heat" transfer in the adiabatic process, $\Delta S = 0$. For the isothermal process, $\Delta U = 0$, thus $\Delta U(c \rightarrow a) = 0$. For the isometric process, $\Delta V = 0$, thus the work done from b to c equals zero: $W(b \rightarrow c) = 0$. These 6 zero values can be entered in the table in the figure.

The next easiest quantity to calculate is $\Delta U(a \rightarrow b)$, since $\Delta U = (3/2)nR\Delta T$ for a monatomic gas. We just need to calculate n and T_b . $n = (P_a V_a)/(RT_a) = (10^5)(2 \times 10^{-3})/(8.314)/(300) \approx 0.08$ moles. To obtain T_b we need to use the properties of the adiabatic process. For an adiabatic process, $PV^\gamma = \text{constant}$. Since $P = nRT/V$, we have $TV^{\gamma-1} = \text{constant}$:

$$\begin{aligned} T_b V_b^{\gamma-1} &= T_a V_a^{\gamma-1} \\ T_b V_b^{2/3} &= T_a V_a^{2/3} \\ T_b &= T_a 2^{2/3} \\ &= (300)2^{2/3} \\ T_b &\approx 476.2^\circ K \end{aligned}$$

since $\gamma = 5/3$ for a monatomic gas. Now that the temperatures are know, we can calculate ΔU . $\Delta U(a \rightarrow b) = (3/2)nR\Delta T = 1.5(0.08)(8.314)(476.2 - 300) \approx 176$

2.7



$$n = \frac{PV}{RT} = \frac{10^5 (2 \times 10^{-3})}{8.314 (300)} \approx .08 \text{ moles}$$

$Q_{a \rightarrow b} = 0$
 $W_{By} (b \rightarrow c) = 0$
 $\Delta U_{c \rightarrow a} = 0$

$$\Delta U_{a \rightarrow b} = \frac{3}{2} n R \Delta T \approx \frac{3}{2} (.08) (8.314) (176.2) \approx 176 \text{ J}$$

$$W_{By} (c \rightarrow a) = nRT \ln\left(\frac{V_f}{V_i}\right) = P_c V_c \ln 2 = 200 \ln 2 \approx 139 \text{ J}$$

$$\Delta S (c \rightarrow a) = \frac{Q}{T} \approx \frac{139 \text{ J}}{300 \text{ K}} \approx .46 \text{ J/K}$$

Process	W_{By}	Q	ΔU	ΔS
$a \rightarrow b$	-176 J	0	176 J	0
$b \rightarrow c$	0	-176 J	-176 J	-0.46 J/K
$c \rightarrow a$	139 J	139 J	0	+0.46 J/K
cycle	-37 J	-37 J	0	0

Joules. With this value of $\Delta U(a \rightarrow b)$, we can determine $W_{by}(a \rightarrow b)$: $W_{by}(a \rightarrow b) = Q - \Delta U = -176$ Joules.

The next quantity we can calculate is the work done by the gas from c to a. $W_{by}(c \rightarrow a) = \int PdV$. Since the temperature is constant, $P = nRT/V$, and we can calculate the integral.

$$\begin{aligned}
 W_{by}(c \rightarrow a) &= \int PdV \\
 &= \int_{V_i}^{V_f} nRT_c \frac{dV}{V} \\
 &= nRT_c \ln \frac{V_f}{V_i} \\
 &= P_c V_c \ln(2) \\
 &= (2 \times 10^5)(10^{-3}) \ln(2) \\
 &\approx 139 \text{ Joules}
 \end{aligned}$$

Since $\Delta U(c \rightarrow a) = 0$, $Q(c \rightarrow a) = \Delta U + W_{by} = 139$ Joules. We can add up the columns to find Q and W_{by} for the complete cycle. This yields $Q = W_{by} = -37$ Joules for the whole cycle.

In computing the entropy changes, it is easiest to calculate the entropy change along the isothermal. This is because T is constant for an isothermal process:

$$\Delta S(c \rightarrow a) = \int \frac{dQ}{T_c} = \frac{1}{T_c} \int dQ = \frac{Q_{total}}{T_c} \quad (20)$$

We could also have used T_a in this equation, since $T_a = T_c$. For the isothermal process c to a, $Q_{total} = Q(c \rightarrow a) \approx 139$ Joules. Thus,

$$\Delta S(c \rightarrow a) = \frac{Q_{total}}{T_c} \approx \frac{139}{300} \approx 0.46 \text{ Joules}/^\circ K \quad (21)$$

Since the entropy change for the whole cycle is zero, we must have $\Delta S(b \rightarrow c) \approx -0.46$ Joules/K. We can now fill in the whole chart for Phil's cyclic process.

Exercise 2.8

Little Roni has a balloon filled with helium. The balloon has a volume of 0.01 m^3 , and contains 0.8 moles of helium. The balloon is initially at a temperature of $300^\circ K$. While she is sitting in her pre-school classroom, the balloon suddenly pops. She immediately becomes concerned about the entropy increase of the helium that was in

the balloon, and wants us to calculate the increase of entropy. The classroom has a volume of 100 m^3 and the helium gas comes to equilibrium in the classroom with a final temperature of 300°K . Can we help her?

Yes, we can. Eventhough the process is not a reversible process, we can still calculate the difference in entropy between the initial state and the final state. The initial state consists of 0.8 moles of helium at a volume of 0.01 m^3 , and a temperature of 300° . In the final state the same 0.8 moles of helium has a volume of 100 m^3 and a temperature of 300° . For the actual process, one cannot draw a curve in the $P - V$ plane that connects these two states. However, the change in entropy of the system only depends on the initial and final states, and not on the process connecting the two states. So, to find the change in entropy, we can choose any process that connects the two states. An isothermal process is probably the easiest one to choose, since the initial and final temperatures are the same.

The change in entropy from an initial state i to a final state f is

$$\Delta S = \int \frac{dQ}{T} \quad (22)$$

For an isothermal process, the temperature is constant and can be taken out of the integral:

$$\Delta S = \frac{1}{T_i} \int dQ = \frac{Q}{T_i} \quad (23)$$

where Q is the total amount of "heat" energy transferred. For an isothermal process for a ideal gas, $\Delta U = 0$, so $Q = W_{by}$. The work done by an ideal gas for an isothermal process is

$$W_{by} = \int p dV = nRT \int_{V_i}^{V_f} \frac{dV}{V} \quad (24)$$

Carrying out the integral yields

$$W_{by} = nRT_i \ln\left(\frac{V_f}{V_i}\right) = nRT_i \ln(10^5) \quad (25)$$

Putting all the equations together, we obtain for the entropy change

$$\begin{aligned} \Delta S &= \frac{1}{T_i}(nRT_i) \ln(10^5) \\ &= nR \ln(10^5) \end{aligned}$$

$$\begin{aligned}
&= (0.8)(8.314)5 \ln(10) \\
&\approx +76.6 J/^\circ K
\end{aligned}$$

That is a large entropy increase, so you had better not pop too many balloons.

Exercise 2.9

While walking along the beach one day, Sandy spots a crumpled up aluminum can whose mass is 100 grams. It has been sitting in the sun all day and is at a temperature of 80°C . Sandy is not thinking about the environment and throws the hot aluminum can into the ocean, which has a temperature of 20°C . After the aluminum is finished transferring energy into the ocean, how much did the entropy of the universe change?

We can calculate the entropy change of the aluminum can, and then the entropy change of the ocean. First the aluminum can. As the can loses energy, its temperature continuously changes. Therefore, we must integrate $(\Delta Q)/T$ since T changes:

$$\Delta S_{can} = \int \frac{dQ}{T} \quad (26)$$

The temperature change ΔT is related to ΔQ by $\Delta Q = mc\Delta T$, so we have

$$\Delta S_{can} = \int_{T_i}^{T_f} \frac{mc}{T} dT \quad (27)$$

We must use the Kelvin scale for temperatures, which gives

$$\Delta S_{can} = mc \int_{353}^{293} \frac{dT}{T} = mc \ln(293/353) \quad (28)$$

The heat capacity of aluminum is $0.21 \text{ cal/g/}^\circ\text{C} = 890 \text{ J/kg/}^\circ\text{C}$, so

$$\Delta S_{can} = (0.1)(890) \ln(293/353) \approx -16.6 \text{ J/}^\circ\text{K} \quad (29)$$

The change in the entropy of the ocean is easier to calculate since the temperature of the ocean essentially does not change during the process. It remains at $20^\circ\text{C} = 293^\circ\text{K}$. Thus, the change in entropy of the ocean is

$$\Delta S_{ocean} = \frac{Q_{total}}{293} \quad (30)$$

The total energy transferred to the ocean is $Q = mc\Delta T$, where the quantities on the right side are for aluminum. That is, $Q = 0.1(890)(60) = 5340 \text{ Joules}$. So the entropy change of the ocean is

$$\Delta S_{ocean} = \frac{5340}{293} = +18.2 \text{ J}/^\circ K \quad (31)$$

The net change in the entropy of the universe is

$$\Delta S_{net} = 18.2 - 16.6 = 1.6 \text{ J}/^\circ K \quad (32)$$

So not only has Sandy polluted the environment, she also increased the entropy of the universe. If she would have thrown the can into the recycle bin, at least she would have helped the environment.