Exercises on Work, Energy, and Momentum

Exercise 1.1
Consider the following two vectors:

$\vec{A}$: magnitude 20, direction 37° North of East
$\vec{B}$: magnitude 10, direction 45° North of West

Find the scalar product $\vec{A} \cdot \vec{B}$.

One could solve this problem two ways. First, the scalar product is equal to $|\vec{A}|\ |\vec{B}|\ cos\theta$ where θ is the angle between the two vectors. The angle between these two vectors is 98°. So we have

$$\begin{aligned}\vec{A} \cdot \vec{B} &= 20(10) cos 98^\circ \\ \vec{A} \cdot \vec{B} &\approx -28\end{aligned}$$

Another way to solve this problem is to express the vectors in terms of the unit vectors:

$\vec{A} \approx 16\hat{\imath} + 12\hat{\jmath}$
$\vec{B} \approx -7.07\hat{\imath} + 7.07\hat{\jmath}$

The components are found by taking the appropriate sin and cos functions. Now, we can find the scalar product by must multiplying the respective components:

$$\begin{aligned}\vec{A} \cdot \vec{B} &= A_x B_x + A_y B_y \\ \vec{A} \cdot \vec{B} &\approx 16(-7.07) + 12(7.07) \\ \vec{A} \cdot \vec{B} &\approx -28\end{aligned}$$

Exercise 1.2
Consider the following two forces:

$$\begin{aligned}\vec{F}_1 &= (2\hat{\imath} + 4\hat{\jmath})\ N \\ \vec{F}_2 &= (5\hat{\imath} - 1\hat{\jmath})\ N\end{aligned}$$

What is the angle between these two forces?

We can use the relationship $\vec{F_1} \cdot \vec{F_2} = |\vec{F_1}||\vec{F_2}|cos\theta$, where θ is the angle between the two vectors. For our vectors, we have $|\vec{F_1}| = \sqrt{2^2 + 4^2} = \sqrt{20}$; $|\vec{F_2}| = \sqrt{5^2 + 1^2} = \sqrt{26}$. The scalar product is just $\vec{F_1} \cdot \vec{F_2} = 2(5) + 4(-1) = 6$. Therefore:

$$\begin{aligned}
\vec{F_1} \cdot \vec{F_2} &= |\vec{F_1}||\vec{F_2}|cos\theta \\
6 &= \sqrt{20}\sqrt{26}cos\theta \\
cos\theta &= \frac{6}{\sqrt{20}\sqrt{26}} \\
cos\theta &\approx 0.263 \\
\theta &\approx 74.7^\circ
\end{aligned}$$

Exercise 2.1
A block is free to slide without friction on a horizontal table. William exerts a constant force of 5 Newtons on the block. The direction of the force is 37° with respect to the horizontal. William keeps applying the force on the block as it moves a distance of 10 meters on the table top.

a) How much work was done by the William's force?

Since William's force is constant and the block moves in a straight line, the amount of work done by his force is just the component of the force in the direction of motion times the distance traveled. So $W_{William} = 5\ cos37^\circ\ 10 \approx 40$ Joules.

b) How much work was done by the force of gravity?

None! Since the force of gravity acts perpendicular to the direction of the path, there is no component of the gravitational force in the direction of the motion.

c) How much work was done by the force that the table exerts on the block?

None! Since there is no friction, the table can only exert a force on the block perpendicular to the surface. This force has no component in the direction of the motion.

d) What is the net work done on the block?

The net work is the sum of the work done by William's force, the force of gravity, and the table's force. Thus, $W_{net} = W_{William} + W_{gravity} + W_{table} = 40 + 0 + 0 = 40$ Joules.

Exercise 2.2
Eddie has made a long gun. The bullet has a mass of 10 grams. A force of 3 Newtons pushes the bullet down the barrel, but the barrel has a frictional force of 1 Newton in the opposite direction. The barrel is 5 meters long and the bullet starts off at rest. What is the final speed of the bullet as it leaves the barrel?

Since we are only interested in the final speed of the bullet, we can use the work-energy theorem: Net Work = change in Kinetic Energy. Since the Net Force equals 3 - 1 = 2 Newtons, the Net Work is $W_{net} = 2(5) = 10$ Joules of work. So we have

$$\begin{aligned} 10 &= \frac{m}{2}v_f^2 - 0 \\ 10 &= (0.01/2)v_f^2 \\ v_f &\approx 44.7\ m/s \end{aligned}$$

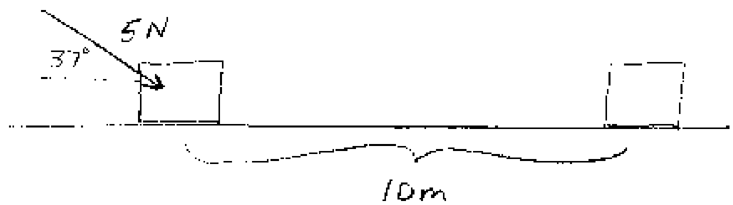
Note that the work done by the 3 Newton force equals 15 Joules, and the work done by friction equals -5 Joules. The Net Work is $15 - 5 = 10$ Joules.

Exercise 2.3
Stella is sitting on top of a big slide. The slide has a vertical drop of 10 meters. If there is no friction between her and the slide, what is her speed at the bottom of the slide?

We can use the Work-Energy theorem for this problem, since we are only interested in her final speed and not on the time it takes her to reach the bottom. There are two forces acting on her as she slides down: the force of gravity and the force of the slide on her. So

$$W_{gravity} + W_{slide} = change\ in\ Kinetic\ Energy \qquad (1)$$

2.1



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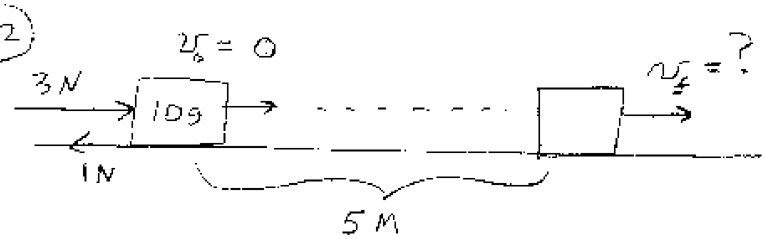

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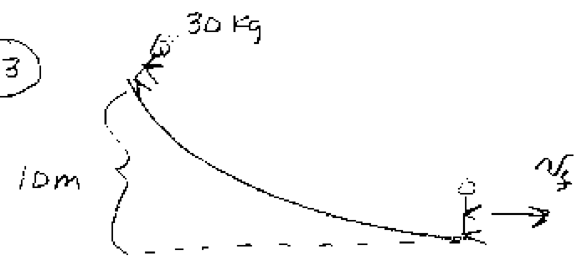
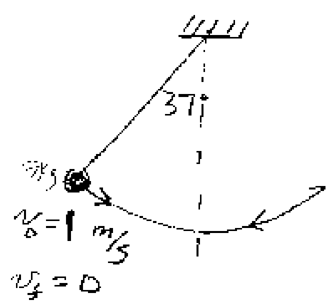


2.4



The work done by gravity is just $mg(y_i - y_f)$, which in our case is just $10mg$. Since there is no friction, the force that the slide exerts on Stella is perpendicular to its surface, and hence perpendicular to the direction of her velocity at all times during her slide. Thus, the work done by this "normal" force is zero:

$$\begin{aligned} 10mg + 0 &= \frac{m}{2}v_f^2 - 0 \\ 10(9.8) &= \frac{v_f^2}{2} \\ v_f^2 &= 196 \\ v_f &= 14\ m/s \end{aligned}$$

Note that this result does not depend on her mass, nor does it depend on the shape of the slide. As she slides down, $\vec{N}$ can change, and the component of $m\vec{g}$ parallel to the slides surface can change. However, the net work is simply $mg(y_i - y_f)$.

Wow, 14 m/s is quite fast. Stella tries out the slide and measures her speed at the bottom. She finds that her speed is only 8 m/s. How much work was done by the force of friction?

Even if friction is present, the Work-Energy Theorem is still valid. Nowever, now the net work includes the work done by friction:

$$W_{gravity} + W_{slide} + W_{friction} = change\ in\ Kinetic\ Energy \qquad (2)$$

To find the work done by friction we need to know Stella's mass, which is 30 kg.

$$\begin{aligned} 10mg + 0 + W_{friction} &= \frac{m}{2}v_f^2 - 0 \\ 10(30)9.8 + W_{friction} &= \frac{(30)8^2}{2} \\ W_{friction} &= -1980\ Joules \end{aligned}$$

The work done by friction is negative since the frictional force is in the opposite direction to the motion. Since the slide is a curved surface, the frictional force changes as she moves down the slide. To carry out the actual integral for the work done by friction, $\int \vec{f} \cdot \vec{dr}$ would be quite difficult. However, we can use the Work-Energy theorem to get this result without doing any integrals.

Exercise 2.4
An absent-minded professor does the "pendulum-of-death" demonstration. He takes a bowling ball of mass 3 Kg and ties it to a long rope. For the demonstration he is supposed to let the ball go at his nose with no speed. The ball swings across the room and returns stopping at his nose. However, he forgets and gives the ball an initial speed of 1 m/s. He is lucky however. There is a lot of air friction and the ball still returns just in front of his nose. How much work was done by air friction?

We can apply the work energy theorum, since we are not interested in the time the ball took to swing back and forth, but only in the initial and final speeds. There are three forces acting: gravity, the tension in the rope, and friction. So we have:

$$W_{gravity} + W_{tension\ force} + W_{friction} = change\ in\ Kinetic\ Energy \tag{3}$$

The work done by the tension force of the rope is zero because the tension acts in a direction perpendicular to the ball's velocity. The work done by gravity is also zero, since the initial and final heights of the ball are the same. That is, $mgy_f = mgy_i$. So the equation becomes

$$\begin{aligned} 0 + 0 + W_{friction} &= \frac{m}{2}v_f^2 - \frac{m}{2}v_i^2 \\ W_{friction} &= 0 - \frac{3}{2}(1)^2 \\ W_{friction} &= -1.5\ Joules \end{aligned}$$

Note that the length of the rope did not enter in this calculation.

Exercise 2.5
Dora wants to determine the coefficient of friction between a block and a particular surface. She places a block of mass m on an inclined plane. the plane makes an angle of 37° with respect to the horizontal. The block starts from rest a distance of 10 meters up the incline. she lets the block go, and it slides down the plane and along a table top. The block comes to rest 10 meters from the start of the incline. The coefficient of friction is the same on the incline and the tabletop. What is the coefficient of friction?

We will use the Work-energy theorem to solve the problem. There are three forces acting: gravity, friction, and the force normal to the surfaces. Actually the last two are forces from the surface to the block. We need to find the work done by each of these forces, along the path taken by the block. $W_{normal} = 0$, since the normal force acts perpendicular to the direction of the motion. $W_{gravity} = mg(y_i - y_f) = mg(10)sin37° = 6mg$. The force of friction does work on the block as it slides down the incline and as it slides across the table top. The work done by friction equals $W_{friction} = -mg(cos37°)\mu(10) - mg(10)\mu$, where the first term is the work done by friction on the incline and the second term is the work done by friction on the table top. Substiting the values for the trig functions gives $W_{friction} = -mg(0.8)10\mu - mg(10)\mu = -18mg\mu$. The work-energy theorem yields:

$$\begin{aligned} W_{gravity} + W_{normal} + W_{friction} &= change\ in\ Kinetic\ energy \\ 6mg + 0 - 18mg\mu &= 0 - 0 \end{aligned}$$

Solving for μ gives $\mu = 1/3$.

Exercise 2.6
We'll do the classic "loop-the-loop" demonstration in lecture. A block slides without friction down an incline and enters a circular path at the bottom of the incline. The radius of the loop is r. How far up the incline should the block be placed so that it does not leave the track as it travels in the loop?

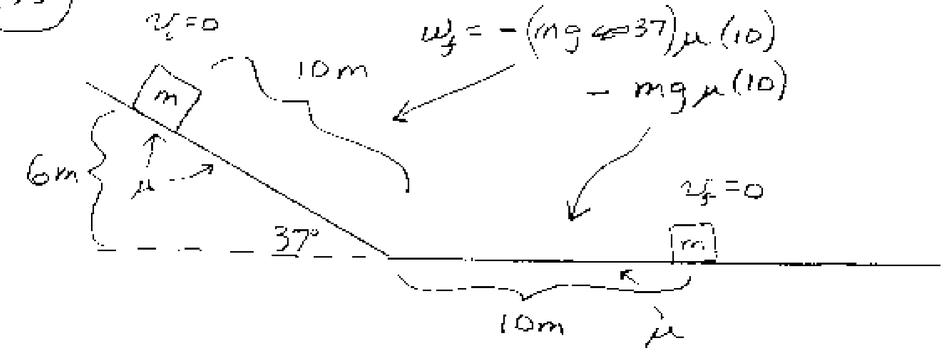
We'll first determine the speed the block needs at the top of the loop such that it stays on the track. Then we will determine how far up the incline it needs to start such that the block will have this speed at the top of the loop. While traveling in the loop, the block is moving in a circle. At the top of the loop, all the forces acting on the block are vertical, since there is no friction. The track can only exert a force perpendicular to its surface, and gravity acts downward. Let N be the force that the track exerts on the block. If the block is moving fast enough to stay on the track at the top, we must have

$$mg + N = m\frac{v^2}{r} \tag{4}$$

Both $m\vec{g}$ and $\vec{N}$ point downward at the top of the loop. We can solve this equation for N:

(2.5)

$$w_f = -(mg\cos 37)\mu(10) - mg\mu(10)$$



$$w_f = -mg\mu(10\cos 37 + 10) = -mg\mu(8+10)$$

$$w_g + w_N + w_f = \frac{m}{2}v_f^2 - \frac{m}{2}v_i^2$$

$$mg6 + 0 - 18mg\mu = 0 - 0$$

$$\mu = \frac{6}{18} = 1/3$$

(2.6)

$$N + mg = \frac{mv^2}{r}$$

$$N = \frac{mv^2}{r} - mg \geq 0$$

$$\boxed{v^2 \geq rg}$$

$$W_g + W_N = \frac{m}{2}v_f^2 - \frac{m}{2}v_i^2$$

$$mg(h-2r) + 0 = \frac{m}{2}(rg) - 0$$

$$\boxed{h \geq \frac{5}{2}r}$$

$$N = m\frac{v^2}{r} - mg \tag{5}$$

The faster the block moves, the larger N will be. If $v^2/r = g$, then N will be zero. This means that the track will exert no force on the block at the top of the loop. This is the critical speed for the block to stay on the track. If v is slower than $\sqrt{rg}$ the track would have to "pull up" on the block to keep it on the track, which it cannot do. Thus, $v_{critical} = \sqrt{rg}$.

As the block slides down the incline, the only force that does work is gravity, since there is no friction. The force of the incline on the block is perpendicular to the blocks motion and does no work. Thus, we have

$$\begin{aligned}
W_{gravity} + W_{normal} &= m\frac{v_f^2}{2} - m\frac{v_i^2}{2} \\
mg(h - 2r) + 0 &= m\frac{rg}{2} - 0 \\
h &= \frac{5}{2}r
\end{aligned}$$

Exercise 2.7

Alan pushes on a 10 Kg block on a frictionless surface in the "x" direction. The block starts from rest at $x = 0$, and Alan pushes with a force given by $F_x = 6x$ Newtons, where x is in meters. From this formula, one can see that Alan's force increases linearly with the distance that the block is from the origin. He keeps applying this force for 2 meters, i.e. till $x = 2$. What is the speed of the block at $x = 2$ meters?

We can use the Work-energy theorem to find the final speed of the block. The only force that does work is Alan's force. However, we cannot find the work done by his force by simply multiplying (Force) times (distance). This is because the force is not constant. We must integrate:

$$\begin{aligned}
W_{net} &= \int_0^2 F_x dx \\
&= \int_0^2 6x\ dx \\
&= 3x^2|_0^2 \\
W_{net} &= 12\ Newtons
\end{aligned}$$

Now we can use the Work-energy theorem:

$$
\begin{aligned}
W_{net} &= m\frac{v_f^2}{2} - m\frac{v_i^2}{2} \\
12 &= 5v_f^2 - 0 \\
v_f &\approx 1.55\, m/s
\end{aligned}
$$

Exercise 2.8 Lance, 80 Kg, rides his bike up a steep hill at a constant speed. The hill makes an angle of 10° with the horizontal. If he rides a distance of 12 Km up the incline find:

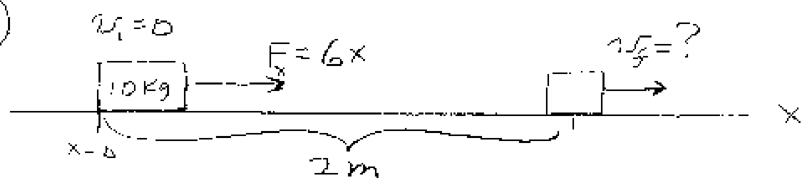
a) The work done by gravity.
The work done by gravity is just $mg(y_i - y_f)$. $(y_i - y_f)$ is equal to $-mg(12000 sin 10°) = -80(9.8)(12000 sin 10°) \approx -1.63 \times 10^6$ Joules. The work done by gravity is negative because the component of the gravitational force along the incline is in the *opposite* direction to the motion.

b) The work done by Lance, W_L.
Since the **velocity** of Lance is **constant**, there is no change in kinetic energy. Thus, **the Net Work is zero**. The normal force does no work, so we must have $W_L + W_{gravity} = 0$. So, $W_L \approx 1.63 \times 10^6$ Joules.

c) If Lance can complete the 12 Km in one hour (a speed of 12 Km/hr), what is his power output?
Power is equal to (energy transfer)/time. So his power output is $P \approx (1.63 \times 10^6\ Joules)/(3600\ sec) \approx 453\ Watts$. The average person can only ride a bike aerobically at a power output of 100 Watts.

Exercise 2.9
Uli the eskimo is sitting on top of his igloo. He starts sliding down the side. There is no friction between Uli and the igloo. As he slides down the side, his speed increases and he leaves the surface of the igloo. At what angle θ, shown in the figure, does he leave the surface of the igloo?

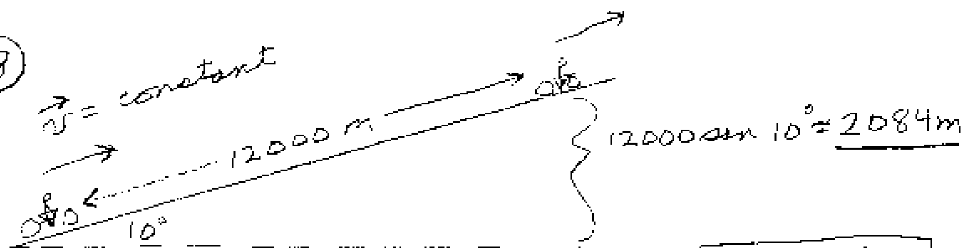
(2.7)



$$W = \int_0^2 6x\,dx = 3x^2\Big|_0^2 = 12 - 0 = 12 \text{ Joules}$$

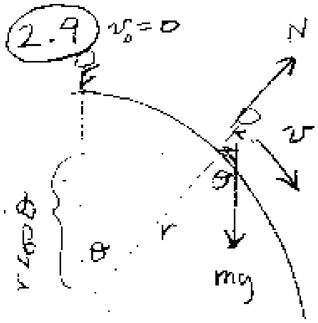
$$12 = \frac{10}{2} v_f^2 - 0 \quad \Rightarrow \quad v_f \simeq 1.55\, m/s$$

(2.8)



$$W_g = -mg(2084m) = -80(9.8)(2084) \simeq \boxed{-1.63\times 10^6 \text{ J}}$$

(2.9)



$$mg\cos\theta - N = \frac{mv^2}{r}$$

$$N = mg\cos\theta - \frac{mv^2}{r} \geq 0$$

$$mg\cos\theta - \frac{m}{r} v_{max}^2 = 0$$

$$mg(r - r\cos\theta) = \frac{m}{2} v^2$$

$$v^2 = 2gr(1-\cos\theta)$$

$$\rightarrow v_{max}^2 = rg\cos\theta$$

$$2gr(1-\cos\theta) = rg\cos\theta$$

$$\cos\theta = 2/3$$

$$\theta = 48.2^\circ$$

We can use the work energy theorem to determine Uli's speed after he has slide down the side to an angle θ. There are two forces acting on Uli, the force of gravity and the force that the surface of the igloo exerts on him. Since there is no friction, the force of the igloo on Uli is perpendicular to Uli's motion, and hence does no work. Thus, only the force of gravity can do work on Uli, so we have

$$\begin{aligned} W_{net} &= m\frac{v_f^2}{2} - m\frac{v_i^2}{2} \\ mg(r - rcos\theta) &= m\frac{v^2}{2} - 0 \\ v^2 &= 2gr(1 - cos\theta) \end{aligned}$$

As seen in the figure, the change in height of Uli is equal to $r - rcos\theta$. Uli's motion is *circular* as he slides down the side. If he is moving on the surface of the igloo, the net force towards the center of the circle must be mv^2/r. The net force is the component of $m\vec{g}$ towards the center ($mg\ cos\theta$) minus the normal force N that the surface exerts on Uli.

$$mg\ cos\theta - N = m\frac{v^2}{r} \tag{6}$$

Since the normal force **N must be positive**, the largest speed v_{max} that Uli can go is

$$v_{max}^2 = gr\ cos\theta \tag{7}$$

Combining this result with $v^2 = 2gr(1 - cos\theta)$ gives

$$\begin{aligned} 2gr(1 - cos\theta_{max}) &= gr\ cos\theta_{max} \\ cos\theta_{max} &= \frac{2}{3} \\ \theta_{max} &\approx 48.2^\circ \end{aligned}$$

Note that the result does not depend on Uli's mass, the radius of the igloo, or g! All eskimo's all over the universe will leave the surface of their igloos at an angle of 48.2°.

Exercise 2.10

You need to design a swing, and must know the maximum tension that the rope will experience. If a girl of mass m starts from rest at an angle of θ_0, what is the force that the ropes exert when she is at the bottom of the swing?

This is a two step problem. First we will determine the tension of the rope in terms of her speed at the bottom using Newton's second law. Then we will determine what her speed is if she starts at an angle of θ_0 using the work-energy theorem.

At the bottom of the swing, the girl is traveling in a circle with speed v. Her acceleration toward the center of the circle is v^2/l, where l is the length of the rope. Her net force is therefore, $F_{net} = mv^2/l$. At the bottom of the swing, the two forces that are acting on the girl, $m\vec{g}$ and $\vec{T}$, are both vertical. Thus, we have

$$\begin{aligned} T - mg &= m\frac{v^2}{l} \\ T &= mg + m\frac{v^2}{l} \end{aligned}$$

where T is the force the ropes exert on the girl. Actually, the tension in each rope is $T/2$. This equation makes sense. If her speed is zero, then $T = mg$. As T increases, the tension in the ropes increase and add a force equal to mv^2/l.

To determine what her speed is at the bottom of the swing if she starts at an angle of θ_0, we can use the work energy theorem:

$$\begin{aligned} W_g + W_T &= m\frac{v^2}{2} - 0 \\ mgh + 0 &= m\frac{v^2}{2} \\ v^2 &= 2gh \end{aligned}$$

The work done by T is zero, since $\vec{T}$ is perpendicular to the direction of the girl's velocity. We can express h in terms of l and θ_0: $h = l - lcos\theta_0$. Therefore, $v^2 = 2gl(1 - cos\theta_0)$. Substituting into the equation for T gives

$$\begin{aligned} T &= mg + \frac{m}{l}(2gl)(1 - cos\theta_0) \\ &= mg + 2mg(1 - cos\theta_0) \\ T &= mg(3 - 2cos\theta_0) \end{aligned}$$

If the girl starts at 90°, then the force of the ropes at the bottom are equal to $3mg$.

Exercise 2.11
You want to throw your physics book of mass m far from earth, to a distance r_{max} from the earth's center. What speed v_0 do you need so that it will travel out to r_{max}?

To solve this problem, one needs to know what the force of gravity is when the book is a distance r from the center of the earth. Let R be the radius of the earth. The gravitational force decreases as $1/r^2$ (inverse square law) for $r > R$. So the magnitude of the gravitational force that the earth exerts on an object of mass m a distance r from its center is:

$$|\vec{F}_g| = mg\frac{R^2}{r^2} \tag{8}$$

Note, this formula is only true if $r > R$. The direction of the force is towards the center of the earth. We can use the work-energy theorem to solve the problem. Since the speed of the book at r_{max} is zero:

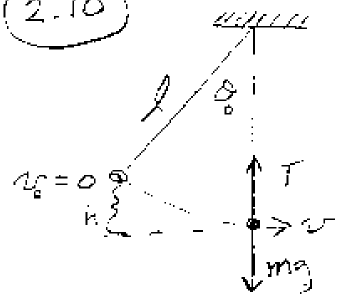
$$W_g = 0 - m\frac{v_0^2}{2} \tag{9}$$

To calculate W_g, we need to integrate, because the gravitational force is not constant. If an object travels far above the earths surface, then the decrease in the gravitational force must be considered. In this case $W_g \neq mgh$. The work done by gravity is given by

$$\begin{aligned} W_g &= \int_R^{r_{max}} (-\frac{mgR^2}{r^2})dr \\ &= \frac{mgR^2}{r}|_R^{r_{max}} \\ W_g &= \frac{mgR^2}{r_{max}} - mgR \end{aligned}$$

The negative sign in front of mgR^2/r^2 is due to the fact that the gravitational force is towards the center of the earth, but the path from R to r_{max} is away from the center of the earth. The work-energy theorem gives

2.10



$$T - mg = \frac{mv^2}{l}$$

$$T = mg + \frac{mv^2}{l}$$

$$W_g + W_T = \frac{m}{2}v^2 - 0$$

$$mgh + 0 = \frac{m}{2}v^2$$

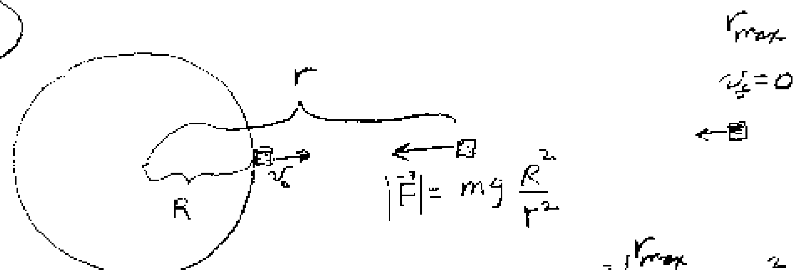
$$v^2 = 2gh = 2g(l - l\cos\theta_0)$$

$$T = mg + 2mg\frac{l}{l}(1 - \cos\theta_0)$$

$$T = mg(1 + 2(1 - \cos\theta_0))$$

$$\boxed{T = mg(3 - 2\cos\theta_0)}$$

2.11



$$|\vec{F}| = mg\frac{R^2}{r^2}$$

$$W_g = \int_R^{r_{max}} \left(-\frac{mgR^2}{r^2}\right)dr = \frac{mgR^2}{r}\Bigg|_R^{r_{max}} = \frac{mgR^2}{r_{max}} - mgR$$

$$\frac{mgR^2}{r_{max}} - mgR = 0 - \frac{m}{2}v_0^2$$

$$\boxed{v_0^2 = 2gR - \frac{2gR^2}{r_{max}}}$$

$$
\begin{aligned}
\frac{mgR^2}{r_{max}} - mgR &= 0 - m\frac{v_0^2}{2} \\
v_0^2 &= 2gR - \frac{2gR^2}{r_{max}}
\end{aligned}
$$

The larger that r_{max} is, the faster v_0 must be. If you want to get rid of your physics book forever, then $r_{max} = \infty$. In this case, $v_0 = \sqrt{2gR}$. Since the radius of the earth is 6.37×10^6 meters, we have $v_0 = \sqrt{2(9.8)(6.37 \times 10^6)} \approx 11174\ m/s$. This speed is called the escape velocity of the earth. Objects at the earth's surface with speeds greater than the excape velocity will never return. They will go up but never come down. I think you will have a hard time getting rid of your physics book.

Exercise 3.1

There is a giant burrito, mass 10 Kg, moving with a speed of 1 m/s, and a big olive, mass 1 Kg, moving at a speed of 6 m/s. You apply a force of 2 Newtons to both of them. Which one will stop in the shortest time? Which one will stop in the shortest distance?

Since force times distance is the change in kinetic energy, the object with the greatest kinetic energy will travel the longest distance. Since force times time is the change in momentum, the object with the greatest momentum will travel the longest time. Let's make a table of the properties of the two objects:

Object	K. E. (Joules)	Momentum (Kg-m/s)	stopping distance (m)	stopping time (s)
burrito	5	10	2.5	5
olive	18	6	9	3

The kinetic energy is calculated from $K.E. = mv^2/2$. The momentum is calculated from $momentum = mv$. The stopping distance equals $K.E./F$, and the stopping time is just $momentum/F$. The olive has more kinetic energy and requires a longer distance to stop. The burrito has more momentum and takes a longer time to stop. I'd rather get hit with the burrito since it has less kinetic energy.

Exercise 3.2

a) A bouncy ball of mass 100 grams is thrown against a wall with a speed of 2 m/s.

After hitting the wall, the ball rebounds with a speed of 2 m/s. If the ball was in contact with the wall for 0.1 seconds, what is the average force that the ball exerted on the wall?

Let the x-direction be the initial direction of the ball. The average force in the x-direction *that the ball experiences* can be calculated using $F_x(ave) = \Delta P_x/\Delta t$, where ΔP_x equals the change in momentum in the x-direction:

$$\begin{aligned} \Delta P_x &= P_x(final) - P_x(initial) \\ &= (0.1)(2)(-\hat{\imath}) - (0.1)(2)\hat{\imath} \\ \Delta P_x &= -0.4\hat{\imath}\ Kg\ m/s \end{aligned}$$

With this value for ΔP_x, the average force that the ball experiences can be calculated to be $F_x(ave) = -0.4/0.1\hat{\imath} = -4\hat{\imath}$ Newtons. The wall feels this force in the opposite direction, so the wall feels a force of $4\hat{\imath}$ Newtons.

b) Now a ball of soft puddy, also of mass 100 grams is thrown against the wall with a speed of 2 m/s. After hitting the wall, the ball sticks to it. It takes 0.2 seconds for to come to rest after hitting the wall. What is the average force that the ball exerted on the wall in this case?

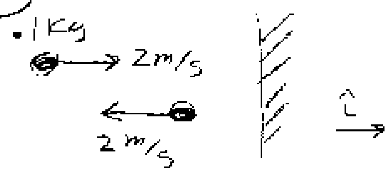
Let the x-direction be the initial direction of the ball. As before, the average force in the x-direction *that the ball experiences* can be calculated using $F_x(ave) = \Delta P_x/\Delta t$. In this case,

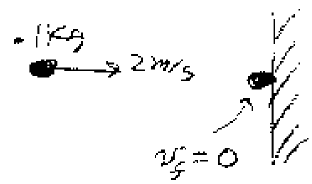
$$\begin{aligned} \Delta P_x &= P_x(final) - P_x(initial) \\ &= 0 - (0.1)(2)\hat{\imath} \\ \Delta P_x &= -0.2\hat{\imath}\ Kg\ m/s \end{aligned}$$

So the average force that the puddy feels is $F_x(ave) = -0.2/0.2\hat{\imath} = -1\hat{\imath}$ Newtons. Since the wall feels this force in the opposite direction, the wall feels a force of $1\hat{\imath}$ Newton.

Exercise 3.3
Wolfram was driving his Volkswagon, mass 500 kg, and accidentally hit a parked

(3.2)





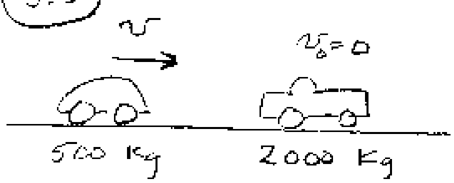
$F_{ave} = \dfrac{\Delta P}{\Delta t}$

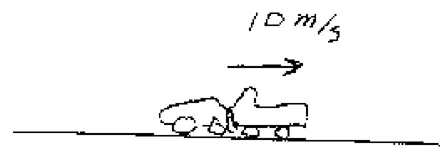
$F_{ave} = \dfrac{-(.1)(2) - (.1)2}{.1} = -4N\,\hat{i}$

$F_{ave} = \dfrac{\Delta P}{\Delta t}$

$F_{ave} = \dfrac{0 - (.1)(2)}{.2} = -1N\,\hat{i}$

(3.3)





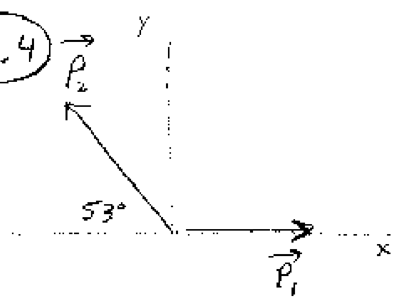
$P(\text{Before}) = 500v + 0$

$P(\text{after}) = (500 + 2000)10$

$500v = 2500(10)$

$\boxed{v = 50\ m/s}$

(3.4)



$\vec{P}_1 = 2(20)\,\hat{i} = 40\,\hat{i}$ Kg m/s

$\vec{P}_2 = 3(-10\cos 53°\,\hat{i} + 10\sin 53\,\hat{j})$

$\vec{P}_2 = (-18\,\hat{i} + 24\,\hat{j})$ Kg m/s

$\vec{P}(\text{TOTAL}) = \vec{P}_1 + \vec{P}_2$

$\boxed{\vec{P}(\text{TOTAL}) = (22\,\hat{i} + 24\,\hat{j})\ \text{Kg m/s}}$

BMW, mass 2000 Kg. After the collision, the two cars travel off together, stuck together. CSI has figured out that the speed of the two cars after the collision was 10 m/s. The speed limit is 40 m/s, and Wolfram is also charged with speeding. He has come to you to help him prove that he wasn't speeding. Can you help him?

We'll neglect friction and treat the problem as an inelastic collision between two objects. We will assume that there are no external forces, which results in the *conservation of the total momentum of the system*. That is, the total momentum before the collision is equal to the total momentum after the collision:

$$\begin{aligned} P_{total}(before) &= P_{total}(after) \\ (m_1v_1 + m_2v_2)_{before} &= (m_1v_1 + m_2v_2)_{after} \\ 500(v) + 0 &= 500(10) + 2000(10) \end{aligned}$$

We can solve this equation for v, which yields:

$$v = \frac{25000}{500} = 50\ m/s \tag{10}$$

So Wolfram was traveling 50 m/s before the collision and was therefore speeding. Sorry, we cannot help him; neither can the best lawyer. We cannot change the Laws of Nature.

Exercise 3.4

A block (block 1) of mass 2 Kg is traveling due east with a speed of 20 m/s. Another block (block 2) of mass 3 Kg is moving in a direction of 53° north of west with a speed of 10 m/s.

a) What is the total momentum of the system (i.e. the two blocks)?

The total momentum is just the **vector sum** of the individual momenta. Let's use unit vectors. The magnitude of momentum is (mass)(speed):

$$\begin{aligned} \vec{P}_1 &= 2(20)\hat{i} = 40\hat{i}Kg(m/s) \\ \vec{P}_2 &= 3(-6\hat{i} + 8\hat{j}) = (-18\hat{i} + 24\hat{j})Kg(m/s) \end{aligned}$$

The total momentum of the system is the vector sum of these two vectors:

$$
\begin{aligned}
\vec{P}_{total} &= \vec{P}_1 + \vec{P}_2 \\
&= ((40-18)\hat{i} + 24\hat{j})Kg(m/s) \\
\vec{P}_{total} &= (22\hat{i} + 24\hat{j})Kg(m/s)
\end{aligned}
$$

The purpose of this exercise is to remind us that **momentum is a vector**.

b) What is the velocity of the center of mass of the system?

The velocity of the center of mass, $\vec{v}_{cm}$ is given by:

$$
\begin{aligned}
\vec{v}_{cm} &= \frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1 + m_2} \\
&= \frac{\vec{P}_{total}}{m_1 + m_2} \\
&= \frac{22\hat{i} + 24\hat{j}}{5}\ m/s \\
&= (4.4\hat{i} + 4.8\hat{j})\ m/s
\end{aligned}
$$

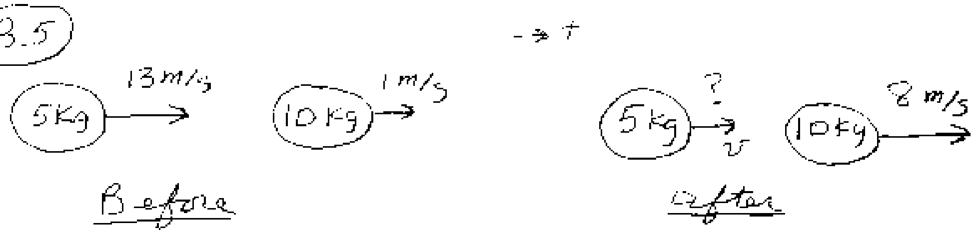
Exercise 3.5
An object of mass 5 Kg collides with an object of mass 10 Kg. Initially, the 5 Kg mass is moving to the right with a speed of 13 m/s, and the 10 Kg mass is moving to the right with a speed of 1 m/s. The 5 Kg mass catches up to the 10 Kg mass and collides with it. After the collision, the 10 Kg mass moves to the right with a speed of 8 m/s.

a) What is the final velocity of the 5 Kg mass?

Since there are no external forces, the total momentum of the system is conserved. Since all velocities are to the left or right, this is a one dimensional collision problem. If we let the "+" direction be to the right, then we have:

$$
\begin{aligned}
P_{total}(before) &= P_{total}(after) \\
5(13) + 10(1) &= 5(v) + 10(8) \\
75 &= 5v + 80 \\
v &= -1\ m/s
\end{aligned}
$$

(3.5)



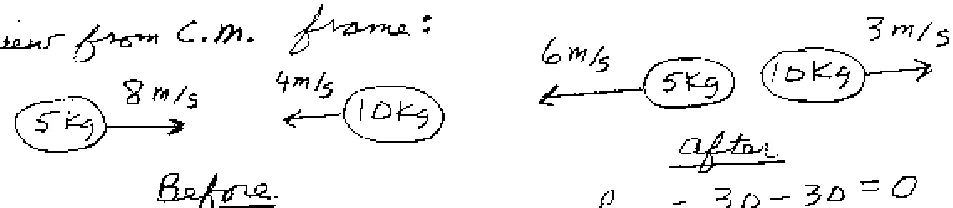
$P_{Before} = 5(13) + 10(1)$
$= 75 \text{ Kg m/s}$

$P_{after} = 5v + 10(8)$
$= 5v + 80 \text{ Kg m/s}$

$75 = 5v + 80 \Rightarrow \boxed{v = -1 \text{ m/s}}$

$v_{cm} = \frac{5(13) + 10(1)}{15} = \boxed{5 \text{ m/s}}$

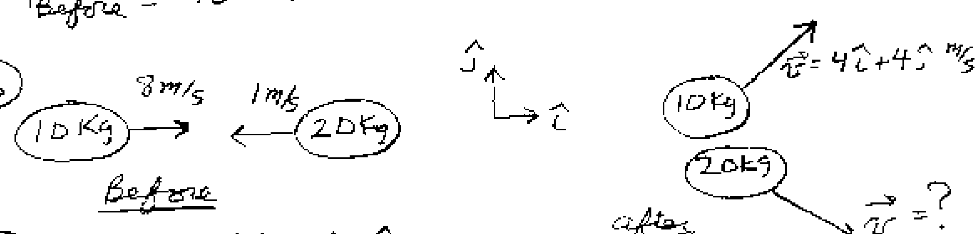
View from C.m. frame:



$P_{Before} = 40 - 40 = 0$

$P_{after} = 30 - 30 = 0$

(3.6)



$\vec{P}_{Before} = (80 - 20)\hat{i} = 60\hat{i}$

$\vec{P}_{after} = 10(4\hat{i} + 4\hat{j}) + 20\vec{v}$

$60\hat{i} = 40\hat{i} + 40\hat{j} + 20\vec{v}$

$\boxed{\vec{v} = \hat{i} - 2\hat{j} \text{ m/s}}$

The negative sign means that the 5 Kg mass is traveling in the "negative" direction, or to the left. So after the collision, the 5 Kg mass travels to the left with a speed of 1 m/s.

b) What is the velocity of the center of mass of the system?

The velocity of the center of mass, $\vec{v}_{cm}$ is given by:

$$\begin{aligned}
\vec{v}_{cm} &= \frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1 + m_2} \\
&= \frac{\vec{P}_{total}}{m_1 + m_2} \\
&= \frac{75\hat{\imath}}{15}\ m/s \\
&= 5\hat{\imath}\ m/s
\end{aligned}$$

Note that we could have used the velocities after the collision just as well, since the center-of-mass velocity doesn't change!

c) Is the collision elastic?

To determine if the collision is elastic or not, we need to calculate the kinetic energy before the collision and compare it with that after the collision. The kinetic energy before the collision is

$$K.E.(before) = \frac{5}{2}(13)^2 + \frac{10}{2}(1)^2 = 427.5\ Joules \tag{11}$$

The kinetic energy after the collision is:

$$K.E.(after) = \frac{5}{2}(-1)^2 + \frac{10}{2}(8)^2 = 322.5\ Joules \tag{12}$$

So we see that the total kinetic energy is not the same after the collision as before. $427.5 - 322.5 = 105$ Joules of kinetic energy were changed into another form of energy. Mechanical kinetic energy is not conserved, so the collision is inelastic.

d) It is interesting to consider the collision as viewed from the center-of-mass reference frame. The center of mass reference frame is one moving at the velocity of the center of mass: $5\hat{i}$ m/s. To find the velocities as observed in this reference frame, one just subtracts 5 m/s from each velocity in the other frame:

Before the Collision: The 5 Kg mass has a velocity of $8\hat{i}$ m/s, and the 10 Kg mass has a velocity of $-4\hat{i}$ m/s. The total momentum before the collision is $\vec{P}_{total} = 40\hat{i} - 40\hat{i} = 0$.

After the Collision: The 5 Kg mass has a velocity of $-6\hat{i}$ m/s, and the 10 Kg mass has a velocity of $3\hat{i}$ m/s. The total momentum after the collision is $\vec{P}_{total} = -30\hat{i} + 30\hat{i} = 0$.

In the center of mass frame, the total momentum of the system is zero before and after the collision. For the kinetic energies: $K.E.(before) = (5/2)8^2 + (10/2)4^2 = 240$ Joules. $K.E.(after) = (5/2)6^2 + (10/2)3^2 = 135$ Joules. The K.E. that was changed was $(240 - 135) = 105$ Joules as before. The actual values of momentum and kinetic energy depends on ones reference frame. However, in all reference frames (in the absence of external forces) the total momentum before the collision will equal to total momentum after the collision, and the "loss" of kinetic energy will be the same as well.

Exercise 3.6
A 10 Kg object is moving to the right with a speed of 8 m/s, and a 20 Kg object is moving to the left with a speed of 1 m/s. They collide with each other, and after the collision the 10 Kg mass travels off at 45° from its original direction. It's velocity is $\vec{v} = (4\hat{i} + 4\hat{j})$ m/s.

a) What is the final velocity of the 20 Kg mass?

Since there are no external forces, the total momentum of the system is conserved. That is, it is the same before, after, and during the collision.

$$\begin{aligned}\vec{P}_{total}(before) &= \vec{P}_{total}(after) \\ 10(8)\hat{i} - 20(1)\hat{i} &= 10(4\hat{i} + 4\hat{j}) + 20\vec{v} \\ 60\hat{i} &= 40\hat{i} + 40\hat{j} + 20\vec{v} \\ \vec{v} &= (1\hat{i} - 2\hat{j})\ m/s\end{aligned}$$

So the 20 Kg mass travels off with a speed of $v = \sqrt{1^2 + 2^2} = \sqrt{5}$ m/s, at an angle of

$\theta = tan^{-1}2 \approx 63°$ south of east.

b) Is the collision elastic?

To answer this question, we need to calculate the K.E. before and after the collision and compare. $K.E.(before) = (10/2)8^2 + (20/2)(-1)^2 = 330$ Joules. $K.E.(after) = (10/2)(4\sqrt{2})^2 + (20/2)(\sqrt{5})^2 = 210$ Joules. K.E. is not conserved, so the collision is inelastic.

c) What is the velocity of the center of mass?

The velocity of the center of mass is given by $\vec{P}_{total}/(m_{total})$, so

$$\begin{aligned}
\vec{v}_{cm} &= \frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1 + m_2} \\
&= \frac{\vec{P}_{total}}{m_1 + m_2} \\
&= \frac{60\hat{i}}{30}\ m/s \\
&= 2\hat{i}\ m/s
\end{aligned}$$

We would get the same result if we would have used the velocities after the collision.

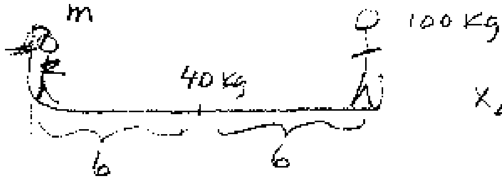
Exercise 3.7

Jane and Austin are sitting in a canoe. Austin wants to know Jane's mass, but she won't tell him. He tricks her into switching positions with him in the canoe. After the switch, he notices that the canoe moves 2 feet. He knows that his mass is 100 Kg. If the canoe is 12 feet long and has a mass of 40 Kg, what is her mass?

We will assume there are no external forces on the system. If this is true, then the velocity of the center of mass is constant. Since the center of mass velocity is initially zero, it will remain zero. Thus, the **the position of the center of mass** will remain the same after Jane and Austin exchange places. Setting up our coordinate system as shown in the figure, we have before the exchange:

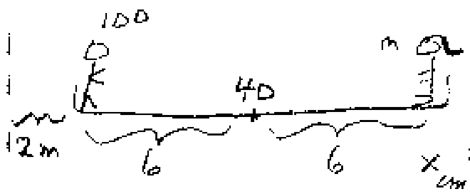
$$x_{cm} = \frac{m(0) + 40(6) + 100(12)}{m + 40 + 100}$$

(3.7)



$$x_{cm} = \frac{m(0) + 40(6) + 100(12)}{m + 40 + 100}$$

$$x_{cm} = \frac{1440}{m + 140}$$



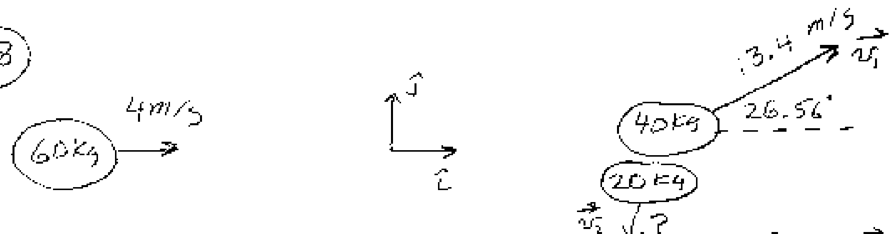
$$x_{cm} = \frac{100(2) + 40(8) + m(14)}{m + 40 + 100}$$

$$x_{cm} = \frac{520 + 14m}{m + 140}$$

$$520 + 14m = 1440$$

$$\boxed{m \approx 65 \text{ Kg}}$$

(3.8)



$\vec{P}_{TOT}(\text{Before}) = 240 \text{ Kg m/s } \hat{\imath}$

$\vec{P}_{TOT}(\text{after}) = m_1\vec{v}_1 + m_2\vec{v}_2$

$\vec{v}_1 = 13.4 \cos 26° \,\hat{\imath} + 13.4 \sin 26° \,\hat{\jmath}$ m/s

$\vec{v}_1 = 12\hat{\imath} + 6\hat{\jmath}$

$$240\hat{\imath} = 40(12\hat{\imath} + 6\hat{\jmath}) + 20\vec{v}_2$$

$$20\vec{v}_2 = (240 - 40(12))\,\hat{\imath} - 40(6)\,\hat{\jmath}$$

$$\boxed{\vec{v}_2 = (-12\hat{\imath} - 12\hat{\jmath}) \text{ m/s}}$$

$|\vec{v}_2| \approx 16.97$ m/s at 45° S of W

$$= \frac{1440}{m + 140}$$

After the exchange, the canoe has moved 2 feet to the right. Now we have

$$\begin{aligned} x_{cm} &= \frac{100(2) + 40(8) + m(14)}{m + 40 + 100} \\ &= \frac{520 + 14m}{m + 140} \end{aligned}$$

Equating the two x_{cm}, we have

$$\begin{aligned} 520 + 14m &= 1440 \\ m &\approx 65\ Kg \end{aligned}$$

Moral of the story: Beware of Physics Majors.

Exercise 3.8
A 60 Kg bomb is initially moving to the right with a speed of 4 m/s. It explodes into two pieces, one of mass 40 Kg, and the other of mass 20 Kg. After the explosion, the 40 Kg piece flies away with a speed of 13.4 m/s at an angle of 26.56° from the original direction as shown in the figure. What is the velocity of the 20 Kg piece?

Since there are no external forces in this problem, we will assume that **the total momentum of the system is the same before as after the explosion**. Choosing $\hat{\imath}$ to point to the right (east), the initial momentum is $\vec{P}_{tot}(initial) = 4(60) = 240\hat{\imath}$ Kg(m/s). In terms of the unit vectors shown in the figure, the final velocity of the 40 Kg mass is $\vec{v}_1 = 13.4cos(26.56°)\hat{\imath} + 13.4sin(26.56°)\hat{\jmath} \approx (12\hat{\imath} + 6\hat{\jmath})$ m/s. Equating the total momentum before the explosion with that after the explosion:

$$\begin{aligned} 240\hat{\imath} &= 40(12\hat{\imath} + 6\hat{\jmath}) + 20\vec{v}_2 \\ 240\hat{\imath} &= 480\hat{\imath} + 240\hat{\jmath} + 20\vec{v}_2 \\ \vec{v}_2 &= (-12\hat{\imath} - 12\hat{\jmath})\ m/s \end{aligned}$$

So the speed of the 20 Kg mass is $|\vec{v}_2| = \sqrt{12^2 + 12^2} \approx 17$ m/s. The direction of $\vec{v}_2$ is 45° south of west.

b) How much kinetic energy is released in the explosion?
The kinetic energy released is the difference in the final kinetic energy and the initial kinetic energy. The initial kinetic energy is:

$$K.E.(initial) = (60/2)4^2 = 480\ Joules$$

The final kinetic energy is:

$$K.E.(final) = (40/2)13.4^2 + (20/2)16.97^2 \approx 6471\ Joules$$

Therefore, $6471 - 480 = 5991$ Joules were released in the explosion.

Exercise 3.9

Jane thinks that Tarzan has been gaining too much mass, however, he doesn't want to tell her what he weighs. One day Tarzan is swinging in the forest. He starts (from rest) from one side at an angle of 53° from the vertical. He swings down and grabs Jane at the bottom of the swing. They swing together on the other side up to a maximum angle of 37°. Jane knows that her mass is 50 Kg. What is Tarzans's mass?

To solve this problem, we need to apply momentum conservation during the collision, and the work-energy theorem during the swinging down and up parts. Let's first consider the collision at the bottom of the swing. Let m_T be Tarzan's mass, and m_J be Jane's mass. Let the initial speed of Tarzan before the collision be v_T, and the speed after the collision be v_f. Since there are no external forces during the collision, the total momentum is the same before as after the collision:

$$m_T v_T = (m_T + m_J) v_f \tag{13}$$

After the collision they both have the same velocity, v_f. We need to relate the speeds at the bottom with the maximum angle that they swing up to. Using the Work Energy theorem for Tarzan's initial swing down to Jane:

$$\begin{aligned}
Net\ Work &= change\ in\ Kinetic\ Energy \\
W_g + W_T &= \frac{m}{2} v_T^2 - 0 \\
mg(l - lcos(53°)) &= \frac{m}{2} v_T^2 \\
mgl(0.4) &= \frac{m}{2} v_T^2 \\
v_T &= \sqrt{2gl(0.4)}
\end{aligned}$$

where W_T is the work done by the tension in the rope, which equals zero. Here we have used $cos 53° \approx 0.6$. After the collision, they both swing up together to a maximum angle of 37°. Using the work energy theorem for this motion:

$$\begin{aligned}
Net\,Work &= change\ in\ Kinetic\ Energy \\
W_g + W_T &= 0 - \frac{m}{2}v_f^2 \\
-mg(l - lcos(37°)) &= -\frac{m}{2}v_f^2 \\
mgl(0.2) &= \frac{m}{2}v_f^2 \\
v_f &= \sqrt{2gl(0.2)}
\end{aligned}$$

Substituting these two speeds into the momentum conservation equation yields:

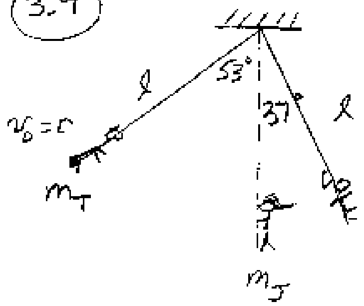
$$\begin{aligned}
m_T v_T &= (m_T + m_J)v_f \\
m_T\sqrt{2gl(0.4)} &= (m_T + m_J)\sqrt{2gl(0.2)} \\
m_T &= \frac{m_J}{\sqrt{2} - 1} \approx 121\ Kg
\end{aligned}$$

Note that the length of the rope and g cancel out in the equation. 121 Kg is around 265 pounds. Tarzan needs to exercise more.

Exercise 3.10
A block of mass m_0 starts from rest on a frictionless incline. It is initially a height h_0 above the ground. The block slides down the incline and collides with another block of mass $4m_0$. After the collision, the $4m_0$ block travels forward up an incline plane, and the m_0 block slides backwards up the incline from which it started. They both slide up their inclines to the same height h above the ground. All surfaces are frictionless. Determine what the height h is in terms of h_0.

This problem is similar to the Tarzan problem, except that the objects do not stick together after the collision. As before, we can analyze the problem in two parts: the collision, and the sliding up and down the inclines. First the collision. Let v_0 be the velocity of the m_0 block just before the collision. Since both blocks rise to the same height h after the collision, they must have the same speed v after the collision. Since there are no external forces, the total momentum of the system is conserved:

(3.9)



$v_T = \sqrt{2g\ell(1-\cos 53°)}$

$v_T = \sqrt{2g\ell(.4)}$

Before collision

$v_f = \sqrt{2g\ell(1-\cos 37°)}$

$v_f = \sqrt{2g\ell(.2)}$

after collision

momentum is conserved in the collision:

$$m_T v_T = (m_T + m_J) v_f$$

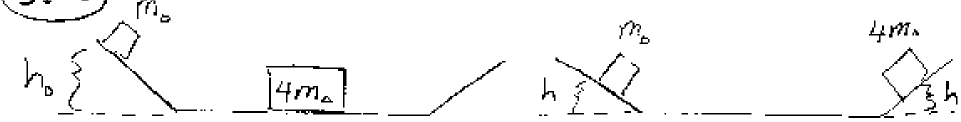
$$m_T \sqrt{2g\ell(.4)} = (m_T + m_J)\sqrt{2g\ell(.2)}$$

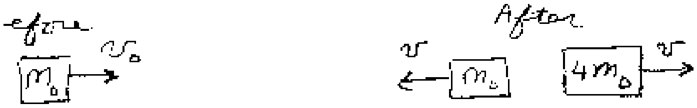
$$m_T \sqrt{2} = m_T + m_J$$

$$m_T = \frac{m_J}{\sqrt{2}-1} = \frac{50\text{ kg}}{.41} \simeq \boxed{121\text{ Kg}}$$

or 265 lbs

(3.10)





$$m_0 v_0 = m_0(-v) + 4m_0 v$$

$$\boxed{v = v_0/3}$$

$mgh = \frac{mv^2}{2}$

$h = \frac{v^2}{2g}$

$mgh_0 = \frac{m}{2}v_0^2$

$h_0 = \frac{v_0^2}{2g}$

$$\frac{h}{h_0} = \frac{v^2/2g}{v_0^2/2g} = \frac{v^2}{v_0^2} = \left(\frac{1}{3}\right)^2 = \boxed{\frac{1}{9}} \quad \boxed{h = \frac{h_0}{9}}$$

$$
\begin{aligned}
\vec{P}_{total}(before) &= \vec{P}_{total}(after) \\
m_0 v_0 &= m_0(-v) + 4m_0 v \\
v &= \frac{v_0}{3}
\end{aligned}
$$

The block m_0's velocity is negative because it is moving in the opposite direction as the $4m_0$ block. Since there is no friction, the only force that does work is gravity. Thus, we have $mgh = (m/2)v^2$, or $h = v^2/(2g)$ both before and after the collision:

$$
\begin{aligned}
h &= \frac{v^2}{2g} \\
&and \\
h_0 &= \frac{v_0^2}{2g}
\end{aligned}
$$

Taking the ratio of these equations gives

$$
\begin{aligned}
\frac{h}{h_0} &= \frac{v^2}{v_0^2} \\
&or \\
h &= \frac{h_0}{9}
\end{aligned}
$$

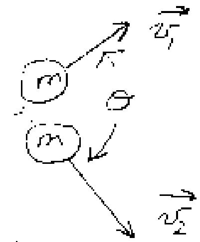
since $v = v_0/3$.

Exercise 3.11

Consider the collision on a pool table between the cue ball and another ball. Initially the cue ball has a velocity $\vec{v}_0$, and the other ball is at rest. After the collision, the cue ball travels off with a velocity $\vec{v}_1$ and the other ball with a velocity $\vec{v}_2$. Both balls have a mass m. If we assume that the collision is elastic, what is the angle between the recoiling pool balls, i.e. the angle θ in the figure?

To solve the problem, we need to assume that the collision is elastic, and the balls just slide on the table top. Since there are no external forces, the total momentum of the system is the same before as after the collision:

(3.11)

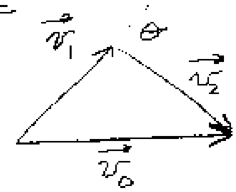


Before

after

$m\vec{v}_0 = m\vec{v}_1 + m\vec{v}_2$ } momentum conservation ⇒

$\vec{v}_0 = \vec{v}_1 + \vec{v}_2$



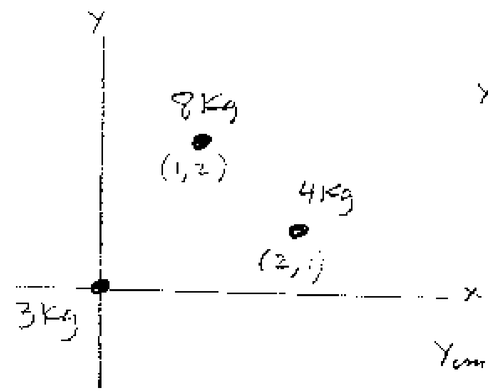
Law of cosines $v_0^2 = v_1^2 + v_2^2 + 2v_1v_2\cos\theta$

K.E. conservation: $\frac{m}{2}v_0^2 = \frac{m}{2}v_1^2 + \frac{m}{2}v_2^2 \Rightarrow v_0^2 = v_1^2 + v_2^2$

$0 = 2v_1v_2\cos\theta$

$\boxed{\theta = 90^\circ}$

(3.12)



$$X_{cm} = \frac{3(0) + 8(1) + 4(2)}{3 + 8 + 4}$$

$$\boxed{X_{cm} = \frac{16}{15}}$$

$$Y_{cm} = \frac{3(0) + 8(2) + 4(1)}{3 + 8 + 4}$$

$$\boxed{Y_{cm} = \frac{20}{15} = \frac{4}{3}}$$

$$\begin{aligned}\vec{P}_{total}(before) &= \vec{P}_{total}(after)\\ m\vec{v}_0 &= m\vec{v}_1 + m\vec{v}_2\\ \vec{v}_0 &= \vec{v}_1 + \vec{v}_2\end{aligned}$$

Note that these three velocity vectors form a triangle as shown in the figure. Applying the law of cosines for this triangle,

$$v_0^2 = v_1^2 + v_2^2 + 2v_1v_2cos\theta \tag{14}$$

The plus sign on the right side of the equation is because θ is on the outside of the triangle. To proceed further, we need to either know one of the final velocities or the kinetic energy "lost" in the collision. If we assume that the collision is elastic, then there is no kinetic energy loss. That is, kinetic energy is conserved:

$$\begin{aligned}K.E.(before) &= K.E.(after)\\ \frac{m}{2}v_0^2 &= \frac{m}{2}v_1^2 + \frac{m}{2}v_2^2\\ v_0^2 &= v_1^2 + v_2^2\end{aligned}$$

Substituting v_0^2 into the momentum conservation equation gives us

$$\begin{aligned}v_1^2 + v_2^2 &= v_1^2 + v_2^2 + 2v_1v_2cos\theta\\ cos\theta &= 0\\ \theta &= 90^\circ\end{aligned}$$

Experienced pool players know that the cue ball travels off 90° from the direction of the ball that was hit. Now you know the physics of why this is true. Hope your pool game improves.

Exercise 3.12
Find the center of mass for the mass distribution shown in the figure.

We just need to apply the formula for the center of mass:

$$\vec{r}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3}{m_1 + m_2 + m_3} \tag{15}$$

This is a vector equation. We can each component separately. First the x components:

$$\begin{aligned}
x_{cm} &= \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3} \\
x_{cm} &= \frac{3(0) + 8(1) + 4(2)}{3 + 8 + 4} \\
x_{cm} &= \frac{16}{15}
\end{aligned}$$

Then the y components:

$$\begin{aligned}
y_{cm} &= \frac{m_1y_1 + m_2y_2 + m_3y_3}{m_1 + m_2 + m_3} \\
y_{cm} &= \frac{3(0) + 8(2) + 4(1)}{3 + 8 + 4} \\
y_{cm} &= \frac{4}{3}
\end{aligned}$$

Exercise 3.13
Where is the center-of-mass of the bent wire shown in the figure? Each side has a mass m and a length l.

The center of mass of each side is at the center of the side. So the bent wire has the same center of mass as the three masses in the figure next to it: a mass m at $(0, l/2)$, a mass m at $(l/2, l)$, and a mass m at $(l, l/2)$. The x coordinate of the center of mass is

$$\begin{aligned}
x_{cm} &= \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3} \\
x_{cm} &= \frac{m(0) + m(l/2) + m(l)}{3m} \\
x_{cm} &= \frac{l}{2}
\end{aligned}$$

The y coordinate of the center of mass is

$$\begin{aligned} y_{cm} &= \frac{m_1 y_1 + m_2 y_2 + m_3 y_3}{m_1 + m_2 + m_3} \\ y_{cm} &= \frac{m(l/2) + m(l) + m(l/2)}{3m} \\ y_{cm} &= \frac{2}{3} l \end{aligned}$$

Exercise 3.14
One way to measure the speed of a bullet is to use a "ballistic pendulum". The ballistic pendulum consists of a block of wood suspended by a string. The bullet is shot into the wood. The wood and bullet swing up to a maximum height h. Let m be the mass of the bullet, and M be the mass of the wood. What is the speed v_0 of the bullet in terms of h, m, M and g?

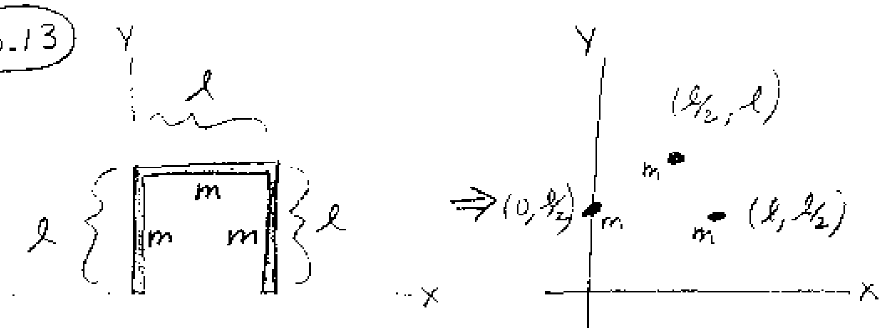
This problem is best solved in two steps. First the collision between the bullet and the block. The second part is the swinging up of the block and the bullet. Since there are no external forces during the collision, momentum is conserved when the bullet collides with the block:

$$\begin{aligned} \vec{P}_{total}(before) &= \vec{P}_{total}(after) \\ m v_0 &= (M + m) v \\ v &= \frac{m}{M + m} v_0 \end{aligned}$$

where v is the speed of the block (plus bullet) just after the collision. To express the height h in terms of v, we can use the work-energy theorem. As the block (plus bullet) swings up, two forces are acting: the force of gravity and the tension in the string.

$$\begin{aligned} W_g + W_T &= \textit{change in kinetic energy} \\ -(M + m) g h + 0 &= 0 - \frac{M + m}{2} v^2 \\ h &= \frac{v^2}{2g} \end{aligned}$$

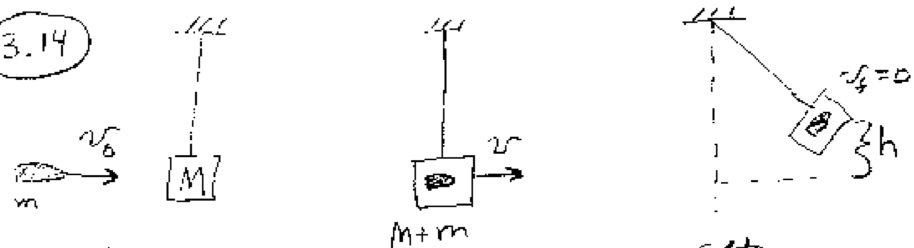
(3.13)



$$X_{cm} = \frac{m(0) + m\left(\frac{\ell}{2}\right) + m\ell}{3m} = \boxed{\frac{\ell}{2}}$$

$$Y_{cm} = \frac{m\left(\frac{\ell}{2}\right) + m\ell + m\left(\frac{\ell}{2}\right)}{3m} = \boxed{\frac{2}{3}\ell}$$

(3.14)



Before   after

$$m v_0 = (M+m) v \qquad W_g + W_T = 0 - \frac{(M+m)}{2} v^2$$

$$v = \left(\frac{m}{M+m}\right) v_0 \qquad -(M+m) g h + 0 = -\frac{(M+m)}{2} v^2$$

$$h = \frac{v^2}{2g}$$

$$h = \left(\frac{m}{M+m}\right)^2 \frac{v_0^2}{2g}$$

$$\text{or} \quad \boxed{v_0 = \sqrt{2gh}\left(\frac{M+m}{m}\right)}$$

The work W_T done by the tension in the string is zero, since the direction of this force is perpendicular to the direction of the block's velocity. The work W_g done by gravity is negative since the block moves upward in the opposite direction of the gravitational force. Substituting for v in the second equation gives

$$\begin{aligned} h &= (\frac{m}{M+m})^2 \frac{v_0^2}{2g} \\ &\textit{or} \\ v_0 &= \sqrt{2gh}\frac{M+m}{m} \end{aligned}$$

By measuring h, M, and m, one can determine the speed of the bullet.