Having Fun with Error Analysis

Peter Siegel, California State Polytechnic University, Pomona, Pomona, CA

We present a fun activity that can be used to introduce students to error analysis: the M&M game. Students are told to estimate the number of individual candies plus uncertainty in a bag of M&M's. The winner is the group whose estimate brackets the actual number with the smallest uncertainty. The exercise produces enthusiastic discussions and serves as a good "mixer" for the first day of a laboratory class.

Estimating uncertainties in measurements and propagation of uncertainties are important laboratory skills that should be taught to all students of physics at some level.[1] However, for the most part, students are more interested in the value of the measurement itself than in its uncertainty. They tend to view error analysis as tedious busy work, which takes away from the enjoyment of the experiment. This is unfortunate but understandable since in undergraduate laboratory experiments, uncertainties usually seem neither particularly relevant nor exciting to determine. To best motivate the students, activities should be designed for which the uncertainty is as important as the value of the measurement itself.

We would like to share a fun activity that has been quite successful in getting our incoming physics students to think about uncertainties. On the first day of the laboratory class, we play the "M&M game."[2] The goal is to estimate the number of candies in a particular bag of plain M&M's. We divide the students into groups of two to four members. Each team needs to come up with a number and an uncertainty for the number of candies in the bag, for example 101 ± 4. Alternatively, the students can decide on a range for the number of candies. To make their estimates, the only tool the students are allowed to use is a balance. Each group is given an unopened bag of M&M's, an empty wrapper of the same size, and five to 10 loose M&M candies. After everyone has made their estimates with uncertainties, we count the actual number in the bag. The winner is the team with the smallest uncertainty (or range) whose estimate brackets the actual number. If there is a tie under these rules, the group closest to the actual number wins. The prize is a new bag of M&M's for each member of the winning team.

This exercise leads to a lot of discussion within each team. Most students agree that the best way to obtain *N,* the number candies in the bag, is by the following formula: $N = (M_{bag} - M_{wrapper})/m$, where M_{bag} is the mass of the bag, $M_{wrapper}$ is the mass of the wrapper, and m is the average mass of an individual candy. Most groups (but not all) will determine an average value for m by finding the mass of all 10 candies and dividing by 10.

The real decision making is over what value to choose for the uncertainty ΔN. If ΔN is very large, the true number will most likely fall into their range, but they will probably not win since other groups might have a smaller ΔN. If ΔN is too small (i.e., 1) then their chances of having the true number fall with their range is small. Thus, the chance of winning is reduced if ΔN is too large or too small, and it is critical to choose an appropriate value for ΔN. Since there is a prize to be won, the students are motivated to choose

DOI: 10.1119/1.2715421

the best value for ΔN. During the discussion phase of the exercise, we usually suggest that they choose ΔN such that they might bracket the actual number 68% of the time. However, even with this advice some groups decide to take a chance and choose ΔN very small.

When the students are finished with their analysis, they write their estimates on the board. Some estimates from a recent class of engineering/science students are: 103 ± 2, 104 ± 1, 105 ± 1, 106 ± 2, 107 ± 3, and 109 ± 6. At this stage, we often have a classroom discussion on how each team decided on their uncertainties. After hearing each group's comments, the instructor can describe a "textbook" method, which we present in the next paragraph. Finally, the bag is opened, the excitement mounts as the candies are counted, and the winner(s) are determined amidst cheers. For this class, there were 107 M&M's in the bag. The winning group was the one that estimated 106 with an uncertainty of 2 since they bracketed the value with the smallest uncertainty. Although one group's estimate was 107, the true value, this group did not win the contest because their uncertainty was larger, namely 3. However, they also received a bag of M&M's as a prize for their accurate measurements.

After the groups have reported on how they obtained their errors, a "textbook" method for the determination of the uncertainty is explained as follows. The number N is given by: $N = M/m = (M_{bag} - M_{wrapper})/m$. We need to estimate the uncertainty for each of the three measurements and propagate the errors. The absolute uncertainty in M is the sum of the absolute uncertainties in M_{bag} and $M_{wrapper}$, i.e., $\Delta M = \Delta M_{bag} + \Delta M_{wrapper}$. The relative uncertainty in N is the sum of the relative uncertainties in M and m, i.e., $\Delta N/N = \Delta M/M + \Delta m/m$. Using our balance, whose smallest division on the scale is 0.1g, the engineering students first estimated an instrument uncertainty of 0.1 g for a single mass measurement:[1] $M_{bag} = (90.2 \pm 0.1)$ g and $M_{wrapper} = (2.2 \pm 0.1)$ g. This gives $M = (88.0 \pm 0.2)$ g, or a 0.25% error in M. One group determined the mass of 10 candies to be (8.5 ± 0.1) g, giving a 1.2% percent error in the mass of 10 M&M's. The same relative error applies to a single candy; thus, the largest percent uncertainty is in m. Adding the two percent errors results in a 1.45% error or ± 1.5 in N. Since N for the different groups varied by more than one M&M, students felt that this error estimate was too small. Realizing there may be systematic errors, they decided to use 0.15 g instead of the smallest scale increment, which gives a 1.8% error in m and a 0.4% error in M. After some discussion, the class as a whole was comfortable with a 2.2% error or an uncertainty in N of 2 or 3.

We next calculate the class average and standard deviation of the six values of N, which is 105.7 ± 2.0 for this class. Finally, we briefly discuss some possible systematic errors. Some students remark that 10 candies are not necessarily a good representation of the whole batch. This is an opportunity to discuss the difference between instrument uncertainty and sample uncertainty in estimating m. In this case, the sample uncertainty is caused by the variation in mass of each M&M. The uncertainty is statistical and results from using a small random sample to predict the properties of a larger one, a practice done in opinion polls. Another systematic uncertainty might be due to the fact that we are all using the same type of balance. The final consensus is that we would use an uncertainty of 2 or 3 if we were to publish our results.

There are many variations of this activity. To increase the uncertainty, peanut M&M's and/or a larger bag of candies can be used. Individual peanut M&M's have a larger variation than plain ones. We have tried M&M bags with as many as 460 candies inside, and the students enjoyed the challenge of estimating this large number. Giving the students a smaller number of individual candies also increases the error in measuring N. We tell the students that skill plays a big role in winning the prize. The group that makes the most accurate measurements of M_{bag}, $M_{wrapper}$, and m has the best chance of winning. A correct estimate of ΔN will also improve their chance of winning. To reward more students, we usually give a prize to the group who comes closest to the actual number.

Although the candy exercise does not pertain to a physical phenomenon, it is a simple scientific problem in which the students can use skills they already possess, e.g., measuring and logical reasoning. The instructor can give the students little or no initial guidance and instead let them come up with their own method to obtain the solution. The goal is to measure a definite integer value that will be determined at the end of the exercise, rather than being compared to a value from the literature. The answer is unknown to everyone and is found at the end by opening the bag. The nicest aspect of the problem is that it leads to a lot of classroom discussion in which the students can share their ideas in estimating the uncertainty. We use this activity on the first day of lab, where it also serves as a "mixer" and as a sweet way to show that error analysis can be fun.

References

1. Saalih Allie et al., "Teaching measurement in the introductory physics laboratory," *Phys. Teach*. **41**, 394–401 (Oct. 2003).
2. Candy can also be used for statistical activities; see Chuck Stone, "'Sweetening' technical physics with Hershey's Kisses," *Phys. Teach*. **41**, 234–237 (April 2003).

PACS codes: 01.50.Pa, 06.00.00

Peter Siegel *is a professor of physics at Cal Poly Pomona. His areas of interest include medium energy physics, nonlinear dynamics, and physics education.*

Physics Department, California State Polytechnic University, Pomona, Pomona, CA; pbsiegel@csupomona.edu

etcetera...

etcetera... Editor *Albert A. Bartlett*
Department of Physics
University of Colorado
Boulder, CO 80309-0390

Many Kinds of Eyes: The Eyes Have It

"In the animal kingdom, there are diverse types of camera eyes. For example, fish eyes have a spherical gradient index lens. The bird eye has the added control of reshaping and deforming the cornea as well. Brucke's muscles attached to bony ossicles in reptiles and birds actively change the lens thickness. Birds have an additional muscle, Crampton's muscle, which can alter the shape of the cornea. In contrast, the whale eye uses hydraulics to move the lens itself closer or farther from the retina; a chamber behind the lens is filled or emptied with fluid depending on the focal length needed. This design allows for good vision in and out of the water, and compensates for the increased pressure in deeper aquatic environments. The protractor lentis in some amphibian eyes moves a fixed-shape lens closer or farther from the retina for accommodation."[1]

1. L.P. Lee and R. Stevens, "Inspirations from Biological Optics for Advanced Photonic Systems," *Sci.* **310**, 1148–1150 (Nov. 18, 2005).