one needs to know whether $\Phi^{\times}$ is too big for the system under considera-tion. Work on this problem of "tight rigging" is still in progress. Our results can be extended to more complicated quantum mechanical systems with spectrum-generating algebra given by the noncompact SU(2, 2) {isomorphic to SO(4, 2)} and the Schrodinger group[5].

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MAXIMIZING THE SPINNING TIME OF A KELANTAN TOP

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Summary - various goals for a student project are discussed. A research project involving the analysis of a spinning top is presented which includes finding the equation of motion for a flat disc top spinning vertically in air and consideration for maximizing the spinning time for a particular thrower.

1. Introduction

It if often supposed that special or expensive equipment is needed to carry out a worthwhile Physics project, when in fact there are many interesting applications of Physics around which are often overlooked due to their relative simplicity. Experiments which can be related to everyday experience can help to stimulate an interest in Physics and make the subject less abstract and more practical. The research project examined in this paper deals with an old Malaysian cultural activity from the state of Kelantan. Quite simply, this activity consists of contests which are held to see who can spin a top the longest. The project is as follows: a) analyse the motion of a flat disc top spinning in air (equation of motion), and (b) find a method for maxi-mizing the spinning time by considering changes in the size and weight of the top.

In order to evaluate the worth of a student project or experiment, the important aspects or goals need to be stated. Some of the important things that a student can learn from a project are as follows:-

I) The ability to come up with a general approach to a problem, and deciding which parameters are to be measured.

II) The ability to design a method for measuring the desired quantities with the least experimental error using the available equipment.

III) The ability to derive a model or theory to explain the experimental results.

Even though these same procedures are used in advanced research, advanced equipment is not needed to instruct a student in these techniques. These ideas can be developed with a simple project that is geared to the interest and level of the student. The following Kelantan Top Project is such an example.

2. Analysis of Motion

2(I) General Approach

The long spinning Kelantan top, called "Gasing Uri", is a flat disc which spins on a short spike of hard steel as shown in Fig. 1. Although the size and weight of the top may vary, the basic shape does not. Also, since the top spins vertically for up to 1½ hours (sleeping top), and only spends 2-3 minutes precessing before it dies, it will suffice to limit the analysis to the vertical spinning motion of the top and describe its motion in terms of its angular velocity, ω. The torque, τ, is along the axis of rotation, so the equation of motion is:

$$I\frac{d\omega}{dt} = \tau(m, a, \omega)$$

where I is moment of inertia, m is the mass, and a is the radius of the top. Thus an understanding of the torque, $\tau(m, a, \omega)$, in terms of mass, radius, and angular velocity will provide us with an equation of motion for any flat disc top. $\tau(m, a, \omega)$ can be found by analysing $I\, d\omega/dt$ for various tops, and this is the approach used.

2(II) Measurement and Data

In order to measure $\omega(t)$ as accurately as possible with the available equipment, a stroboscope and a pulse counter were used. With the stroboscope set at a fixed frequency, the time was recorded when the top appeared stationary, then the number of light pulses from the stroboscope was counted for a 60 second interval. After the counting was completed, the strobe was reset to find ω at a different time. With this method, the error involved in the measurement of ω was 1% and of the time, 1 second.

Four different sized tops were used in the experiment. Their specifications are listed in table I. The tops were initially spun by hand, and sped up with the aid of an electric motor. $\omega(t)$ was then measured on various surfaces, an example of which is shown in Fig. 2.

2(III) Equation of Motion

A satisfactory model for the torque can be found by considering the total torque as the sum of a term due to the surface friction at the contact point, τ_s, and a term due to air frictional drag, τ_a. It is also plausible to assume that τ_s varies negligibly with ω and is only a function of the top's mass, and that τ_a depends only on ω and a:

$$\tau(m, a, \omega) = \tau_s(m) + \tau_a(a, \omega) \qquad \ldots\ldots (1)$$

The two torque contributions can be determined by graphing I $d\omega/dt$ vs. ω as shown in Fig. 3. The steel spike is made of the same material for all the tops, and was filed to the same sharpness, so the graph of τ_s vs m in Fig. 4 will give a rough idea of $\tau_s(m)$. From these results, it is reasonable to assume $\tau_s \propto m$, or $\tau_s = \mu m$. However, this simple relationship does not hold if the top penetrates the spinning surface. In that case, τ_s is much larger and will not be constant throughout the spinning time, but will increase with penetration depth. Thus for consistent results it is necessary that the spinning surface be harder than the hard steel tip. The smallest value for μ obtained from readily available materials was 0.5 dyne-cm/gm for a high speed carbon steel spinning surface.

The dependence of τ_a on ω and a can be derived[1] from the Navier-Stokes equations, the continuity equation, and the boundary conditions of a rotating flat disc imposed, with the results:

$$\tau_a = (0.616)\pi\rho\nu^{\frac{1}{2}}a^4\omega^{3/2} \qquad \ldots\ldots (2)$$

where ρ is the density of the fluid and ν is the kinematic viscosity. This theoretical prediction is verified by plotting $\ln\tau_a$ vs $\ln\omega$ as shown in Fig. 5. The linear relationship, with a slope of 3/2, confirms $\tau_a \propto \omega^{3/2}$. Finally, the a^4 dependence is examined by computing $\tau_a/(a^4\omega^{3/2})$ as done in table II. Thus, from the above results, an approximate expression for the torque on a disc shaped top (Gasing Uri) can be written as:

$$\tau = -\mu m - 10^{-3}a^4\omega^{3/2} = I\frac{d\omega}{dt} \qquad \ldots\ldots (3)$$

This relationship has been verified experimentally for values of $\omega a^2 < 10^4$, and for spinning surfaces of hard steel as used with the Gasing Uri. The spinning time of the top can now be solved from equation 3 to give:

$$t = \frac{33.3I}{a^4D^{1/3}}\left[\frac{1}{6}\ln\frac{D+\omega^{3/2}}{(D^{1/3}+\omega^{\frac{1}{2}})^3} + \frac{1}{\sqrt{3}}\tan^{-1}\frac{2\omega^{\frac{1}{2}}-D^{1/3}}{D^{1/3}\sqrt{3}}\right]_{\omega_f}^{\omega_o} \qquad \ldots\ldots (4)$$

where t is the spinning time in minutes, $D = 10^3\mu m/a^4$, ω_o is the initial angular velocity, and ω_f is the final angular velocity. Thus, the spinning time can be computed once m, a, I, μ, ω_o, and ω_f are known. The computed time agrees quite well with the experimental results as shown in table III.

3. Maximizing the Spinning Time

In the last section, the equation of motion for a flat disc spinning top was determined, and an expression was derived to predict the spinning time once the relevant parameters were known. The next problem to consider is how to choose the proper sized top for a thrower so that the spinning time is maximized. To do this analysis properly, knowledge of $\omega_0(m, a, I)$ is necessary. That is, how fast can the thrower spin different tops of various sizes and weights. With this information, the optimum specifications to maximize the spinning time can be calculated. However, the function $\omega_0(m, a, I)$ varies from person to person, as well as depending on personal factors within the individual thrower, and thus involves many variables which are not necessarily consistent. Hence, an analysis of this complicated function is more a problem of Kinesiology, not Physics. Nonetheless, it is an interesting excercise to examine $\omega_0(m, a, I)$ for various tops and a particular thrower on a particular day. The services of an experienced and consistent top thrower was obtained to help us with our experiment. The results are listed in table IV.

The results show that the rotational energy which the thrower can impart to the top is roughly the same for tops of equal radii. This is the case for the tops of radii 9.5 cm and 9.0 cm as shown. Thus a plausible relationship is that within certain ranges of mass and radius, the energy imparted to the top depends only on the radius. The present rules of the top spinning contests require a top in the competition to have a radius of 9.5 cm, so an approach to maximizing the spinning time is to consider the initial energy as a constant. With this constraint, plus limiting the top to wood and lead construction with the lead filling the outer region of the disc, the spinning time as a function of the top's mass can be computed. The results for an initial energy of 260 Joules are listed in table V. It can be seen here that the best top for this thrower to choose is one of mass 3000 gms. However, since this is the maximum of a very gentle peak, and since a lot of other human factors have been discarded, the above constant energy analysis can only indicate that a top of proper mass is somewhere in the range of 2500 gms to 3500 gms.

The top that lasted the longest for this thrower was of mass 3200 gms and spun for approximately 90 minutes before falling. The above considerations then suggest that the East Coast Malaysians have, through trial and error, designed the longest spinning flat disc top with the available materials and human energy.

4. Acknowledgement

The authors wish to thank thrower Abdul Rahman (Pahang State Champion), scooper Wan Omar, and winder Mohamad Chik from Kampung Cherating Dalam Kuantan, for the use of their expert skills at top throwing.

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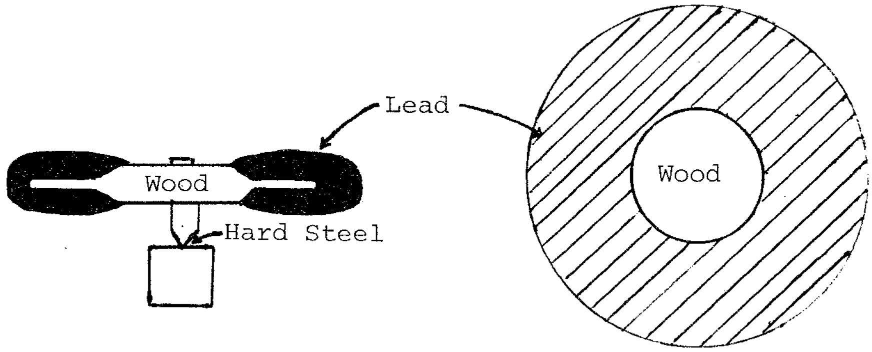

Figure 1 - Gasing Uri

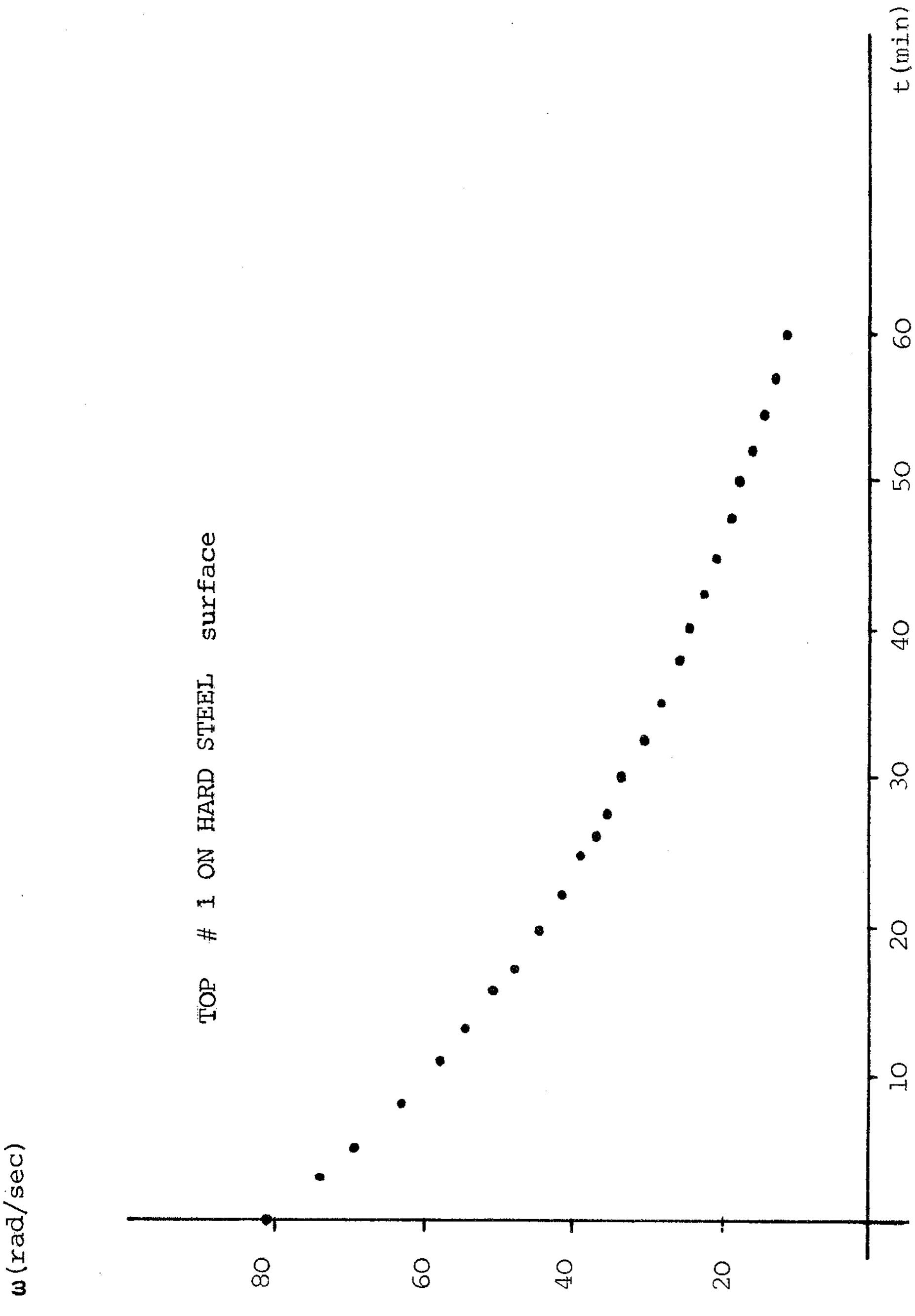

FIG. 2 - Variation of angular velocity with time.

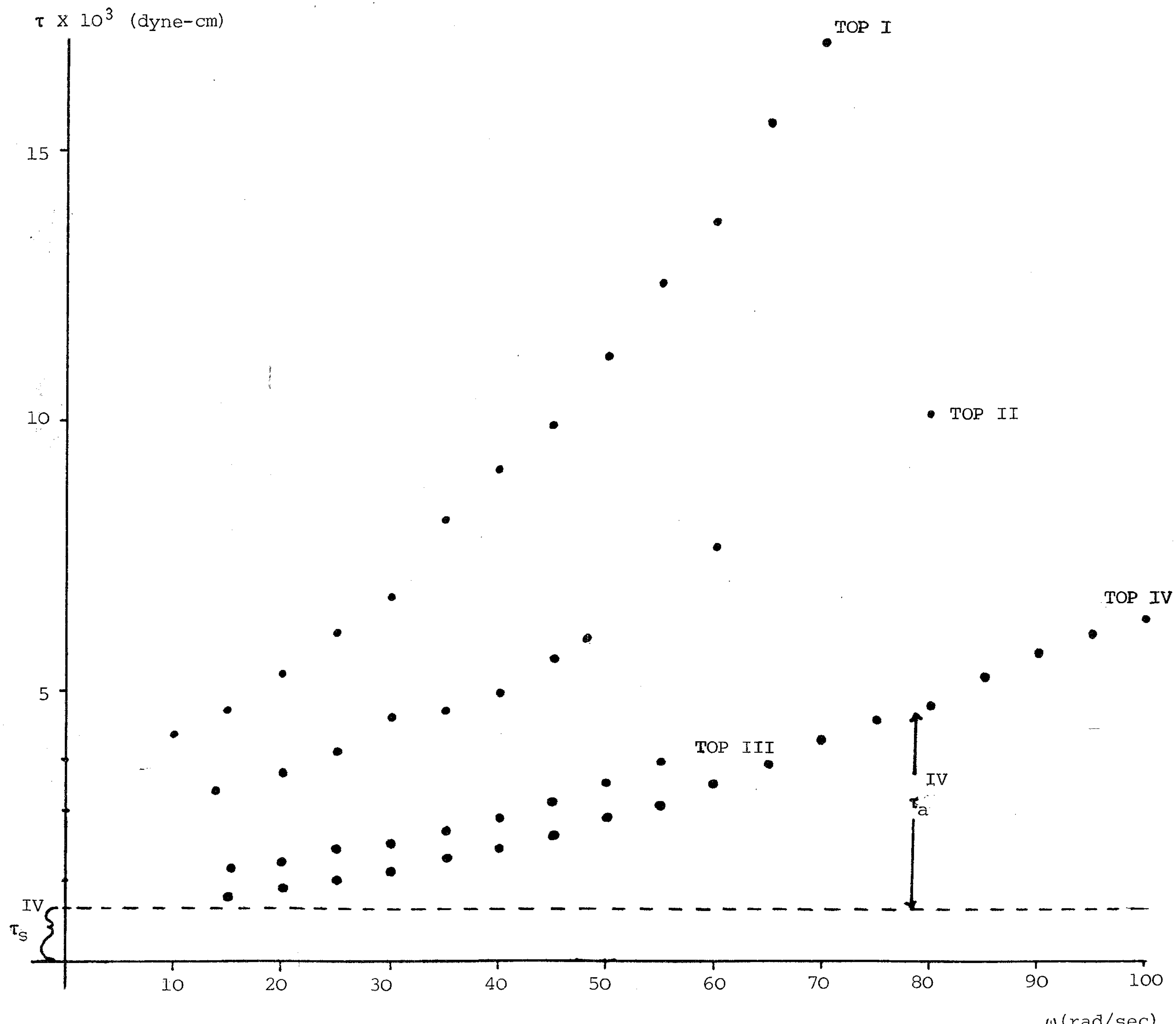

FIG. 3 - Torque as function of angular velocity

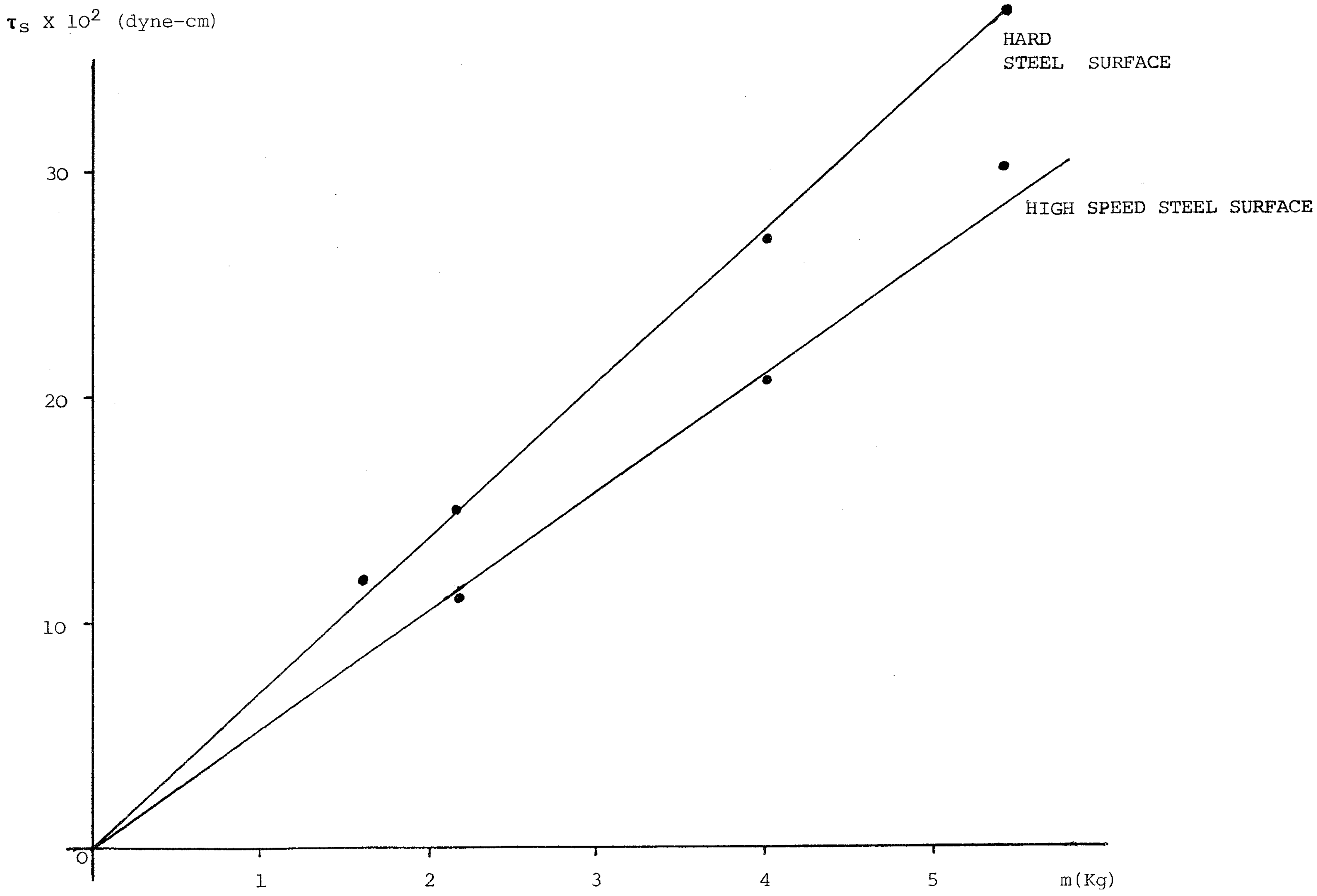

FIG. 4 - Torque due to surface friction vs mass

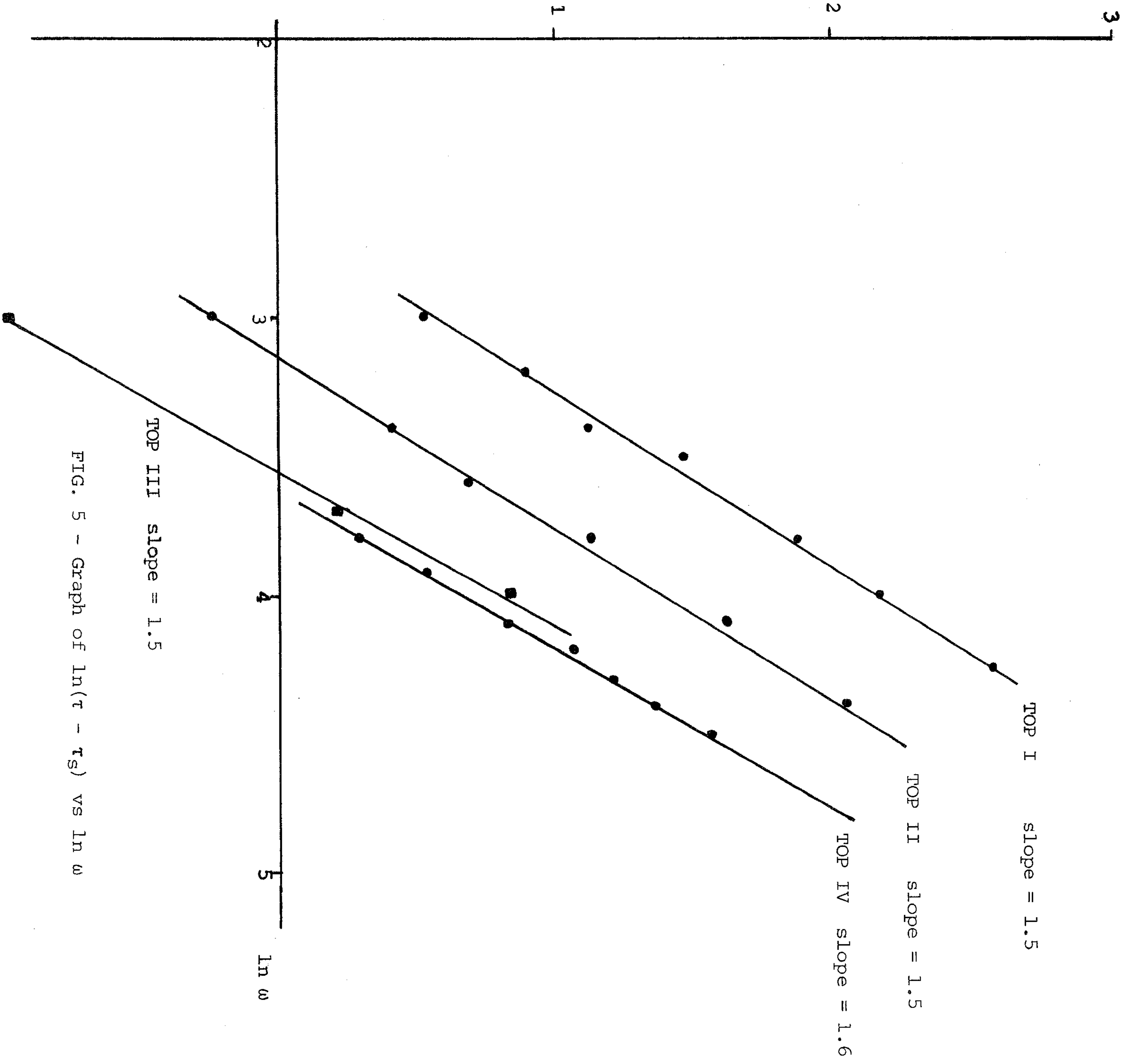

FIG. 5 - Graph of $\ln(\tau - \tau_s)$ vs $\ln \omega$

TABLE I

TOP SPECIFICATIONS

	m (Kg)	a (cm)	$I \times 10^4$ (gm-cm^2)
TOP I	5.42 ± 0.04	11.8 ± 0.1	47.0 ± 2.0
TOP II	4.00 ± 0.04	10.3 ± 0.1	27.0 ± 2.0
TOP III	2.17 ± 0.02	8.8 ± 0.1	11.5 ± 0.5
TOP IV	1.60 ± 0.1	7.7 ± 0.1	6.5 ± 0.5

TABLE II

$\tau_a/(a^4\omega^{3/2})$ computed

TOP I	TOP II	TOP III	TOP IV	$(0.616)\pi\rho\nu^{\frac{1}{2}}$
1.1×10^{-3}	0.95×10^{-3}	1.0×10^{-3}	1.1×10^{-3}	0.9×10^{-3}

TABLE III

Computed Time

	TOP IV	TOP III	TOP II	TOP I
COMPUTED t (min)	37	38	40	70
EXPERIMENTAL RESULT (min)	36	38	40	68

TABLE IV

ω_0 (m, a, I)

MASS (Kg)	Radius (cm)	I x 10^4 (gm-cm^2)	ω_0 (rad/sec)	E (Joules)
5.42	11.8	47.0	88	182
4.00	10.3	27.0	138	257
3.30	9.5	18.9	163	251
3.20	9.5	18.6	163	247
2.55	9.5	16.0	176	248
1.90	9.0	7.7	226	197
2.17	8.8	11.5	188	203
0.60	8.2	2.1	301	95
1.10	7.5	4.5	213	102

TABLE V

Computed Spinning Time

$E = 260$ Joules, $\omega_f = 6$ rad/sec

m (gms)	t (min)
1500	78.1
2100	86.3
2500	88.9
2700	89.5
2800	89.7
2900	89.8
3000	89.8
3100	89.7
3200	89.5
3500	88.6
4000	85.9