The fact that admissible boundary conditions may break symmetries was studied by Capri.[13]

The primary conclusion of this note is that Hamiltonians like the ones given by Eqs. (1)–(4) do not, by themselves, specify the system. Only after the boundary conditions have been specified can one study their properties.

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[1]C. Cisneros, R. P. Martinez-y-Romero, H. N. Nunez-Yepes, and S. L. Salas-Brito, "Comment on 'Quantum mechanics of the $1/x^2$ potential,' by Andrew M. Essin and David J. Griffiths [Am. J. Phys. **74**(2), 109–117 (2006)]," Am. J. Phys. **75**(10), 953–955 (2007).

[2]A. M. Essin and D. J. Griffiths, "Quantum mechanics of the $1/x^2$ potential," Am. J. Phys. **74**, 109–117 (2006).

[3]P. Kurasov, "Response to 'Comment on On the Coulomb potential in one dimension' by W. Fisher, H. Leschke, and P. Müller," J. Phys. A **30**, 5583–5589 (1997).

[4]The potential studied in Ref. 2 is given by $V(x<0)=\infty$ and $V(x>0)=\gamma/x^2$. This system is equivalent after a change of variables to the s-wave part of the three-dimensional potential $-\gamma/r^2$.

[5]K. Connolly and D. J. Griffiths, "Critical dipoles in one, two, and three dimensions," Am. J. Phys. **75**(6), 524–531 (2007).

[6]G. Bonneau, J. Faraut, and G. Valent, "Self-adjoint extensions of operators and the teaching of quantum mechanics," Am. J. Phys. **69**(3), 322–331 (2001).

[7]V. S. Araujo, F. A. B. Coutinho, and J. F. Perez, "Operator domains and self-adjoint operators," Am. J. Phys. **72**(2), 203–213 (2004).

[8]I. Tsutsui, T. Fülöp, and T. Cheon, "Connection conditions and the spectral family under singular potentials," J. Phys. A **36**, 275–287 (2003).

[9]W. Fischer, H. Leschke, and P. Müller, "The functional-analytic versus the functional-integral approach to quantum Hamiltonians: The one-dimensional hydrogen atom," J. Math. Phys. **36**, 2313–2323 (1995).

[10]R. J. Loundon, "One-dimensional hydrogen atom," Am. J. Phys. **27**, 649–655 (1959).

[11]W. Fischer, H. Leschke, and P. Müller, "Comment on 'On the Coulomb potential in one dimension,' by P. Kurasov," J. Phys. A **30**, 5579–5581 (1997).

[12]F. A. B. Coutinho, Y. Nogami, L. Tomio, and F. M. Toyama, "The Fermi pseudo-potential in one dimension," J. Phys. A **37**, 10653–10663 (2004).

[13]A. Z. Capri, "Self-adjointness and spontaneously broken symmetry," Am. J. Phys. **45**(9), 823–825 (1977).

Dead time correction via the time series

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We present a method that allows us to correct for dead time and check for the proper operation of a radiation detector while recording data. The method is based on the exponential probability density of the time interval between successive detector pulses, and involves examining the ratio of the moments of the time series with a delay time.

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I. INTRODUCTION

Accounting for the dead time of a detector is an important consideration in radiation measurements and a subject that is usually covered in the undergraduate physics curriculum. The dead time, τ, is the minimum amount of time between two pulses so that they are recorded as separate pulses. A common approach for measuring τ is to use the "two-source" method.[1] Once the dead time is known, the corrected count rate C is determined from the measured count rate c using the relation $C=c/(1-c\tau)$.[1] The two-source method is conceptually simple, but it does not generally yield accurate results for the dead time because of the need to measure the difference between two large numbers that are nearly equal.[1]

In recent years advancements in electronics have made it relatively easy to measure the time between successive Geiger counter pulses.[2,3] The series of times between successive pulses, which we label as t_i for the ith time, contains useful information. An excellent exercise is to make a frequency plot (or histogram) of the time intervals.[2] Students can observe the lack of times less than τ, and can fit the histogram plot with an exponential function. Tests for randomness[3] and the goodness of the exponential fit support the statistical nature of the decay and enable students to study the physics of the decay process. In addition, because the histogram is fitted to the exponential function NCe^{-Ct}, where N is the total number of counts, we can obtain the corrected counting rate C from the fit. However, it is not practical to use this method to determine C. For accurate results we need a large number of times and a bin size that produces an exponential function with a large number of counts in each bin. Because the plot typically involves using a spreadsheet, it takes students some time to carry out the analysis. Usually this exercise is performed only once during a class session.

In this note we present a simple method that determines properties similar to the frequency plot, works with a relatively small number of interval times, and does not involve plotting data. This procedure can therefore be employed in real time.

II. TIME SERIES METHOD

If a detector is working correctly, the probability that it detects a particle between time t and $t+\Delta t$ is $P(t)\Delta t = Ce^{-Ct}\Delta t$. The exponential probability density $P(t)$ has the desirable property that the expectation value of t^m has the simple form:

$$\langle t^m \rangle = \int_0^\infty t^m C e^{-Ct} dt = \frac{m!}{C^m}. \quad (1)$$

Because $\langle t \rangle = 1/C$, we can form the dimensionless combination:

Table I. The moment ratios $\langle t^m \rangle / \langle t \rangle^m$ for m= 2, 3, and 4 for different delay times D for a series of 10 000 Geiger counter intervals. The statistical uncertainties in the last row are approximately the same for each number in the column.

$D(\mu s)$	$\langle t^2 \rangle / \langle t \rangle^2$	$\langle t^3 \rangle / \langle t \rangle^3$	$\langle t^4 \rangle / \langle t \rangle^4$	Count rate (counts/s)	N
0	1.87	5.34	19.6	171.7	10 000
100	1.90	5.42	20.6	174.7	10 000
200	1.93	5.61	21.7	177.9	10 000
300	1.96	5.81	22.9	181.1	10 000
400	2.00	6.02	24.2	184.3	9996
500	2.01	6.07	24.4	185.0	9849
600	2.00	6.06	24.4	184.9	9660
800	2.00	6.05	24.3	184.8	9303
1000	2.01 ± 0.02	6.06 ± 0.19	24.4 ± 1.7	185 ± 2	8975

$$\frac{\langle t^m \rangle}{\langle t \rangle^m} = m\,! \,. \tag{2}$$

The value $m!$ is unique to an exponential probability density. We refer to the left-hand side as the mth moment ratio.

Because our data consist of times t_i between successive recordings of the detected signals, the integral in Eq. (1) reduces to a sum over the t_i. If the detector is working properly, then $(\Sigma_{i=1}^N t_i^m / N)/(\Sigma_{i=1}^N t_i / N)^m$ should equal $m!$, where N is the total number of times in the series. There are two factors that will modify this ratio: statistical uncertainty due to N being finite, and the dead time of the detector. If we subtract a delay time D from each of the t_i, then the sum of Eq. (2) should equal $m!$ if D is greater than τ and the detector is working properly. We must be careful to reject the time in the sum if $(t_i - D) < 0$.

In Table I we list values of the ratio

$$\frac{\Sigma_{i=1}^N (t_i - D)^m / N}{(\Sigma_{i=1}^N (t_i - D)/N)^m} \tag{3}$$

for different values of D for data collected with a Geiger counter. We used the same setup as in Fig. 1 of Ref. 3 with the LED emitter-detector pair replaced by a comparator chip and the parallel port polled instead of a digital input card. We list values of m=2, 3, and 4 for an initial total of N =10 000. As seen in Table I, the moment ratios are close to $m!$ for values of $D \geqslant 400$ μs. If a finer mesh were used, we would see that the effective dead time of this Geiger counter is around 380 μs, because the moment ratios deviate from $m!$ by greater than the statistical error.

The statistical uncertainty of the counting rate decreases as $1/\sqrt{N}$, so N=10 000 results in a 1% statistical error for C. The statistical uncertainty of the moment ratios can be derived by doing the appropriate integrals. The variance of a term in the numerator is $\Delta_m^2 = \langle t^{2m} \rangle - \langle t^m \rangle^2$, where $\langle t^{2m} \rangle = \int t^{2m} C e^{-Ct} dt$. Hence, $\Delta_m / \sqrt{N} = \langle t \rangle^m m!\sqrt{(2m)!/(m!)^2 - 1}/\sqrt{N}$ for the standard deviation of the numerator in Eq. (2). The uncertainty of the denominator in Eq. (2) can be found from $(\langle t \rangle \pm \langle t \rangle / \sqrt{N})^m \approx \langle t \rangle^m (1 \pm m/\sqrt{N})$, with the approximation becoming better as N increases. We cannot add the fractional uncertainties of the numerator and denominator in quadrature because the same t_i appear in both expressions. However, the fractional uncertainty in the moment ratios are to a good approximation uncorrelated to the fractional uncertainties in $\langle t \rangle^m$, because the moment ratios do not depend on $\langle t \rangle$. Hence, the fractional uncertainty of the moment ratios adds via quadrature to that of $\langle t \rangle^m$ to give the fractional uncertainty of $\langle t^m \rangle$. Thus, the fractional uncertainty of a moment ratio is obtained by subtracting the squares of the fractional uncertainties of the numerator and denominator: $((2m)!/(m!)^2 - 1) - m^2)/N$ and taking the square root. The resulting fractional uncertainty for the mth moment ratio is $\sqrt{(2m)!/(m!)^2 - (m^2+1)}/\sqrt{N}$. We verified the validity of this expression using numerical simulation and found the fractional uncertainties agreed with this expression to within 3%. For m=2, the moment ratio with uncertainty is $2(1 \pm 1/\sqrt{N})$. For m=3, we have $6(1 \pm \sqrt{10}/\sqrt{N}) \approx 6(1 \pm 3.2/\sqrt{N})$, and for m=4 we obtain $24(1 \pm \sqrt{53}/\sqrt{N}) \approx 24(1 \pm 7.3/\sqrt{N})$. Thus for N=10 000, the second moment ratio should be 2 to within 1%, the third should be 6 to within 3%, and the fourth should be 24 to within 7%.

III. COMMENTS

Although the determination of the dead time is interesting, there are other lessons to be learned from this determination. The result that the moment ratios are equal to $m!$ within the statistical uncertainty for $D \geqslant 400$ μs demonstrates that the detector is most likely working properly and $P(t)$ is exponential. We have not done a complete check, because we have only used values of m equal to 2, 3, and 4. Because the $m!$ result holds only for an exponential probability density, examining these three cases is good evidence for its applicability. The result that the moment ratios are approximately $m!$ and the counting rate is the same for all values of $D \geqslant 400$ μs demonstrates that there is no memory in nuclear decay.[4] That is, the probability to decay per unit time is a constant and does not depend on past history.

In Table I we give the corrected counting rate C, which equals $1/\langle t \rangle$. Note that the measured counting rate c $(D=0)$ of 172 counts/s is less than the corrected rate of 185 counts/s. The desirable feature of determining the corrected counting rate in this way is that we do not need to know the value of the dead time nor use the correction formula $C = c/(1 - c\tau)$. An added bonus is that the calculations can be done in real time because the sums can be computed quickly. If the counter is connected to a computer, then two or three lines of the table can be printed as the data is being collected. Displaying a few lines of Table I in real time allows the user to determine the corrected counting rate directly and check if the detector is working properly.

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[1]Glenn F. Knoll, *Radiation Detection and Measurement* (Wiley, Hoboken, NJ, 1989), 3rd ed.
[2]F. Arqueros, F. Blanco, and B. Jimenez de Cisneros, "Studying the statistical properties of particle counting with a very simple device," Eur. J. Phys. **25**, 399–407 (2004).
[3]R. Aguayo, G. Simms, and P. B. Siegel, "Throwing nature's dice," Am. J. Phys. **64**, 752–758 (1996).
[4]C. Barnett, "Probabilistic description of radioactivity based on the good-as-new postulate," Am. J. Phys. **47**, 173–177 (1979).