Resonances

When examining the cross section of two particles interacting *as a function of energy*, sometimes the cross section is observed to have a peak at a particular energy. The peaks are refered to as resonances, a term borrowed from mechanics when a system responds with a large amplitude at a particular driving frequency. In particle physics, a resonance is often caused by the creation of a particle, whose mass-energy is the energy of the resonance.

This property is shown in the following two graphs. The experiment involves scattering a μ^+ and its anti-particle μ^-. The total cross section, σ is measured as a function of the center-of-mass total invarient mass-energy $\sqrt{s}$. As seen in the figure, there are well defined peaks in the cross section at particular energies. Most of the peaks are identified with quark-antiquark ($q\bar{q}$) bound states. One peak is caused by the Z particle. Note that some of the $q\bar{q}$ peaks are associated with excited states, and others with the ground $q\bar{q}$ state.

In your current assignment, you have data of the total cross section, σ, for electron-antielectron scattering as a function of invarient mass near 90 GeV, the mass of the Z. You are to try and fit the cross section peak with a Lorentzian function. Why do we expect the shape of the peak to be a Lorentzian function? Our current theory of particle physics describes all interactions using vertices, propagators and coupling constants. The reasoning behind this picture requires a study of relativistic quantum mechanics. Here, we will give an overview of two ways resonances can occur: an s-channel resonance and a t-channel resonance.

s-channel resonances

From quantum field theory, the contributions to the probability amplitude density for a particular scattering process can be represented by diagrams, Feynman diagrams. The complete scattering probability amplitude is the sum of all such diagrams possible in going from the initial state to the final state of the two particles.

For example, suppose particle A and B scatter off of each other elastically. One contribution to the scattering process is described in the diagram. The way one would interpret the diagram is as follows. A and B come together and form a new particle, particle C. Particle C exists for a while, then decays into A and B. In quantum field theory there are rules that guide one in calculating the probability amplitude for this contibution to the complete scattering amplitude.

A "coupling constant" is assigned to every vertex, in this case the vertex of A, B, and C. Note, that there are two ABC vertices. Each of these two vertices will have the same coupling constant, say g. The particle C "propagates" in space and time before it decays. There is an expression representing this propagation: $1/(s - m_C^2)$. Here, s is the invarient energy of particle C, $s = E_C^2 - p_C^2$. You might think that this expression is trivial. Isn't $E^2 - p^2$ equal to m^2 for every particle? Not necessarily. If the particle is "on-shell" or "real", then $E^2 = m^2 + p^2$. However, in theory a particle can have an energy E and a momentum p such that $E^2 - p^2 \neq m^2$. If $E^2 - p^2 \neq m^2$ the particle is said to be virtual, or off-shell.

In an experiment, one can only directly measure particles that are on-shell. Every time a particle can be directly measured, $E^2 - p^2$ will equal the same value, it's mass m^2. Virtual particles exist in the calculation of measureable quantities. Quarks, for example, have never been observed on-shell. A free quark, i.e. with energy E and momentum p, has never been observed, thus their mass has never been directly measured. Quark masses are parameters in our current theories.

For the diagram in which A and B couple to C, the probability amplitude M is proportional to the product of the coupling constants times the propagator:

$$M \propto \frac{g^2}{s - m_C^2} \tag{1}$$

In the center of mass frame of the two particles, $s = (E_A + E_B)^2$, since the momentum of A is opposite to B. So, in the center of mass frame, the probability amplitude is proportional to:

$$M \propto \frac{g^2}{(E_A + E_B)^2 - m_C^2} \tag{2}$$

This expression by itself is problematic. If $E_A + E_B = m_C$, then the right side of the equation is undefined (the denominator equals zero). Except for this problem, one can see that if $E_A + E_B$ is near m_C then M and consequently the cross section becomes large (i.e. a resonance). This singularity is resolved when all possible diagrams are added up. For example, one such diagram is shown. The particle C decays into A+B, with combines again to form C, which then decays into the final states of A+B.

One way to handle the singularity at $E_A + E_B = m_C$ is to assign a complex number to the mass of the decaying particle. The reason this is justified is as follows. For a free particle that doesn't decay, the time evolution of the state goes (relativistically) as

$$\Psi(t) \propto e^{-i(mc^2/\hbar)t} \tag{3}$$

If a particle or state can decay, then the time evolution of the probability density in time goes as

$$\Psi^* \Psi \propto e^{-t/\tau} \tag{4}$$

where τ is the lifetime of the state. In terms of Ψ the above equation implies that

$$\Psi(t) \propto e^{-t/(2\tau)} \tag{5}$$

Combining the decay property with a transiently free particle yields

$$\Psi(t) \propto e^{-i(m_0 c^2/\hbar - i/(2\tau))t} \tag{6}$$

for the time part of the "wave function" $\Psi(t)$. If we want to assign a mass to a state that decays, the mass must have an imaginary part. In terms of the Full Width at Half Maximum Γ, the imaginary part will be $i\Gamma/2$. Thus the mass of a state that decays can be written as $m = m_0 - i\Gamma/2$. All diagrams that start with A+B to C, and end with C decaying to A+B can be treated as a single diagram with particle C as the propagator with mass $m_C = m_0 - i\Gamma/2$. The real part of C's mass is the energy where the resonance peaks, and the imaginary part is related to the lifetime of the particle.

The lifetime τ and Γ are related to each other. Equating the time dependent parts of the two expressions for $\Psi^*\Psi$ to each other gives

$$\begin{aligned} |e^{-t/(2\tau)}|^2 &\sim |e^{-im_C c^2 t/\hbar}|^2 \\ e^{-t/\tau} &\sim |e^{-i(m_0 - i\Gamma/2)c^2 t/\hbar}|^2 \\ e^{-t/\tau} &\sim e^{-\Gamma t c^2/\hbar} \end{aligned}$$

In units where c is unity, we have

$$\Gamma = \frac{\hbar}{\tau} \tag{7}$$

If a particle is not stable the probability density for it to exist for a time t is proportional to $e^{-t/\tau}$. The average amount of time it will exist is τ with a **standard deviation** also equal to τ. When it exists, one never knows what mass energy it will have. Measuring it many times, the average mass it will have will be m_0 with a **standard deviation** of Γ. These **standard deviations** are related inversely to each other, and we have

$$\Gamma\tau \sim \hbar \tag{8}$$

The relationship between the mass-energy and lifetime standard deviations is refered to as the Heisenberg uncertainty principle for energy and time. If a particle has a lifetime greater than 10^{-10} *sec*, then usually the lifetime of the particle is listed in the data tables. For example the neutron, pion, kaon, lambda, sigma and nuclear excited states have their lifetimes listed. If a particle has a very short lifetime, then the width of the peak, Γ is listed in the tables. For example the ρ and ω mesons, the baryon resonances, and the W and Z_0 particles fall into this catagory. Often it is easier to measure the lifetime, τ, instead of Γ, and sometimes the peak width is easier to measure. It is easy to switch between the two quantities, since $\Gamma = \hbar/\tau$.

The probability amplitude at an energy near the mass of C is therefore approximately

$$M \propto \frac{g^2}{(E_A + E_B)^2 - (m_0 - i\Gamma/2)^2} \tag{9}$$

or

$$M \propto \frac{g^2}{s - (m_0 - i\Gamma/2)^2} \tag{10}$$

Factoring the denominator we have

$$M \propto \frac{g^2}{(\sqrt{s} + (m_0 - i\Gamma/2))(\sqrt{s} - (m_0 - i\Gamma/2))} \tag{11}$$

When the energy is near m_0, i.e. $\sqrt{s} \approx m_0$, the first term in the denominator does not change significantly with energy compared to the second term. So,

$$M \sim \frac{1}{\sqrt{s} - (m_0 - i\Gamma/2)} \tag{12}$$

The cross section is proportional to the square of M:

$$\begin{aligned}\sigma &\sim M^*M \\ &\sim \frac{1}{(\sqrt{s}-m_0)-i\Gamma/2}\frac{1}{(\sqrt{s}-m_0)+i\Gamma/2} \\ &\sim \frac{1}{(\sqrt{s}-m_0)^2+(\Gamma/2)^2}\end{aligned}$$

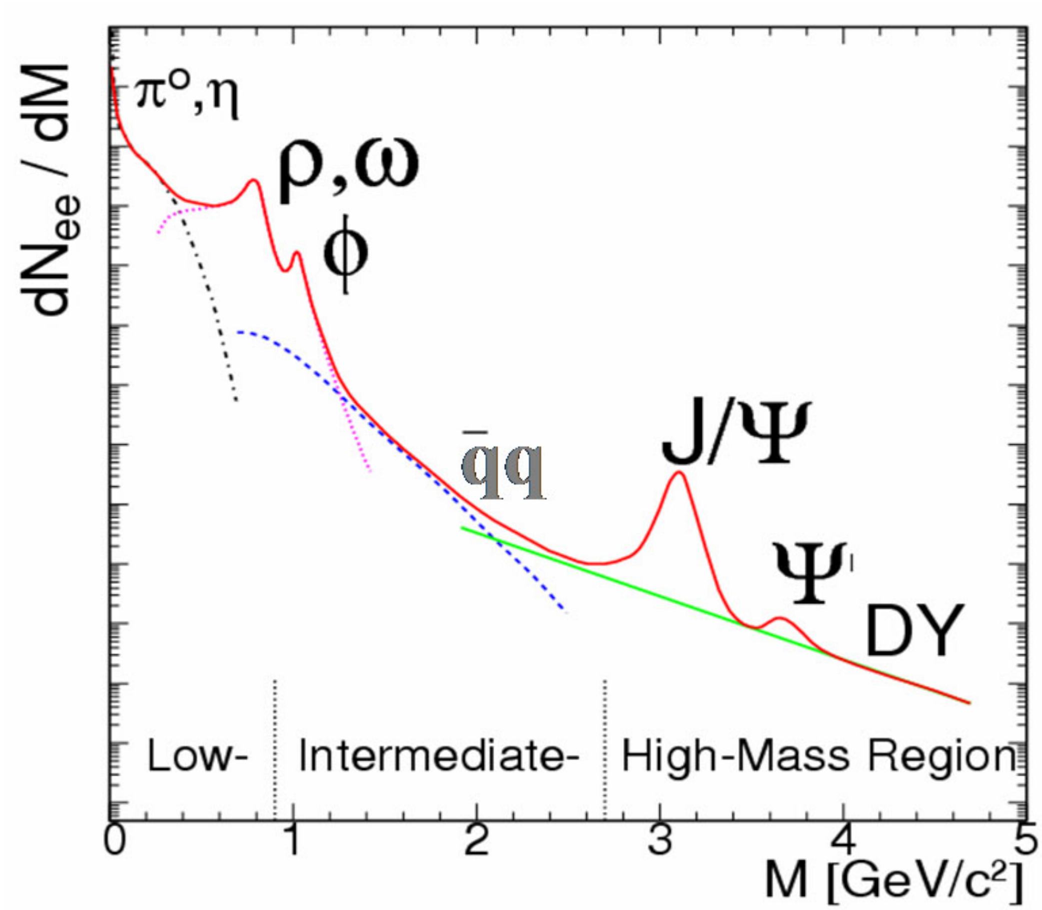
which is a Lorentzian function. Choosing the normalizing constant to be $\sigma_{max}(\Gamma/2)^2$ the cross section can be written in the form

$$\sigma = \sigma_{max}\frac{(\Gamma/2)^2}{(\sqrt{s}-m_0)^2+(\Gamma/2)^2} \tag{13}$$

where σ_{max} is the value of the cross section at the peak.

t-channel Resonances

A resonance, or peak in the cross section, can also occur without forming a "new" particle. The basis of such a process is shown in the figure. Particle A interacts with particle B by the "exchange" of particle D. This diagram by itself cannot produce a resonance. However, one needs to add the diagram that has two particle D's exchanged, plus one with three particle D's exchanged, etc. The infinite sum of all possible "ladder" contributions can cause a resonance peak in the cross section. Using time-dependent perturbation theory, one can show that the infinite sum of the ladder diagrams for an exchange particle with mass m_D is equal (in the non-relativistic approximation) to solving the (non-relativistic) Schroedinger equation with a potential $V(r) \propto e^{-m_D r}/r$.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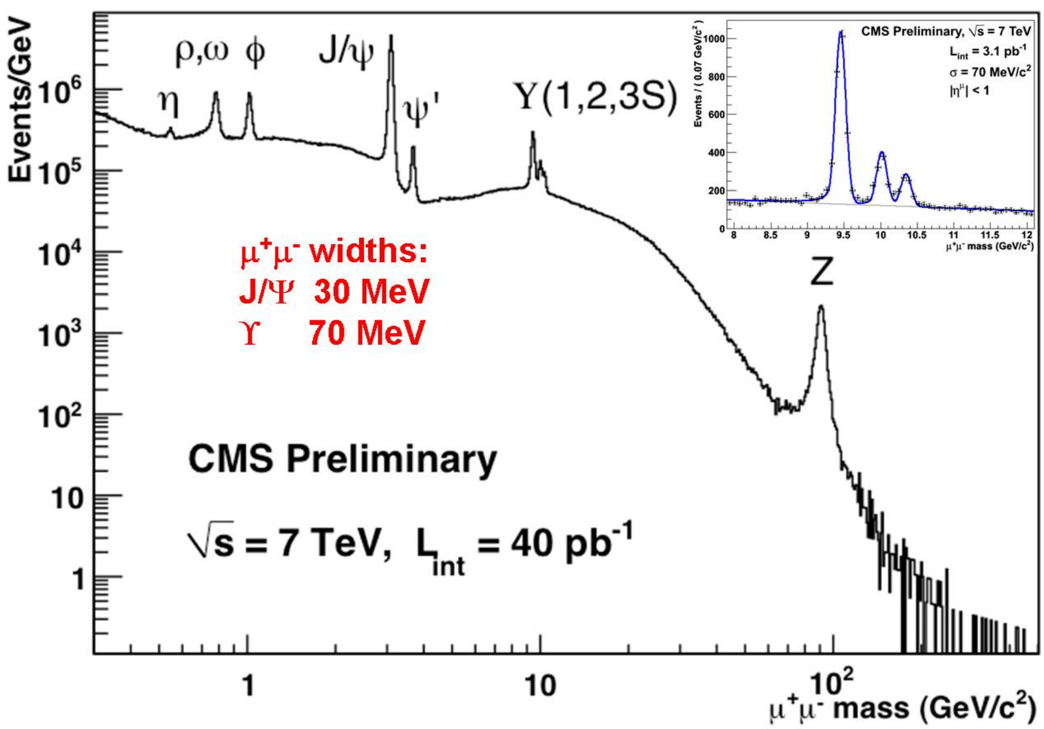
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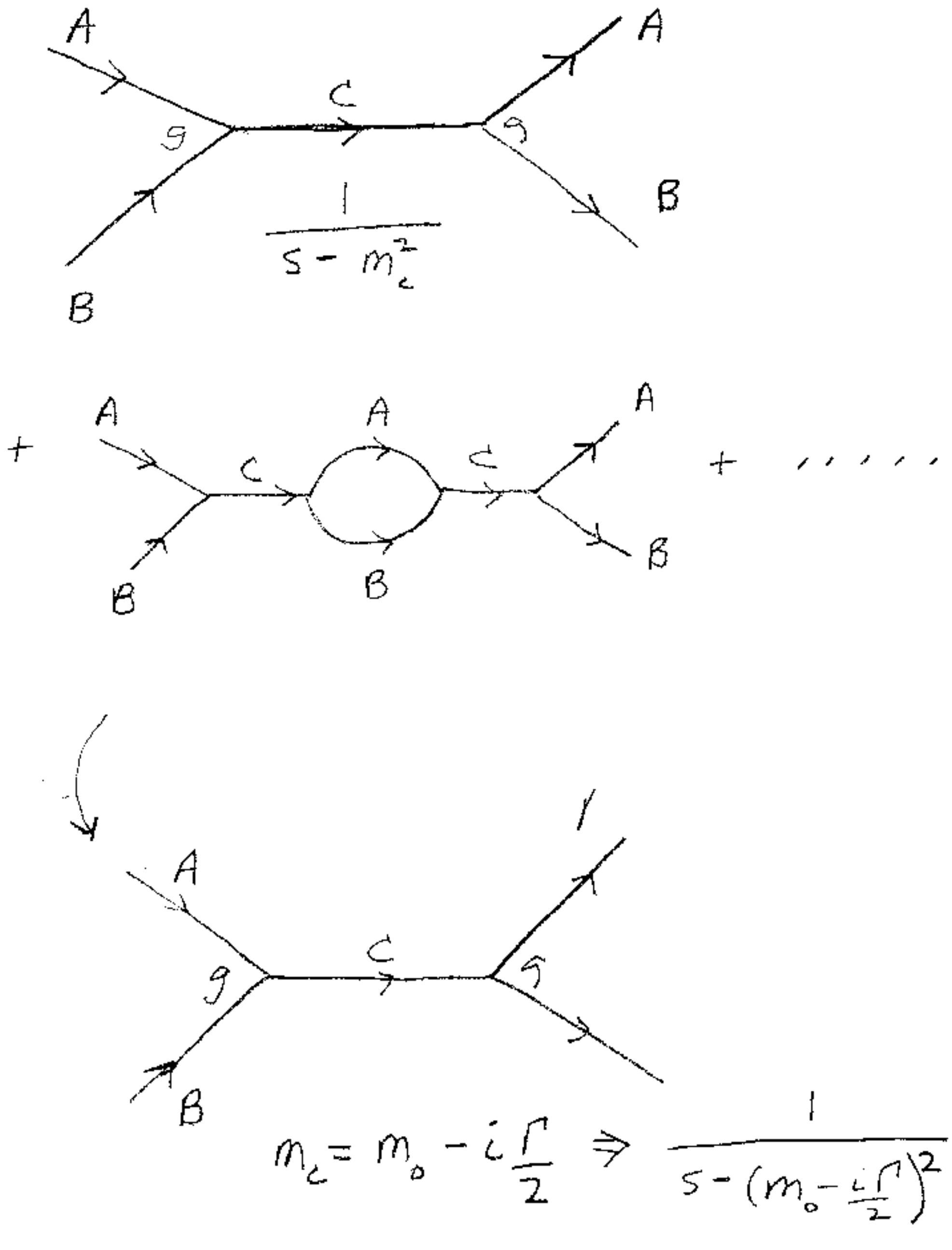
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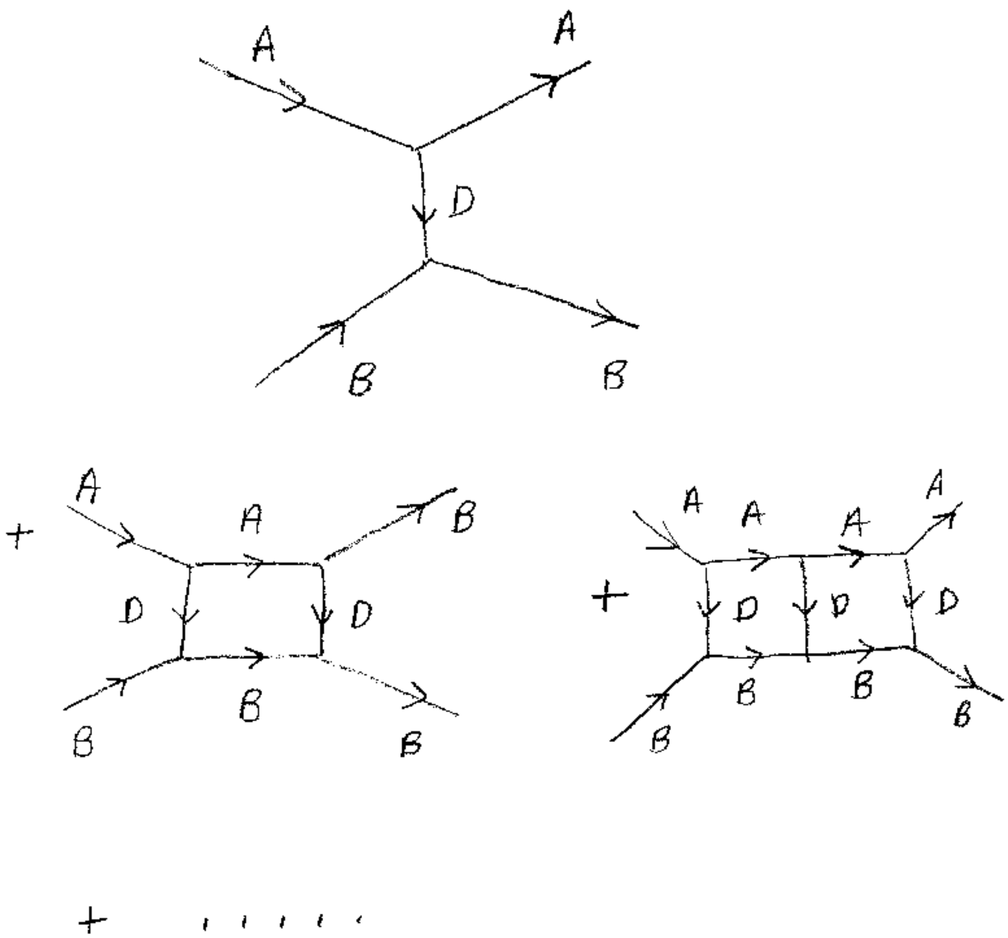
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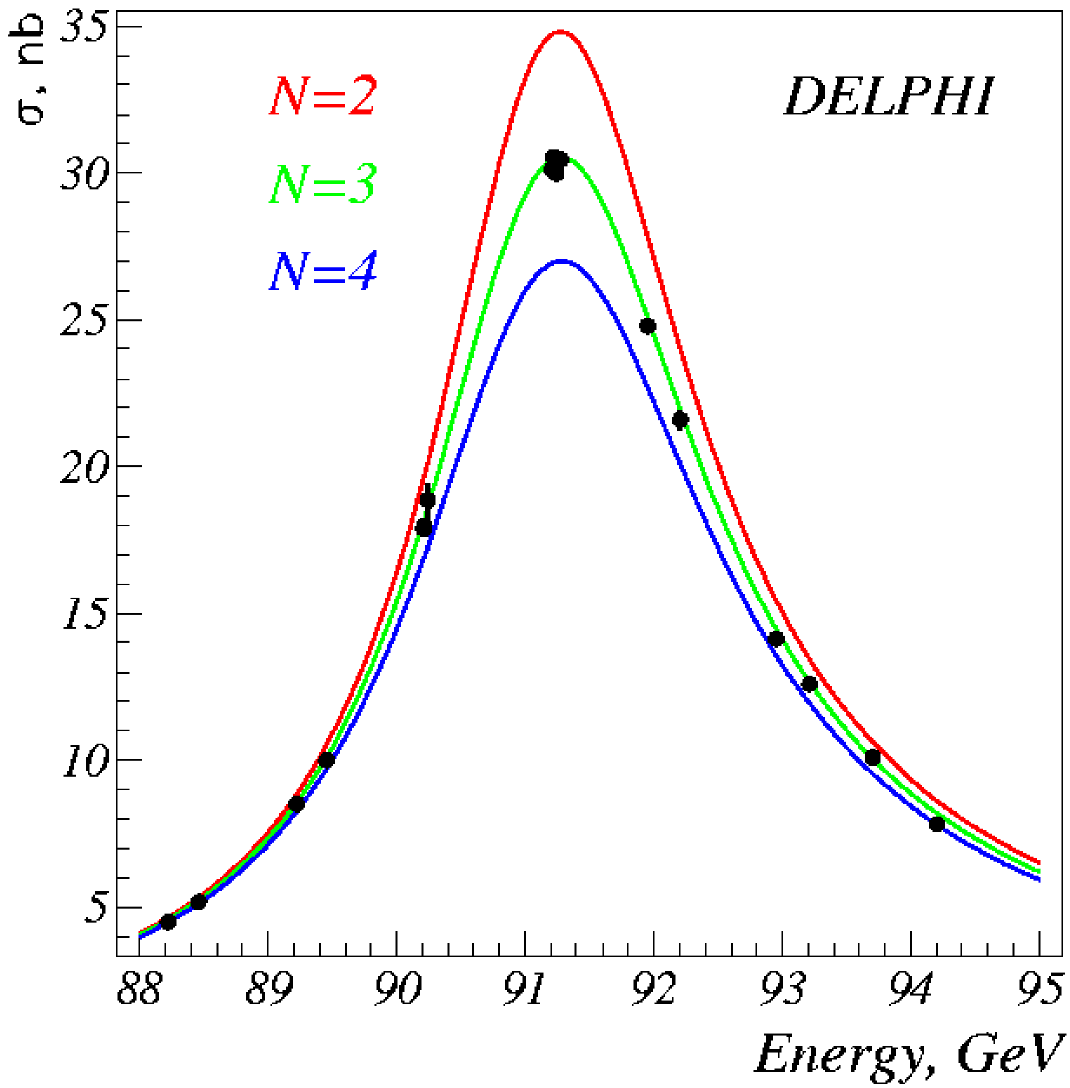
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