

Physics 499

Extra Homework Assignment
Meson-Nucleus elastic scattering

Due Before March 18th

Problem : $K^+ - {}^{12}\text{C}$ elastic scattering

Reference: Phys. Rev. C30, 1256-1266 (1984).

For this assignment you will determine (numerically) the elastic scattering cross section for K^+ scattering off of nuclei, and if you have time determine the best values of the two parameters to match the data. You will use the Born approximation, which involves solving the following integral numerically:

$$\int_0^\infty V(r)r\sin(qr)dr \quad (1)$$

where $q = 2pc/(\hbar c)\sin(\theta/2)$. In this expression, θ is the scattering angle, p is the K^+ momentum. We will take the potential to be that of the spherical square well:

$$\begin{aligned} V(r) &= V_0 \text{ for } r < R \\ &= 0 \text{ otherwise} \end{aligned}$$

The scattering amplitude, $f(\theta)$ from the Born approximation is:

$$f(\theta) = -\frac{1}{\hbar c} \frac{mc^2}{pc} \frac{1}{\sin(\theta/2)} \int_0^\infty V(r)r\sin(qr)dr \quad (2)$$

and the differential cross section is

$$\frac{d\sigma}{d\Omega} = |f(\theta)|^2 \quad (3)$$

I have attached the data for $K^+ - {}^{12}\text{C}$ elastic scattering for a kaon momentum of $800 \text{ MeV}/c$. The rest mass of the K^+ is $493 \text{ MeV}/c^2$.

Your goal is to determine the parameters R and V_0 for a "best fit" of the data, and to graph your calculation with the data. You can build up to this in two parts:

First

Write a computer program that calculates just the differential cross section, $|f(\theta)|^2$

for $K^+ - {}^{12}\text{C}$ elastic scattering at K^+ momentum $800 \text{ MeV}/c$. Your program should ask the user to input the potential strength V_0 and nuclear size R . Your program should then output to the screen the cross section for angles $10^\circ \rightarrow 38^\circ$ at two degree increments.

Second

Add to the first part a publication quality graph of your calculation with the $K^+ - {}^{12}\text{C}$ elastic scattering data plus error bars from the data of Marlow et. al., that is on the next page.

| Angle | | $^{12}\text{C}(K^+,K^-)^{12}\text{C}$ | | | |
|-------|-------|---------------------------------------|--------|-------------------------------|-------|
| lab | c.m. | Elastic c.m. cross section | | 2^+ (4.4 MeV) Cross section | |
| | | (mb/sr) | Error | (mb/sr) | Error |
| 3.5 | 3.79 | 971.00 | 280.62 | | |
| 4.5 | 4.87 | 522.00 | 99.70 | | |
| 5.5 | 5.95 | 343.32 | 44.29 | | |
| 6.5 | 7.03 | 299.51 | 30.25 | | |
| 7.5 | 8.11 | 257.63 | 20.38 | | |
| 8.5 | 9.19 | 214.80 | 16.95 | 1.069 | 1.069 |
| 9.5 | 10.28 | 175.09 | 13.81 | 2.937 | 2.153 |
| 10.5 | 11.35 | 134.36 | 9.83 | 3.061 | 2.103 |
| 11.5 | 12.43 | 104.19 | 7.65 | 3.587 | 1.338 |
| 12.5 | 13.51 | 83.600 | 5.869 | 3.390 | 0.993 |
| 13.5 | 14.69 | 61.899 | 4.407 | 3.613 | 0.954 |
| 14.5 | 15.67 | 50.012 | 3.456 | 5.036 | 0.836 |
| 15.5 | 16.75 | 33.891 | 2.616 | 4.978 | 0.657 |
| 16.5 | 17.83 | 23.335 | 1.862 | 6.027 | 0.711 |
| 17.5 | 18.91 | 16.449 | 1.336 | 5.475 | 0.613 |
| 18.5 | 19.99 | 11.177 | 0.942 | 5.483 | 0.581 |
| 19.5 | 21.06 | 7.380 | 0.652 | 4.944 | 0.514 |
| 20.5 | 22.14 | 4.951 | 0.525 | 4.870 | 0.492 |
| 21.5 | 23.21 | 2.826 | 0.387 | 4.494 | 0.445 |
| 22.5 | 24.29 | 1.744 | 0.286 | 3.639 | 0.368 |
| 23.5 | 25.37 | 1.056 | 0.216 | 3.529 | 0.350 |
| 24.5 | 26.44 | 0.547 | 0.146 | 3.057 | 0.303 |
| 25.5 | 27.51 | 0.302 | 0.106 | 2.587 | 0.264 |
| 26.5 | 28.59 | 0.287 | 0.096 | 2.064 | 0.213 |
| 27.5 | 29.66 | 0.305 | 0.090 | 1.613 | 0.173 |
| 28.5 | 30.73 | 0.215 | 0.069 | 1.310 | 0.143 |
| 29.5 | 31.80 | 0.294 | 0.076 | 0.964 | 0.119 |
| 30.5 | 32.87 | 0.305 | 0.069 | 0.552 | 0.084 |
| 31.5 | 33.94 | 0.291 | 0.066 | 0.421 | 0.073 |
| 32.5 | 35.01 | 0.247 | 0.058 | 0.267 | 0.057 |
| 33.5 | 36.08 | 0.161 | 0.046 | 0.237 | 0.059 |
| 34.5 | 37.14 | 0.222 | 0.509 | 0.181 | 0.056 |
| 35.5 | 38.21 | 0.111 | 0.041 | 0.138 | 0.053 |
| 36.5 | 39.27 | 0.110 | 0.049 | | |
| 37.5 | 40.34 | 0.147 | 0.072 | | |
| 38.5 | 41.40 | 0.023 | 0.026 | | |