

Laboratory Experiment on Natural Radiation using the NaI gamma detector

Your goal for this experiment is to measure the activity of the most common isotopes in our environment, ^{40}K , the ^{238}U series, and the ^{232}Th series. The sample we will be measuring will be a soil sample. You can pick a soil sample from the laboratory or bring in your own soil sample. Note that since we know the natural abundance of ^{40}K , you can also determine the amount of potassium in the soil.

For the analysis we will be using NaI gamma detectors. These detectors are common, rather inexpensive, and portable. The spectrum analysis techniques are similar to those you have used in previous experiments. We would like to let you figure as much out on your own, like a real experimentalist. This manual will give you guidance.

In your soil sample there will be three photopeaks of interest to us. One photopeak will be from ^{40}K at 1460 KeV, another at 1764 KeV from the ^{238}U series, and one at 2614 KeV from the ^{232}Th series. The spectral peaks from these three gamma's are sufficiently far apart so that we can resolve them with our NaI detector. Thus, we can use Gaussian peak fitting to find the area and hence the count rate.

We need to determine the activity from the count rate. The equation that relates activity (decays/time) to (gammas counted)/time is

$$\begin{aligned}\frac{\text{counts}}{\text{time}} &= (\text{Activity})(\text{gamma yield})(\text{efficiency}) \\ \frac{\text{counts}}{\text{time}} &= AY\epsilon(E)\end{aligned}$$

where Activity equals decays/time, and the efficiency ϵ depends on gamma energy E . From our experience with the gamma detector, we can show that to a good approximation $\epsilon(E)$ has a power law dependence on the gamma's energy E : $\epsilon \propto E^b$. Substituting into the above equation for $\epsilon(E)$ we have

$$\begin{aligned}\frac{\text{counts}}{\text{time}} &\propto AYE^b \\ \frac{\text{counts}}{\text{time}} &= AYKE^b\end{aligned}$$

where the constant K is a proportionality constant that depends on the size and shape of the detector and the detector-source geometry. Since the three gammas have different energies, the exponent b is an important parameter. One can measure

b by considering the quantity (counts/yield) for an isotope that emits more than one gamma. If one divides both sides of the equation above by the yield we have:

$$\frac{\text{counts/time}}{Y} = AK E^b \quad (1)$$

If we have a source that emits more than one gamma, then A , K , and b will be the same for each of the gammas emitted: $\text{counts/yield} \propto E^b$. Data of the counts/yield for the various gamma energies will enable a determination of b . We have three disk sources in the laboratory that emit more than one gamma: ^{207}Bi emits three gammas, ^{22}Na emits two gammas, and ^{60}Co which emits two gammas. Although these sources will have different values of AK they will have the same value of b , since they are essentially point sources of radiation without self-absorption. In the Appendix we derive a formula for determining b from the data from the three sources and their total of seven gammas.

We will first use the disk sources, ^{207}Bi , ^{22}Na , and ^{60}Co , to determine b for the soil sample, then analyze the soil sample spectra.

Measuring the exponent b for the soil sample

The gamma's emitted in the soil can be absorbed by the soil before reaching the detector. Since gamma's with lower energy have a higher absorption probability, the exponent b will depend on the density of the soil. To account for this property of "self-absorption" we will determine b with and without soil between the three disk sources and the detector. To approximate the energy dependence of detector efficiency for the soil sample, we will average the two values of b .

1. Set your high voltage and amplifier gain so that the highest energy gamma emitted by ^{207}Bi , 1770 KeV, is measurable. A good choice would be to have the photopeak for this gamma be located around channel 850-900.
2. With the soil sample up against the detector, record the gamma spectra for each of the three disk sources with the disk sources placed just behind the soil sample. A 10-15 minute recording might work for the ^{22}Na and ^{60}Co samples, but a longer recording time, 20-30 minutes, might be necessary for the ^{207}Bi sample since the 1770 KeV gamma has a small yield.
3. Repeat the previous measurement with the soil sample removed. Try to keep the disk sources in the same location that they were with the soil sample present.

Is the attenuation different for the lower energy gammas than the higher energy ones?

4. Measure the area under the photopeaks using the Gaussian curve fitting program mcafit2, which is the same program you used in experiments 2, 3, and 4.
5. Using Excel or the app disksources.html, found on the lab manual web site, determine the exponent b for the case when the soil is present and when it is absent. Place your results on the board for a class discussion.

Efficiency Calibration

In order to obtain the activity of the ^{238}U and ^{232}Th series as well as the amount of potassium in your soil sample, we will need to determine the efficiency of the detector for our soil-detector setup at one energy. It is most convenient to choose the energy of the gamma emitted by ^{40}K , 1460 KeV. This is a good choice since we can prepare a sample of pure KCl that is the same size and shape as our soil sample as a standard.

1. Place the pure KCl sample up against your NaI detector exactly as the soil sample will be placed for the soil measurement. Record the mass of the KCl .
2. Collect a gamma spectra from the KCl sample, and use mcafit2 to determine the counts under the 1460 KeV photopeak.
3. You can now calculate two quantities:

$$\frac{\text{Counts}}{^{40}\text{K Decay}} \quad \text{and} \quad \frac{\text{Counts}}{\text{gram potassium}} \quad (2)$$

in the KCl sample.

Soil Sample Analysis

Your soil sample will be placed in front of a NaI detector with the same detector-sample setup that you have been using. The only difference will be additional lead to shield against background radiation. Data will be collected for 20 hours with your soil sample present. In addition a recording will be done without any soil sample present.

1. Identify the three photopeaks relevant for our analysis. The ^{40}K 1460 KeV photopeak has the most counts, and the 2614 KeV photopeak from the ^{232}Th series is the peak with the largest energy.
2. Measure the area under the three photopeaks relevant for our analysis. To do this you should use the Gaussian fitting program `naisoilfit2`, which is found on the laboratory manual web site, and is designed for the soil spectrum. Two features are different in `naisoilfit2` than `mcafit2`: First, `naisoilfit2` uses a different background function as `mcafit2`. `naisoilfit2` uses a straight line as the background function, one that is a better representation of the background for the soil sample. The straight line background function starts and ends in the window you choose, so choose the window carefully. Second, `naisoilfit2` has a special fit function for the 1764 KeV photopeak region. This spectral peak, from ^{214}Bi , has a main contribution at 1764 KeV and two smaller contributions. The fit function includes all three gammas, and prints only the area under the 1764 KeV peak.
3. From your data, determine the following quantities for your soil:

$$\frac{A_{U238}}{A_{Th232}} \quad ; \quad \frac{A_{U238}}{A_{K40}} \quad ; \quad \frac{A_{Th232}}{A_{K40}}$$

Activity of ^{238}U , ^{232}Th , and ^{40}K in units of Bq/kg
Percent by weight of potassium in the soil

Make sure your lab report for experiment 5 includes:

1. All data from the disk source radiation, with and without the soil attenuation.
2. Graphs of Counts/yield for the 7 disk source energies, and the values of the power exponent b in each case.
3. A discussion of the attenuation properties of the soil.
4. Data and calculations for determining the $\frac{\text{Counts}}{^{40}\text{K Decay}}$ and $\frac{\text{Counts}}{\text{gram potassium}}$ in the KCl sample.

5. All data from the soil spectra, which include the areas under the three photo-peaks with and without the soil present.
6. All calculations for $\frac{AU_{238}}{A_{Th232}}$, $\frac{AU_{238}}{AK_{40}}$, $\frac{A_{Th232}}{AK_{40}}$, the activity of ^{238}U , ^{232}Th , and ^{40}K in units of Bq/kg, and the percent by weight of potassium in the soil.

Appendix

We take the energy dependence for the counts/yield to be $\frac{counts}{yield} = KAE^b$. To measure the exponent b , we use data at 7 energies from three sources: 3 energies from Bi^{207} , 2 energies from Na^{22} , and 2 energies from Co^{60} .

$$\begin{aligned} (C/Y)_i &= KA_{Bi}E_i^b \quad (i = 1, 2, 3) \\ (C/Y)_i &= KA_{Na}E_i^b \quad (i = 4, 5) \\ (C/Y)_i &= KA_{Co}E_i^b \quad (i = 6, 7) \end{aligned}$$

Note that the exponent b is the same for all the sources.

Taking the log of both sides of the equations gives;

$$\begin{aligned} \ln((C/Y)_i) &= b \ln(E_i) + \ln(KA_{Bi}) \quad (i = 1, 2, 3) \\ \ln((C/Y)_i) &= b \ln(E_i) + \ln(KA_{Na}) \quad (i = 4, 5) \\ \ln((C/Y)_i) &= b \ln(E_i) + \ln(KA_{Co}) \quad (i = 6, 7) \end{aligned}$$

We define $y_i \equiv \ln((C/Y)_i)$ for the count data, $x_i \equiv \ln(E_i)$ for the energy data, $k_{Bi} \equiv \ln(KA_{Bi})$, $k_{Na} \equiv \ln(KA_{Na})$, and $k_{Co} \equiv \ln(KA_{Co})$. Then the equations become:

$$\begin{aligned} y_i &= b x_i + k_{Bi} \quad (i = 1, 2, 3) \\ y_i &= b x_i + k_{Na} \quad (i = 4, 5) \\ y_i &= b x_i + k_{Co} \quad (i = 6, 7) \end{aligned}$$

There are 4 free parameters to vary to best fit the data: b , k_{Bi} , k_{Na} , and k_{Co} . To determine the "best fit" values we define the chi-square function χ^2 :

$$\chi^2 \equiv \sum_{i=1}^3 (b x_i + k_{Bi} - y_i)^2 + \sum_{i=4}^5 (b x_i + k_{Na} - y_i)^2 + \sum_{i=6}^7 (b x_i + k_{Co} - y_i)^2 \quad (3)$$

The "best fit" values are the ones that minimize the χ^2 function. At the minimum value of χ^2 , the derivatives with respect to each of the free parameters are zero: $\frac{\partial \chi^2}{\partial b} = 0$, $\frac{\partial \chi^2}{\partial k_{Bi}} = 0$, $\frac{\partial \chi^2}{\partial k_{Na}} = 0$, and $\frac{\partial \chi^2}{\partial k_{Co}} = 0$.

The derivative with respect to b yields:

$$\frac{\partial \chi^2}{\partial b} = \sum_{i=1}^3 2(b x_i + k_{Bi} - y_i)x_i + \sum_{i=4}^5 2(b x_i + k_{Na} - y_i)x_i + \sum_{i=6}^7 2(b x_i + k_{Co} - y_i)x_i = 0 \quad (4)$$

The above equation can be written as

$$bX^2 + k_{Bi}X_{Bi} + k_{Na}X_{Na} + k_{Co}X_{Co} - \sum_{i=1}^7 (y_i x_i) = 0 \quad (5)$$

where $X^2 \equiv \sum_{i=1}^7 x_i^2$, $X_{Bi} \equiv \sum_{i=1}^3 x_i$, $X_{Na} \equiv \sum_{i=4}^5 x_i$, $X_{Co} \equiv \sum_{i=6}^7 x_i$. The derivative with respect to k_{Bi} yields:

$$\frac{\partial \chi^2}{\partial k_{Bi}} = \sum_{i=1}^3 2(b x_i + k_{Bi} - y_i) = 0 \quad (6)$$

$$bX_{Bi} + 3k_{Bi} - Y_{Bi} = 0 \quad (7)$$

where $Y_{Bi} \equiv \sum_{i=1}^3 y_i$. Similarly, for the last two parameters we have

$$bX_{Na} + 2k_{Na} - Y_{Na} = 0 \quad (8)$$

$$bX_{Co} + 2k_{Co} - Y_{Co} = 0 \quad (9)$$

with $Y_{Na} \equiv \sum_{i=4}^5 y_i$ and $Y_{Co} \equiv \sum_{i=6}^7 y_i$

Combining equations 5, 7, 8, and 9, we obtain one equation involving only b :

$$b\left(X^2 - \frac{X_{Bi}^2}{3} - \frac{X_{Na}^2}{2} - \frac{X_{Co}^2}{2}\right) - \sum_{i=1}^7 (x_i y_i) = -\frac{Y_{Bi}X_{Bi}}{3} - \frac{Y_{Na}X_{Na}}{2} - \frac{Y_{Co}X_{Co}}{2} \quad (10)$$

Which can be solved for b

$$b = \frac{X^2 - Y_{Bi}X_{Bi}/3 - Y_{Na}X_{Na}/2 - Y_{Co}X_{Co}/2}{\sum_{i=1}^7 (x_i y_i) - (X_{Bi})^2/3 - (X_{Na})^2/2 - (X_{Co})^2/2} \quad (11)$$

Once b is determined, one can solve for the other three parameters:

$$k_{Bi} = (Y_{Bi} - bX_{Bi})/3$$

$$k_{Na} = (Y_{Na} - bX_{Na})/2$$

$$k_{Co} = (Y_{Co} - bX_{Co})/2$$

You can solve for b using Excel, or use the App found on the laboratory manual web site called disksources.html.