

## Attenuation of Radiation in Matter

In this experiment we will examine how radiation decreases in intensity as it passes through a substance. Since radiation interacts with matter, its intensity will decrease as it travels through a material. The attenuation properties of radiation will effect how much shielding is necessary and how much dosage one receives. Since radiation deposits its energy as it travels through matter, the amount of dosage one receives depends on how the radiation is attenuated. Radiation that attenuates fast, i.e. loses its energy in a short distance, will give a higher dosage (near the surface) than radiation that attenuates slowly, i.e. loses its energy over a long distance in the material.

First we consider how the intensity of gamma particles decreases as they pass through a material. We will see that the attenuation properties of gamma particles can be described fairly simply. The attenuation of  $\beta$  and  $\alpha$  particles is more complicated.

### Attenuation of gamma particles

As gamma particles pass through matter, they interact with the material and get absorbed. Suppose gamma particles are incident from the left on a slab, and the slab has a thickness of  $x$ . After passing through the slab of material, the gamma particles emerge on the right. Let the intensity of the gammas incident from the left be denoted as  $I_0$ , the initial intensity. Let the intensity of the gammas that emerge on the right, after passing through the slab, be denoted as  $I(x)$ , the final intensity. The unit for intensity,  $I_0$  and  $I(x)$ , is (number of gamma particles)/area/time. For units of intensity we could also use energy/area/time. Either type of unit could be used in our applications, but since our detectors measure counts, we will use (number of gammas)/time. As the slab gets thicker,  $x$  gets larger, and  $I(x)$  becomes smaller. As  $x$  increases, more radiation is absorbed in the material and less passes through and emerges on the right side.

How does  $I(x)$  depend on  $x$ ? To a very good approximation the number of gammas that pass through the slab decrease exponentially with thickness:

$$I(x) = I_0 e^{-\alpha x} \quad (1)$$

This equation is referred to as Lambert's law, and is applicable for linear attenuation. The geometry for linear attenuation is that the material is a rectangular slab, and the gamma's are incident perpendicular to the slab. The attenuation is in a "line" and is uniform in the plane of the slab. The units of  $\alpha$  are (1/distance) since

$\alpha x$  is unitless. In practice  $x$  often has units of cm, so  $\alpha$  will have units of  $cm^{-1}$ . The parameter  $\alpha$  is called the linear attenuation coefficient.

The linear attenuation coefficient,  $\alpha$ , will depend on the type of material and on the energy of the gamma radiation,  $E_\gamma$ . When gamma particles travel through matter, any interaction that occurs will be with the electrons in the substance. The higher the density of electrons, the more the gamma particles will attenuate. That is, if you want to shield yourself from gamma radiation, you need to put lots of electrons between yourself and the source. Since the electron density is approximately proportional to the density of the material, the attenuation coefficient  $\alpha$  depends strongly on the density of the substance. For this reason, it is convenient to define a mass attenuation coefficient by dividing  $\alpha$  by the density  $\rho$  of the material. Starting with the equation for linear attenuation:

$$I(x) = I_0 e^{-\alpha x} = I_0 e^{\frac{\alpha}{\rho}(\rho x)} \quad (2)$$

$$I(x) = I_0 e^{-\mu z} \quad (3)$$

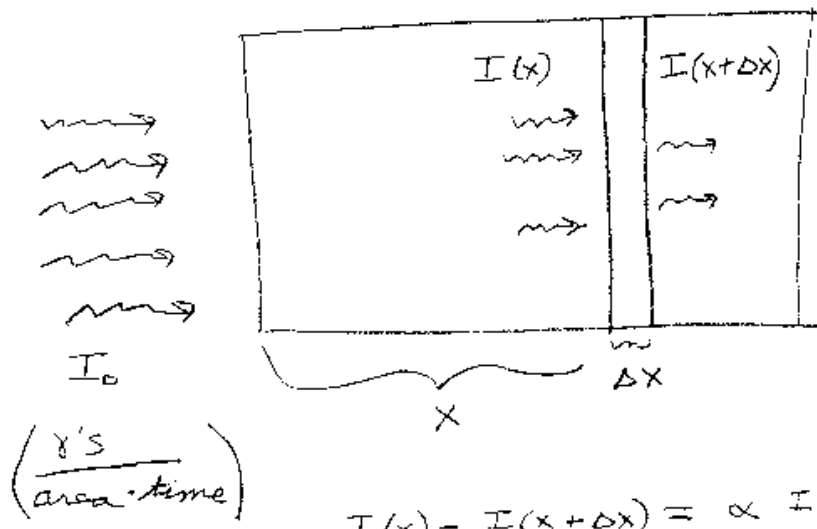
where  $z \equiv \rho x$ , and the mass attenuation coefficient  $\mu$  is defined as  $\alpha/\rho$ . The linear attenuation coefficient is just the mass attenuation coefficient times the density of the material:

$$\alpha = \mu \rho \quad (4)$$

The units of  $\mu$  are (area/mass), and are commonly given in ( $cm^2/g$ ). Since  $\mu z$  is unitless,  $z$  is in units of (mass/area) or ( $g/cm^2$ ). The "thickness"  $z$  can be thought of as the number of grams of "thickness" per square centimeter area of the slab. That is, for one square centimeter of area,  $z$  is the number of grams of the substance that the gamma's travel through. The more mass there is, the more electrons there are, and the more attenuation. Materials with a high density will have a large  $z$  (lots of electrons) for a small thickness, and are good for shielding gammas.

In tables of attenuation coefficients, it is the mass attenuation coefficient that is always listed. This is because the mass attenuation coefficient,  $\mu$ , has the density dependence somewhat factored out. The mass attenuation coefficient mainly depends on energy. For a fixed energy,  $\mu$  is roughly the same for a large class of materials. When performing an attenuation calculation, one looks up the mass attenuation coefficient in the tables, multiplies by the density of the substance to obtain  $\alpha$ , then uses the linear attenuation equation.

Why does the attenuation follow an exponential formula? Such a simple relationship must derive from a simple reason. The derivation of Lambert's law can be obtained



$$I(x) - I(x+\Delta x) = \alpha I(x) \Delta x$$

$$I(x+\Delta x) - I(x) = -\alpha I(x) \Delta x$$

$$\lim_{\Delta x \rightarrow 0} \frac{I(x+\Delta x) - I(x)}{\Delta x} = -\alpha I(x)$$

$$\frac{dI}{dx} = -\alpha I$$

$$I = I_0 e^{-\alpha x}$$

by considering a very thin slice of the slab of thickness  $\Delta x$  as shown in the figure.

Let  $I(x)$  be the intensity of the radiation incident on the thin slab from the left, and let  $I(x + \Delta x)$  be the intensity of the radiation after passing through the thin slab. From quantum mechanics, the interaction of gamma particles (photons) with electrons is probabilistic. Thus, any particular gamma particle has a certain probability of being scattered with an electron in the thin slab. Since the number of electrons the gamma will encounter is proportional to  $\Delta x$ , then for small enough  $\Delta x$  the probability that a gamma will interact with an electron in the thin slab is proportional to its thickness:

$$\textit{Probability for an interaction} = \alpha(\Delta x) \quad (5)$$

As with radioactive decay, probability enters in the description of nature. In radioactive decay, the parameter  $\lambda$  was equal to the probability to decay per unit time. Here, the parameter  $\alpha$  equals the probability for the gamma to interact per unit length. Now if one has  $I(x)$  particles at a distance  $x$ , and  $I(x + \Delta x)$  particles at a distance  $x + \Delta x$  into the slab, the number of particles lost per area is just the difference  $I(x) - I(x + \Delta x)$ . However, the number lost per area equals the number incident times the probability for any one gamma to interact with an electron in the slab:

$$I(x) - I(x + \Delta x) = (\textit{probability for one gamma to interact})I(x) \quad (6)$$

$$I(x) - I(x + \Delta x) = \alpha(\Delta x)I(x) \quad (7)$$

Dividing both sides by  $\Delta x$  and multiplying by -1 gives

$$\frac{I(x + \Delta x) - I(x)}{\Delta x} = -\alpha I(x) \quad (8)$$

Taking the limit as  $\Delta x$  goes to zero gives a simple differential equation for I:

$$\frac{dI}{dx} = -\alpha I \quad (9)$$

The general solution to this differential equation is Lambert's law:

$$I(x) = I_0 e^{-\alpha x} \quad (10)$$

where  $I_0$  is the initial intensity at  $x=0$ . The derivation gives us two results: the reason Lambert's law is valid, and the meaning of the linear attenuation coefficient.

The validity of Lambert's law follows from the fact that the interaction of photons with electrons is probabilistic, and a single interaction removes the gamma from the beam. Since the parameter  $\alpha$  in our derivation is the same as the linear attenuation coefficient, we have a physical interpretation for  $\alpha$ . The linear attenuation coefficient is the probability per unit length that a gamma particle will undergo an interaction in the material. If  $\alpha$  is large (small), the probability of an interaction is high (low) and the beam attenuates in a short (long) distance.

We note that Lambert's law applies to gamma particles of a single energy, monoenergetic gammas. In the derivation, we assumed that once the gamma particle interacted with an electron, it was removed. The gamma is removed when the process is photo-absorption, but if Compton scattering occurs the gamma has not been absorbed. In Compton scattering, the gamma particle is scattered out of the beam with a lower energy. Thus,  $I(x)$  in Lambert's law refers to the intensity of gammas with a particular energy.

The mass attenuation coefficient  $\mu$  does not have a particularly strong dependence on the type of material, but  $\mu$  will vary dramatically with energy. For low energy gamma particles (or X-rays)  $\mu$  will be much larger than for high energy gammas. This is because photo-absorption is much more probable at lower energies.

Since photo-absorption is the process for producing a photopeak in our scintillation detectors,  $\mu$  for photo-absorption is related to the efficiency of our detectors. Note the particularly strong energy dependence of the mass attenuation coefficient for photo-absorption.

## Experiment 4

Your task for this experiment is to measure the mass attenuation coefficient for following:

1. 662 KeV gamma particles in lead (Pb)
2. 32 KeV X-ray in Aluminum (Al)
3. Beta particles in Aluminum (Al)

All of these particles are given off by  $Cs^{137}$ . Although you should be able to do the experiment with no help, here are some tips:

Take a number of spectrum readings using  $^{137}Cs$  as a source. Keep the  $^{137}Cs$  source the same distance from the detector while you place the absorbers of various thicknesses between the source and the detector. Be sure you record data for a long enough time to obtain good statistics. Determine the area under each of the photopeaks for the gamma rays. For the  $\beta$  particles (electrons), you can use the Geiger Counter to record the counts. Then plot your data on semi-log paper, and determine the coefficient from the slope. Note: if your data lie in a straight line, you have demonstrated the validity of the exponential attenuation of these particles. Express your attenuation coefficients in units of  $cm^2/gm$ . For the  $\beta$  particles you will have to subtract off the counts due to the gamma and x-rays. **Include all data, calculations, and graphs in your writeup.**

### Recommended thicknesses:

- a) For the 662 Gamma ray in lead, use the 5 lead absorbers (A, B, C, D, E).
- b) For the 32 KeV X-ray use the Aluminum absorbers in increments of approximately  $0.060 gm.s/cm^2$ .
- c) For the beta particles use the Geiger counter and the 12 thinnest aluminum absorbers.

**Note:** Lambert's law assumed that the beam is well collimated. However, very often in the laboratory these "good geometry" conditions are not satisfied. A photon may be scattered and yet still be recorded by the detector. Photon's may also be scattered off shielding and still be recorded under the photopeak. Thus, absorption may not follow Lambert's law exactly. Particularly with the  $\beta$  particles, since  $\beta$  decay does not emit the electron with the same energy every time, Lambert's law might not follow exactly.

### Writeup for Experiment 4

1. Turn in all data, calculations, and graphs for your measurements of the attenuation coefficients for the three different cases. Be sure to include the correct units on all quantities.
2. (Question): Discuss how well Lambert's law is satisfied for the three cases above: a) 662 KeV gamma's through lead, b) 32 KeV X-rays through aluminum, and c)  $\beta$  particles through aluminum.
3. (Question): Which of these particles travels the furthest distance through matter? travels the shortest distance through matter?
4. How do your values for  $\mu$  for the 662 KeV gamma and the 32 KeV X-ray compare to the values listed in the literature shown below? Since these two energies lie between the values listed in the table, you can interpolate between the values in the table below or graph the values in the table along with your value.

Photon Energy (KeV)	Lead ( $cm^2/g$ )	Aluminum ( $cm^2/g$ )
20	85.7	3.41
30	29.7	1.12
40	14.0	0.567
50	7.81	0.369
60	4.87	0.280
80	2.33	0.203
100	5.4	0.171
150	1.97	0.138
200	.99	0.122
300	0.404	0.104
400	0.231	0.0927
500	0.161	0.0844
600	0.125	0.0780
800	0.0885	0.0684

Table I. Linear mass attenuation coefficients  $\mu$  for photons passing through Lead and Aluminum. The units of  $\mu$  are  $cm^2/g$ .