K^{40} activity and Detector Efficiency

Your goal in this experiment is to determine the activity of a salt substitute purchased in a local store. The salt subsitute is pure KCl. Most of the potassium found on earth is the non-radioactive isotope K^{39}. However, there is some of the isotope K^{40} that is present in all natural potassium. K^{40} is radioactive and emits a gamma with energy 1460 KeV. We will measure the radiation given off by the salt sample and then determine the sample's activity in Decays/sec.

To obtain an accurate measurement of the activity, it is important to know the efficiency of the detector. The most important part of the experiment will be to determine the detector's efficiency. In the next section, we discuss what efficiency is and how to measure it.

Efficiency Calibration of Solid Scintillation Detectors

The efficiency ε of a detector is defined as (the number of particles detected)/ (the number of particles emitted):

$$efficiency = \frac{the\, number\, of\, particles\, detected}{the\, number\, of\, particles\, emitted} \tag{1}$$

The efficiency is a number between zero and one. If we know the efficiency of our detector, then measuring the number of particles detected will allow us to determine the number of particles emitted in our sample. The efficiency of a detector will depend on a few factors, the most important are:

1. **The source-detector geometry**: The number of particles detected will depend on how close the source is to the detector. The closer the source is to the detector, the larger the efficiency will be.
2. **The size of the detector**: Larger detectors will usually be more efficient, since they have a larger volume for the particles to absorbed in.
3. **The energy of the gamma (or X-ray) radiation**: The photopeak is produced by photo-absorption. The photo-absorption process has a strong energy dependence. For high energy photons, photo-absorption has a lower probability to occur than photons of low energy.

For solid scintillation detectors, NaI and Ge, the dependence of ε on energy, number 3 above, is quite large. For example, NaI detectors can detect 100 KeV gammas about 4-5 times more efficiently than 1200 KeV gammas. This means that although a photopeak at 1200 KeV is small compared to one at 100 KeV in a particular spectrum, there might be more 1200 KeV gamma emitted than 100 KeV gammas.

Since the efficiency depends on the three factors listed above, one often keeps the source-detector geometry fixed during a series of experiments. That is, for a series of experiments one places all the samples in the exact location relative to the detector. Also, one uses samples that are all the same size and shape. If this is done, then factors 1 and 2 above are the same for all the samples in a particular experiment. In this case, the only efficiency calibration necessary is the energy dependence of ε. In the last experiment of this class, we will be able to have a standard that has the same geometry as our sample. However, for this experiment, Exp 3, we will need to estimate the geometry factor since our standards are point sources and our sample is cylindrical.

The energy dependence for a particular source-detector geometry is measured by using standardized sources. One can purchase sources in which the activity has been calibrated by the manufacture. If the activity of the source is known, then the number of gamma particles emitted can be calculated. By measuring the number of gammas (of a particular energy) detected during a specific time interval, the efficiency ε can be determined.

Efficiency Calculation including Source-Detector Geometry

The distance from the sample to the detector and the size of the detector are important factors that affect the detector's efficiency. We would like to include these affects in our measurements. The method that seems to work best (i.e. is simple and fairly accurate) with our NaI detectors is to use the ansatz:

$$\frac{Counts\ Detected}{time} = \frac{\gamma' s\ emitted}{time} \left(\frac{\pi r^2}{4\pi (x+d)^2}\right) \epsilon \tag{2}$$

where πr^2 is the cross sectional area of the detector, x is the distance from the source to the front face of the detector, and d is the distance from the front face of the detector to the "effective center" of the detector. The geometry factor $(\pi r^2)/(4\pi(x+d)^2)$ represents the fraction of gammas emitted that hit the front face of the detector. This geometry factor is just an approximation, but usually gives consistent results in our experiments. In our laboratory, the NaI crystal in the detector is cylinderical in shape, with a height of 2 inches and a diameter of 2 inches. Finally, the efficiency, ϵ is the probability of photoabsorption if a gamma enters the detector.

The number of γ's emitted can be determined from the activity, A, of the sample. For a particular γ energy, the number of γ's emitted per second equals the activity times the Yield, Y. The Yield is the probability that a γ is emitted during the nuclear decay. In terms of the activity and yield, the above equation becomes

$$\frac{Counts\ Detected}{sec} = AY(\frac{\pi r^2}{4\pi(x+d)^2})\,\epsilon \tag{3}$$

In the experiment, you will first determine d by collecting data from one source located at different distances x from the detector. A computer program that does a χ^2 fit of the data to determine the best estimate of d is available on the lab computers. The program was written by Sue Hoppe (2003), a Cal Poly Pomona physics major.

Once you have determined d for your detector, you can measure how the efficiency ϵ depends on the energy of the gamma. For standards, we will use Cs^{137} ($E_\gamma =$ 662KeV), ^{60}Co ($E_\gamma = 1173.237$ and 1332.501 KeV), Na^{22} ($E_\gamma = 511$ and 1275 KeV), and Bi^{207}. Once the efficiencies for the energies of the calibration sources have been determined, a graph of efficiency vs. energy, $\epsilon(E)$ can be plotted. As you will see, ϵ has a strong energy dependence. There is no simple theory to use to determine what the shape the calibration curve should be. Usually one makes a log-log plot and assumes a power law relationship. Once the calibration curve has been determined, you can extrapolate to find the efficiency of the detector for the energy of the gamma emitted by K^{40}, 1460 KeV.

Calibration of the efficiency graph is not as accurate as the calibration of energy for the scintillation detector. One problem is that it is expensive to obtain accurate calibrated standards. In our laboratory, the standards are calibrated for activity to within 5%. Errors also enter due to the uncertainty of the geometry factor and extrapolating. Thus the uncertainty in the efficiency calibration can be as large as 10-30%.

Experiment 3

Your goal in this experiment is to determine the activity of a KCl salt substitute sample. The experiment consists of three parts:

1. determining the "effective distance" d for the geometry factor

2. measuring different standards to obtain the energy dependence of the efficiency $\epsilon(E)$

3. measuring the KCl sample.

Measuring the Geometry Factor

In this part, you will measure **one standard** at 4 or 5 different distances from the detector. Place a calibrated source (i.e. ^{137}Cs) in the top slot of your detector. Measure the distance from the source to the side of the detector. Record data for a specific time period. The time period will depend on the activity of the source. With active sources you only need to take data for 1-2 minutes when the source is near the detector and 2-3 minutes or so when it is far away. Save your data and use the Gaussian curve fitting program to measure the area under the photopeak for each different distance setting. Your data table should look something like:

distance x	Counts Recorded	Counting Time	Counting Rate (1/sec)
—	—	—	—

Use the computer program on your lab computer, written by Susan Hoppe, to fit the data and determine the "best fit" value for the effective distance d.

Measuring the Energy Dependence of ϵ

Using the calibrated sources ^{137}Cs, ^{22}Na, ^{60}Co, and ^{207}Bi, determine the efficiency ϵ for the energies of the photopeaks. For best accuracy, you should place all the sources at approximately the same distance x from the detector. Use the Gaussian curvefitting program to determine the counts under the photopeak (area).

Data for Standards

Isotope	half-life	Energy	Yield
Cs^{137}	30 yrs	662 KeV	0.85
Na^{22}	2.62 yrs	511 KeV	1.80
Na^{22}	2.62 yrs	1275 KeV	1.0
Co^{60}	5.2714 yrs	1173.237 KeV	1.0
Co^{60}	5.2714 yrs	1332.501 KeV	1.0
Bi^{207}	31.55 yrs	— KeV	.977
Bi^{207}	31.55 yrs	— KeV	.745

Data for K^{40}

Isotope	half-life	Energy	Yield
^{40}K	1.277×10^9 yrs	1460 KeV	0.1069

Once you have calculated $\epsilon(E)$ for the five standard energies, graph your results. Extrapolate your graph to estimate the efficiency ϵ for the energy of the gamma given off by K^{40} (i.e. 1460 KeV). We will put our results on the board for comparison with everyone in the class.

Measuring the KCl salt sample

The KCl sample will be placed as close to the detector as possible without damaging the detector, i.e. such that the end just touches the detector. Let the length of the sample container be L. Since there is just a little amount of radiation emitted from the sample, we will count for a long time to get good statistics.

Since the salt sample is not a point source, we need to make some approximations for the geometry factor. One approximation is to assume that all the salt is located at the center of the container. That is, use $x = L/2$ for the geometry factor. In this case, the count rate is

$$\frac{Counts\ Detected}{sec} \approx AY(\frac{\pi r^2}{4\pi(L/2+d)^2})\,\epsilon \tag{4}$$

Assuming that all the salt is located at $x = L/2$ will result in too small of a geometry factor. Treating the sample as if all the salt were located at its center would be correct if the geometry factor depended linearly with distance from the detector. However, the geometry factor decreases as $1/r^2$, and the effective center is closer than $L/2$ from the detector. A smaller geometry factor will result in an over estimate of the activity of the salt sample.

An approximation that might be better is to assume that the salt all lies on the axis of the detector. If this is true, we can integrate the geometry factor from $x = 0$ to $x = L$. The geometry integral would be:

$$\frac{Counts\ Detected}{sec} \approx Y\int_0^L \frac{\pi r^2}{4\pi(x+d)^2} A\frac{dx}{L}\,\epsilon \tag{5}$$

Evaluation of the integral results in the following expression

$$\frac{Counts\ Detected}{sec} \approx AY(\frac{\pi r^2}{4\pi d(L+d)})\,\epsilon \tag{6}$$

You should calculate the salt's activity using each of the expressions and comment on your results.

After the counting period is finished, use the Gaussian curvefitting program to measure the counts under the $KCl - \gamma$ photopeak (1460 KeV). Before class, the instructor started a long two hour background measurement. Measure the counts under the $KCl - \gamma$ photopeak for the background. Carry out the necessary calculations to determine the activity of the K^{40} in your salt sample. Estimate the uncertainty of your value.

Laboratory Writeup for Experiment 3

1. Turn in your data and results for measuring the "effective distance" d. Include the graph you used when finding "effective distance" d. That is, try and print the graph from invsq2 (Sue Hoppe's program). Does the data follow the inverse square model? Discuss your results.

2. Turn in and show **all your data and all calculations** for measuring the efficiency ϵ for each of the seven calibration energies.

3. Turn in a graph of ϵ versus E for the seven calibration energies, and explain how you extrapolated to find $\epsilon(1460\,KeV)$.

4. Show all calculations for your values for determining the activity of the salt sample. Show and discuss the results obtained for each of the approximations that we used for the geometry factor.

5. In class you will be shown how to determine the activity of the salt sample another way. Show this calculation. Does this result fall within your estimate of 4 above? Discuss.

6. Evaluate the integral in Eq. 5 and obtain the expression in Eq. 6 for the geometry factor for the salt sample.

7. Question: If a 1000 KeV gamma ray interacts with the NaI detector. Which process (photoelectric effect or Compton Scattering) is most likely to occur? Why?