

**Physics 404**  
**Fifth Homework Assignment**  
**Due Friday June 1**

Choose any 5 of the following 7 problems. The students from Fresno who are going to CERN in the summer need to do the Relativistic Kinematics and the Root programming problems.

**Problem 1:** Chapter 7, Problem 7.9

The charge conjugation operator ( $\hat{C}$ ) takes a Dirac spinor  $\psi$  into the "charge-conjugate" spinor  $\psi_c$ , given by

$$\psi_c = i\gamma^2\psi \tag{1}$$

where  $\gamma^2$  is the third Dirac gamma matrix. Find the charge-conjugates of  $u^{(1)}$  and  $u^{(2)}$  and compare them with  $v^{(1)}$  and  $v^{(2)}$ .

**Problem 2:** Chapter 7, Problem 7.15

Show that the adjoint spinors  $\bar{u}^{(1,2)}$  and  $\bar{v}^{(1,2)}$  satisfy:

$$\begin{aligned}\bar{u}(\gamma^\mu p_\mu - mc) &= 0 \\ \bar{v}(\gamma^\mu p_\mu + mc) &= 0\end{aligned}$$

Hint: Take the transpose conjugate of Equations 7.49 and 7.50; multiply from the right by  $\gamma^0$ , and show that  $(\gamma^\mu)^\dagger\gamma^0 = \gamma^0\gamma^\mu$ .

**Problem 3:** Chapter 7, Problem 7.18

Show that the spinor representing an electron at rest is an eigenstate of the parity operator, P. What is its intrinsic parity? How about the positron? What if you changed the sign convention in Eq. 7.61? Notice that whereas the absolute parity of a spin-1/2 particle is in a sense arbitrary, the fact that the particles and antiparticles carry opposite parity is not arbitrary.

**Problem 4:** Chapter 7, Problem 7.33

Confirm the trace theorems 10, 11, 12, and 13.

**Problem 5:** Chapter 7, Problem 7.36

Evaluate the following traces:

a)  $Tr[\gamma^\mu \gamma^\nu (1 - \gamma^5) \gamma^\lambda (1 + \gamma^5) \gamma_\lambda]$

b)  $Tr[(\not{p} + mc)(\not{q} + Mc)(\not{p} + mc)(\not{q} + Mc)]$ , where  $p$  is the four-momentum of a (real) particle of mass  $m$  and  $q$  is the four-momentum of a (real) particle of mass  $M$ . Express your answer in terms of  $m, M, c$  and  $p \cdot q$ .

**Problem 6: Relativistic Kinematics Problem**

A photon scatters off an electron. Before the collision the electron is at rest and the photon has an energy of  $m_e c^2/3$  in the reference frame of the lab. Here  $m_e$  is the mass of the electron. After the collision the photon scatters off at  $90^\circ$  from its initial direction, as viewed in the lab frame. If  $q^\mu$  is the 4-momentum transferred from the photon to the electron, what is the invariant  $q^2 = q_\mu q^\mu$ ? You can express your answer in terms of  $m_e^2 c^2$ .

**Problem 7: Root Programming Problem:**

You are planning to scatter  $\pi^-$  off of protons. The protons will be at rest in the laboratory. Write a program in root that will print out the invariant energy,  $\sqrt{s}$ , as a function of pion laboratory momentum. Your program should print out the values of  $\sqrt{s}$  in the form of the following chart for the pion momenta (in units of  $MeV/c$ ) listed below:

| $P_\pi$ (MeV/c) | $\sqrt{s}$ (MeV) |
|-----------------|------------------|
| 100             |                  |
| 200             |                  |
| 300             |                  |
| 400             |                  |
| 500             |                  |
| 600             |                  |
| 700             |                  |
| 800             |                  |
| 900             |                  |
| 1000            |                  |