Geiger Counter Experiments

In this laboratory session, we will do different experiments with the Geiger Counter: determining the proper operating voltage, estimating the efficiency, and measuring the half-life of Ba^{137m}.

1. Operating Voltage of the Geiger Counter

First, we will measure the Geiger counter's response as a function of applied voltage.

a) Place a source under the Geiger counter tube.

b) Set the timer to count for ten minutes or longer.

c) Set the voltage to zero first, then slowly turn up the voltage until the counter starts to record counts. This is the "starting voltage".

d) Take 1 minute readings, increasing the voltage by about 10 or 20 volts each time. Make a table of your data.

Note: to prevent damaging the tube, **do not increase the voltage more than 150 volts beyond the starting voltage, and certainly not more than 1000 volts.**

e) Graph your results using Excel or on linear graph paper. Label on your graph the starting voltage and the plateau region. Also label the proper operating voltage on the graph. From your graph, do you think your Geiger counter tube is operating properly? Why or why not?

2. Efficiency of the Geiger Counter

In this part, you will estimate the efficiency of the Geiger-Mueller tube for a particular source. The efficiency of the Geiger counter will depend on the sample, so be sure to record the sample used. From the activity written on the source, use the half-life formula to determine the activity in decays/sec of your source today. Place your source as close to the tube as you can, and count for 2 minutes. Estimate the distance the source is away from the tube. Determine the efficiency of the Geiger-Mueller tube for this positioning of the source. We will define the efficiency ϵ as:

$$\epsilon \equiv \frac{particles\ detected}{particles\ emitted} \tag{1}$$

Note: If you use Cs^{137} as your source, the yields are 100% for beta's, 85% for gamma's and 8% for X-rays. That is for every 100 decays there are 100 beta's, 85 gamma's and 8 X-rays. Since the Geiger counter can not differentiate between the different types of radiation, we will just take the number of particles emitted to be equal to the number of decays.

3. Statistical Analysis of the Decay

Here we will do two exercises in examining the statistical nature of nuclear decay.

1. Place a source under the Geiger-Mueller tube. Collect data for two minutes. Record the number of counts. Repeat the collection 30 times if time permits. Your data will consist of 30 integers, each one being the counts for two minutes. Are all the integers equal (i.e. do you get the same number of counts each time?). Find the average number of counts, and the standard deviation σ. If N_i represents the number of counts in measurement i, then

$$N_{ave} = \frac{\sum_{i=1}^{30} N_i}{30} \tag{2}$$

$$\sigma = \sqrt{\frac{\sum_{i=1}^{30}(N_i - N_{ave})^2}{29}} \tag{3}$$

Once you have found N_{ave} and σ write your results on the board. We will have a classroom discussion of our measurements.

2. Next to the link for this experiment, there is a file called "time interval data". There are 66000 numbers in the file. The numbers are the times between successive Geiger Counter pulses in units of μ-sec (10^{-6} sec). Make a histogram of these times. Initially try 100 μsec as a bin size. From the histogram, estimate the dead-time of the detector. For the times after the dead-time, see if an exponential function fits the data. What "physics" does this data support?

4. Measurement of the half-life of Ba^{137m}.

The half-life of Ba^{137m} is on the order of minutes. In this experiment, we will record the counts for the Ba^{137m} source for a 10 second counting time. We take these readings every 30 seconds. Before you start the experiments make a data table similiar to the form below:

time (sec)	Counts in 10 seconds
0	...
30	...
60	...
90	...
...	...

1. After the instructor places the sample under your Geiger Counter tube, start recording data.

2. After correcting for dead time and background, make a graph of the number of counts/sec as a function of time. Fit the graph with an exponential function. Determine the decay constant (and half-life) from the slope of the graph.

Report for Experiment 1

1. Make a table and graph of Counts vs. voltage for your Geiger counter tube. Label on your graph the starting voltage, operating voltage, and the plateau region.

2. Show your data and calculations for determining the efficiency of your Geiger counter.

3. Show your data and calculations for the 30 "2 minute" recordings. The calculations will be for the average number of counts N_{ave} and the standard deviation σ. Show the histogram plot of the time interval data. Determine the dead time and the parameters of the exponential fit. What "physics" do this data support?

4. For the Ba^{137m} decay, use Excel to make a graph of counts/sec vs. time. Include the corrections for background and dead-time.
a) Is the decay exponential? i.e. does it obey the half-life formula?
b) If it does follow an exponential decay, what is the half-life of the decay?

Decay rate of radioactive nuclei

The exponential decrease of the decay rate as a function of time can be understood from one principle of nature:

Each radioactive nucleus has a certain probability to decay per unit time. This probability does not depend on how long the nucleus has been in its excited state (i.e. radioactive).

The probability to decay per unit time is denoted by the symbol λ, and is called the decay constant of the decay. λ has units of 1/time. The probability that a particular nucleus will decay in the time interval δ is $\lambda\delta$ **in the limit as** $\delta \rightarrow 0$. Our radioactive sample has a large number of radioactive nuclei, N_0. The probability that one nucleus will decay in the time interval δ is $N_0\lambda\delta$ in the limit as $\delta \rightarrow 0$. If the efficiency of our detector is ϵ, then the probability that our Geiger Counter tube will detect a particle in the time interval δ is $\epsilon N_0 \lambda\delta$, in the limit as $\delta \rightarrow 0$. For convenience we define $A \equiv \epsilon N_0 \lambda$. If the decay process is probabilistic, then there exists a probability per unit time A such that the probability that our Geiger Counter tube will detect a particle in the time interval δ is $A\delta$, in the limit as $\delta \rightarrow 0$.

Suppose we have an isotope with a long half-life. Since the half-life is long, N_0 hardly changes during our measurement. We can ask the following question: What is the probability P_{not} that we will not record a count within a time t since the last count was recorded? This can be answered as follows. Divide the time t into N equal segments, each of duration δ. That is $\delta = t/N$. Then P_{not} is given by

$$P_{not} = (1 - A\delta)^N \tag{4}$$

Now we need to take the limit as $\delta \rightarrow 0$, or as $N \rightarrow \infty$:

$$P_{not} = \lim_{N\rightarrow\infty} (1 - \frac{At}{N})^N \tag{5}$$

This limit is the exponential to the base e:

$$P_{not} = e^{-At} \tag{6}$$

The probability that the Geiger counter will receive a signal between the time t and $t + \Delta t$ is therefore:

$$P(t)\Delta t \approx e^{-At} A\Delta t \tag{7}$$

We have used the approximation sign, since A is only defined in the limit as $\delta \to 0$ and Δt is a finite time. If we collect a total of C_{tot} data points, then the number of counts recorded between time t and $t + \Delta t$ is just

$$C(t) \approx C_{tot} A e^{-At} \Delta t \tag{8}$$

As $C_{tot} \to \infty$ and $\Delta t \to 0$ the approximation gets better. Your data are consistent with the probability hypothesis if $C(t)$ decreases exponentially.

Half-Life Decay

We will be recording the detected counts for a time interval of 6 seconds while the isotope decays. In our case, we will be recording data from an isotope that has a short half-life. The total number of radioactive nuclei will decrease throughout the experiment. Suppose we start the experiment at time $t = 0$. Let $N(t)$ be the **average number** of radioactive nuclei present after time t. As before, the probability that one nucleus will decay in a time δ is $\lambda\delta$ in the limit as $\delta \to 0$. In a small time increment δ, the average number of nuclei that decay is:

$$N(t) - N(t + \delta) \approx N(t)\lambda\delta \tag{9}$$

The approximation becomes better as $\delta \to 0$. Taking this limit we have

$$\frac{dN(t)}{dt} = -\lambda N(t) \tag{10}$$

The solution to this equation is

$$N(t) = N_0 e^{-\lambda t} \tag{11}$$

Remember that this equation is for the expected average number of radioactive nuclei, $N(t)$, present at time t. If $N(t)$ is the actual number of radioactive nuclei, then the equation is only an approximation. Since then the left side is an integer, and the right side a continuous function of t.

In our experiment, we will record the times, starting from $t = 0$, when the counter receives signals. We will then determine the number of counts received between (say) $t = 0$ and $t = 10$sec, $t = 10$sec and $t = 20$sec, etc... Call the number of counts received between time t and $t + \Delta t$ as $D(t)$ where Δt could be 10 seconds. Then,

$$D(t) \approx \epsilon N(t)\lambda\Delta t \tag{12}$$

where ϵ is the efficiency of the detector. The approximation is due to two reasons. First, Δt is a finite time. Second, $N(t)$ is an average value, so the above equation represents the expected value for $D(t)$. If we substitute in for $N(t)$ we have:

$$D(t) \approx (\epsilon N_0 \lambda \Delta t) e^{-\lambda t} \tag{13}$$

Thus, we see that $D(t)$ should decrease (approximately) exponentially, with the approximation getting better as Δt gets smaller. One can also cast this equation in terms of base 2:

$$D(t) \approx (\epsilon N_0 \lambda \Delta t) (\frac{1}{2})^{t/\tau} \tag{14}$$

where τ is called the half-life. Equating the two equations gives a relationship between λ and τ:

$$\tau = \frac{ln(2)}{\lambda} \tag{15}$$