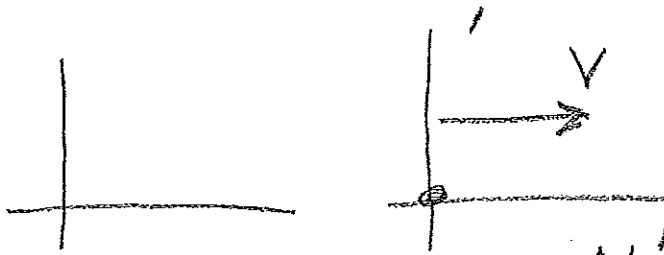


# Solutions to Homework 6

①



Muon  $\Delta t' = 2.2 \times 10^{-6} \text{ sec}$

$$\Delta t = 16 \times 10^{-6} \text{ sec} = \frac{2.2 \times 10^{-6} \text{ sec}}{\sqrt{1 - v^2/c^2}}$$

$$16 = \frac{2.2}{\sqrt{1 - v^2/c^2}}$$

$$\frac{v}{c} = \sqrt{1 - \left(\frac{2.2}{16}\right)^2} \approx .99$$

$$v \approx .99 c$$

②

In your reference frame  $\Delta t' = 1/2 \text{ year}$

In the earth's reference frame  $\Delta t = 500 \text{ years}$

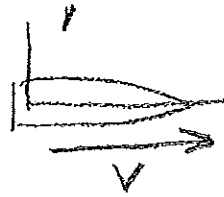
$$500 = \frac{1/2}{\sqrt{1 - v^2/c^2}}$$

$$\frac{v}{c} = \sqrt{1 - \left(\frac{1}{1000}\right)^2}$$

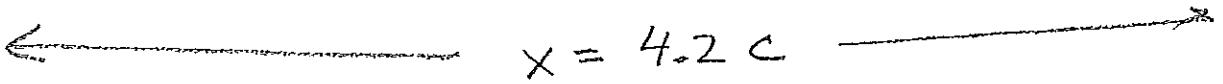
$$\frac{v}{c} \approx .99999995$$

$$v \approx .99999995 c$$

3



$$t' = \frac{1}{12} \text{ yrs}$$



$$x = \frac{vt'}{\sqrt{1 - v^2/c^2}}$$

$$4.2c = \frac{v(1/12)}{\sqrt{1 - v^2/c^2}}$$

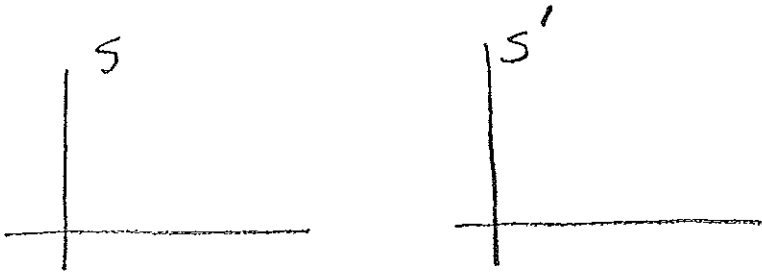
$$50.4 = \frac{v/c}{\sqrt{1 - v^2/c^2}}$$

$$1 - \frac{v^2}{c^2} = \left(\frac{1}{50.4}\right)^2 \frac{v^2}{c^2}$$

$$\frac{v}{c} = \frac{1}{\sqrt{1 + \left(\frac{1}{50.4}\right)^2}} \approx .9998$$

$$v \approx .9998c$$

4



$$v = -0.6c$$

$$x = 3000 \text{ m}$$

$$t = 4 \times 10^{-6} \text{ sec}$$

First event at (0, 0)

$$x' = \frac{3000 - 0.6c(4 \times 10^{-6})}{\sqrt{1 - 0.6^2}}$$

$$x' = \frac{3000 - 720}{.8}$$

$$x' = 2850 \text{ m}$$

4 cont

$$t' = \frac{4 \times 10^{-6} - (.6c)(3000)/c^2}{\sqrt{1 - .6^2}}$$

$$t' = \frac{4 \times 10^{-6} - 6 \times 10^{-6}}{.8} = -2.5 \mu\text{sec}$$

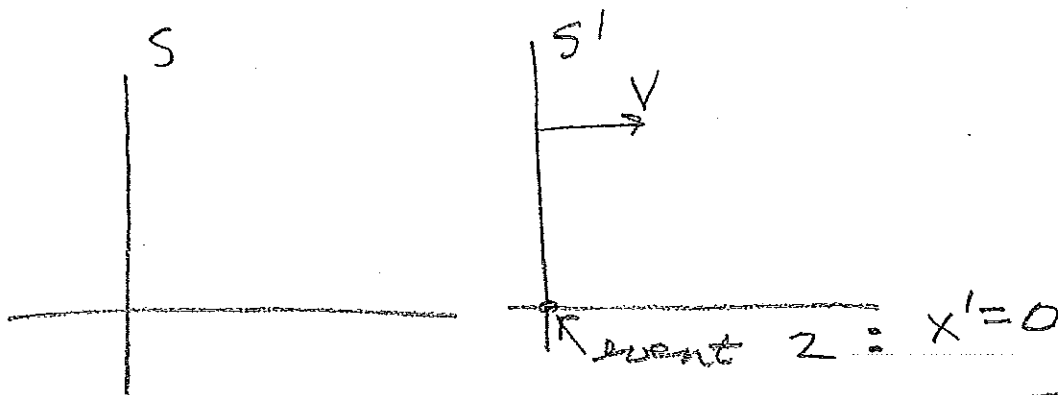
Event 2 occurs at

$$\begin{aligned} X' &= 2850 \text{ m} \\ t' &= -2.5 \mu\text{sec} \end{aligned}$$

Event 2 occurs before event 1 in the  $S'$  frame.

This can happen because the events are spacelike  $\Delta X^2 > c^2 \Delta t^2$

5



$$a) v = \frac{720 \text{ m}}{5 \times 10^{-6} \text{ sec}} = 1.44 \times 10^8 \text{ m/s} = \boxed{.48c}$$

$$b) s^2 = 720^2 - (3 \times 10^8)^2 (5 \times 10^{-6})^2$$

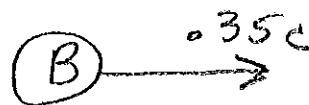
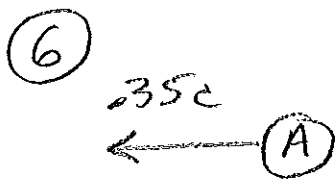
$$s^2 = 720^2 - (1500)^2 = -(1315)^2$$

In the  $S'$  frame  $\Delta X' = 0$

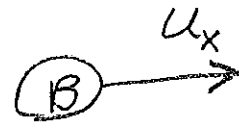
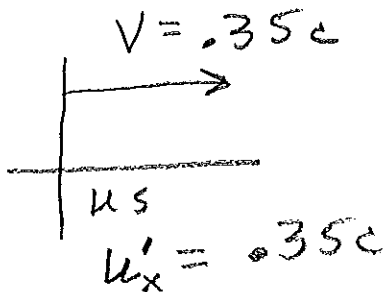
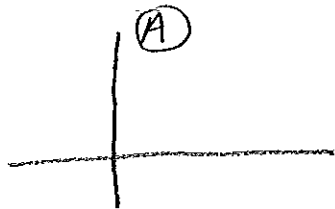
$$-(1315)^2 = 0^2 - (c \Delta t')^2$$

$$\Delta t' = \frac{1315}{c} \approx \boxed{4.38 \mu\text{sec}}$$

Event 1 precedes Event 2 in all frames



⑦ in A's reference frame



⑧ V = 0.35c

⑨  $u_x = \frac{u'_x + V}{1 + u'_x V / c^2} = \frac{0.35c + 0.35c}{1 + (0.35)(0.35)} = \frac{0.7c}{1 + 0.35^2} \approx \boxed{0.624c}$

⑩  $f = 100 \text{ MHz} \sqrt{\frac{1 - 0.9}{1 + 0.9}} = \boxed{22.9 \text{ MHz}}$

⑪  $f = f_0 \sqrt{\frac{1 - v/c}{1 + v/c}}$

$\frac{c}{\lambda} = \frac{c}{\lambda_0} \sqrt{\frac{1 - v/c}{1 + v/c}}$

$\lambda = \lambda_0 \sqrt{\frac{1 + v/c}{1 - v/c}}$

$\lambda = 450 \text{ nm} \sqrt{\frac{1 + 0.2}{1 - 0.2}} \approx \boxed{551 \text{ nm}}$

violet → green