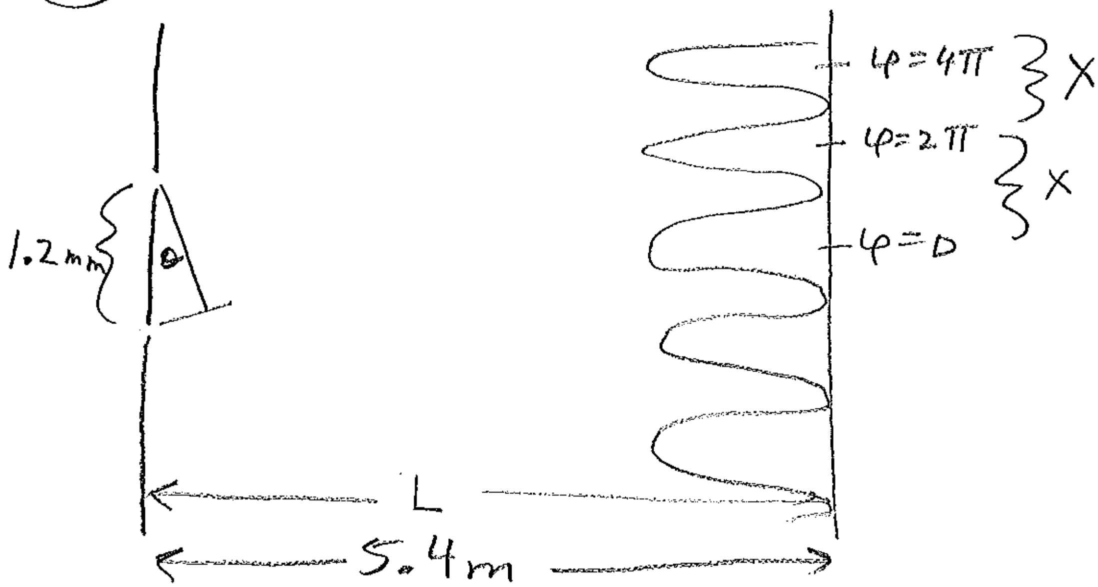
Solutions to HWK 5   PHY234

(1)



$\frac{X}{L} = \tan\theta$

$\tan\theta = \frac{X}{L}$

$$\varphi = \frac{d \sin\theta}{\lambda}(2\pi) = 2\pi \Rightarrow \sin\theta = \frac{\lambda}{d}$$

Using $\sin\theta \approx \tan\theta$, for small θ,

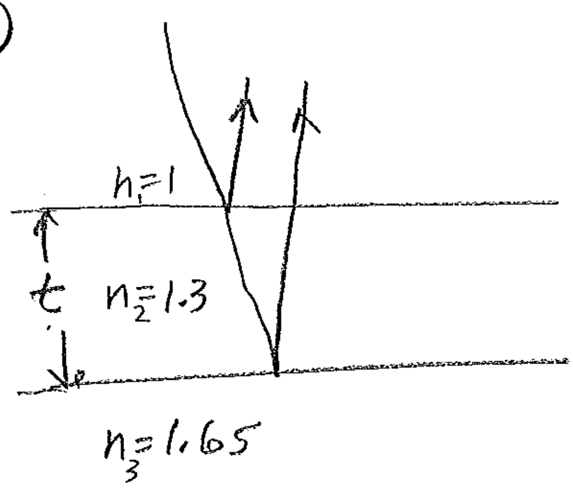
$$\frac{\lambda}{d} \approx \frac{X}{L} \quad \text{or} \quad X \approx \frac{\lambda}{d} L$$

$$X \approx \frac{500 \times 10^{-9}}{1.2 \times 10^{-3}} (5.4\,m)$$

$$X \simeq 2.25 \times 10^{-3}\,m$$

$$\boxed{X \simeq 2.25\ mm}$$

(2)



$$\varphi = \frac{2t}{\lambda_{film}}(2\pi) + \text{phase changes due to boundary}$$

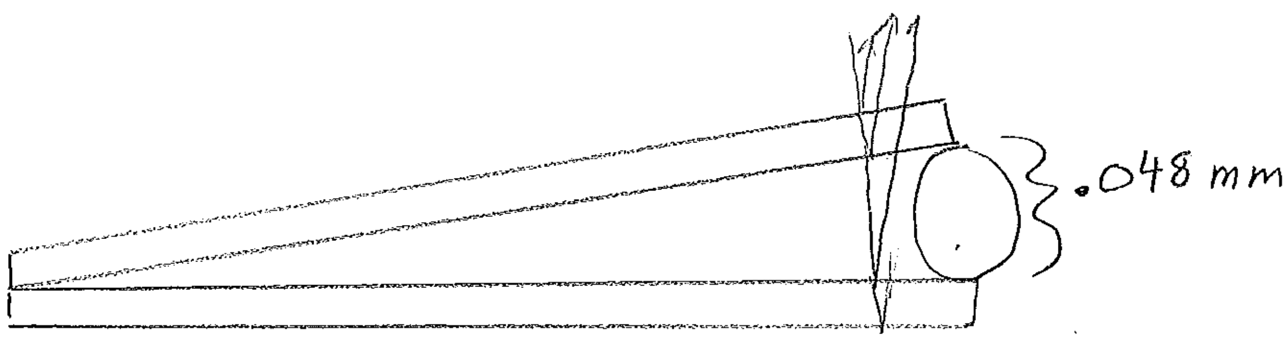
Since both boundarys produce a phase change of π

$$\varphi = \frac{2t}{\lambda_{film}}(2\pi) = 2\pi \quad \text{for constructive interference}$$

$$t = \frac{\lambda_{film}}{2}$$

$$t = \frac{\lambda}{2n_2} = \frac{680\ \text{nm}}{2(1.3)} = \boxed{261.5\ \text{nm}}$$

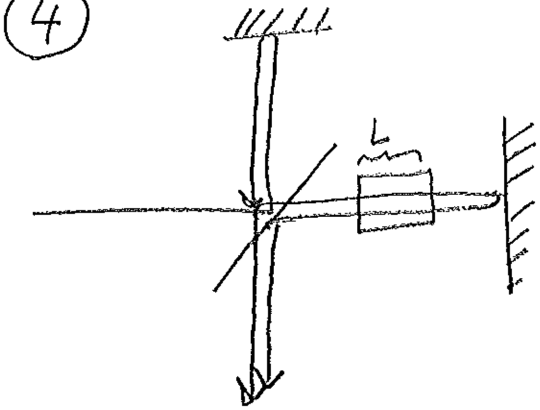
(3)



The extra distance that the reflected wave travels is twice the gap distance. For every additional wavelength a bright fringe will occur.

$$\frac{2(.048 \times 10^{-3})}{680 \times 10^{-9}} = \boxed{141} \text{ bright fringes}$$

(4)



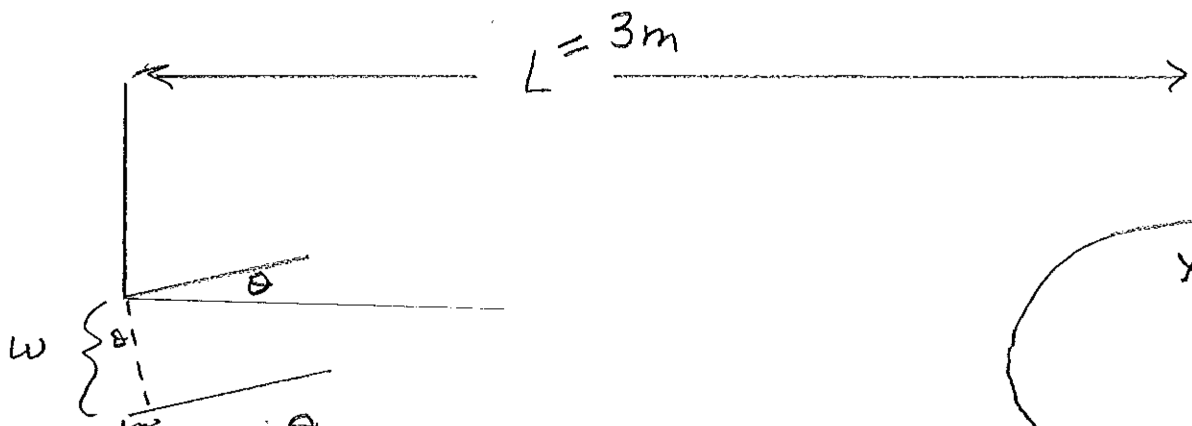
The difference in the number of wavelenghs with air and without air in the chamber will equal the number of fringes observed:

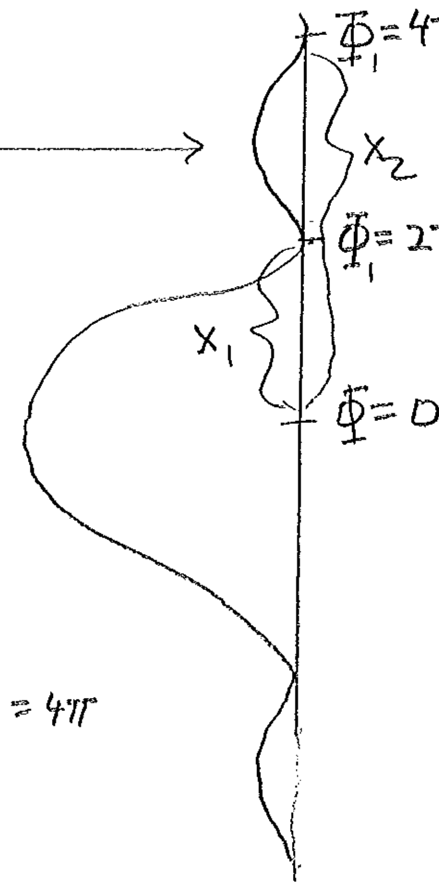
$$\frac{2L}{\lambda_0/n} - \frac{2L}{\lambda_0} = 60$$

$$\frac{2L}{\lambda_0}(n-1) = 60$$

$$n = 1 + \frac{60\,\lambda_0}{2L} = 1 + \frac{60(500\times10^{-9})}{2(.05\,m)} = \boxed{1.0003}$$

(5)





$$\Phi = \frac{w \sin\theta}{\lambda}(2\pi)$$

$$\frac{w \sin\theta_1}{\lambda}(2\pi) = 2\pi \qquad \frac{w}{\lambda}\sin\theta_2 (2\pi) = 4\pi$$

$$\sin\theta_1 = \frac{\lambda}{w} \qquad \sin\theta_2 = \frac{2\lambda}{w}$$

but $\tan\theta_1 = \frac{x_1}{L}$ and $\tan\theta_2 = \frac{x_2}{L}$

for small angles, $\sin\theta \simeq \tan\theta$

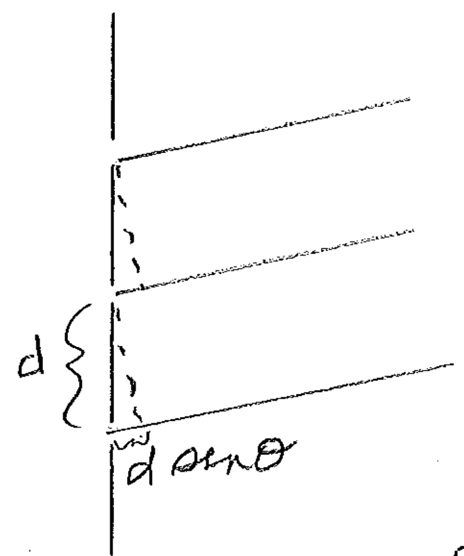
(5 cont.)

So $\frac{x_1}{L} \approx \frac{\lambda}{w}$ and $\frac{x_2}{L} \approx \frac{2\lambda}{w}$

$$x_2 - x_1 \simeq L\left(\frac{2\lambda}{w}\right) - \frac{L\lambda}{w} = \frac{L\lambda}{w} = \frac{(3\,\text{m})(589\times 10^{-9}\,\text{m})}{1\times 10^{-3}\,\text{m}}$$

$$x_2 - x_1 \approx \boxed{1.767\ \text{mm}}$$

(6)

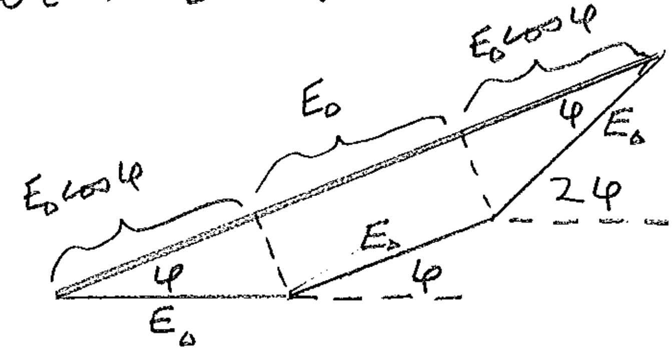


$$E_0 \cos\omega t + E_0 \cos(\omega t + \varphi) + E_0 \cos(\omega t + 2\varphi)$$

where
$$\varphi = \frac{d \sin\theta}{\lambda}\,(2\pi)$$

$$E_{NET} = E_0 \cos\omega t + E_0 \cos(\omega t+\varphi) + E_0 \cos(\omega t + 2\varphi)$$

Using phasors:



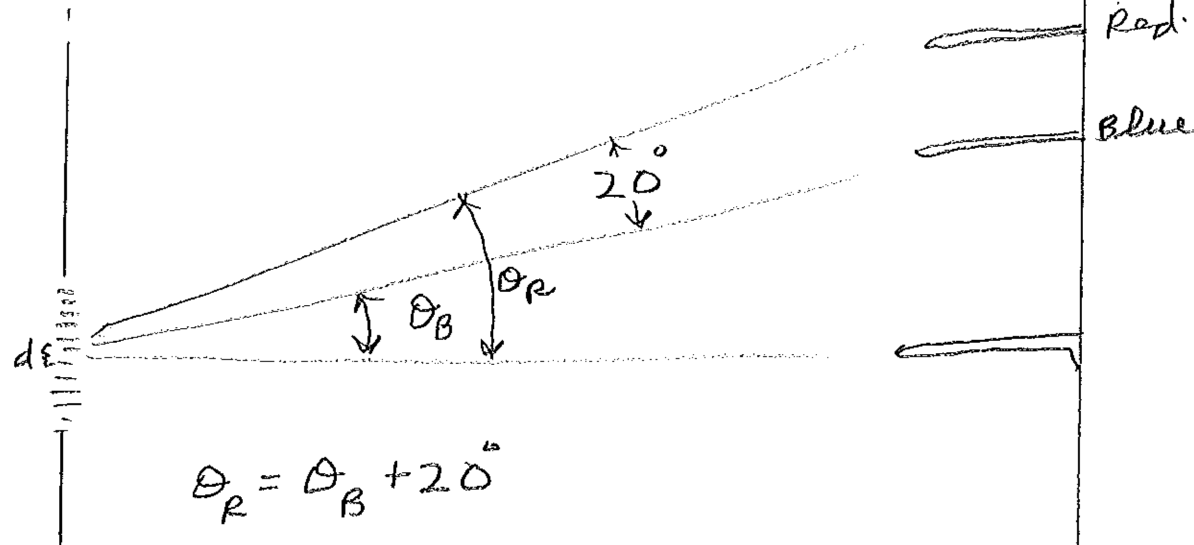
$$E_{NET} = E_0 + E_0 \cos\varphi + E_0 \cos\varphi = E_0(1 + 2\cos\varphi)$$

$$I \propto |E_{NET}|^2 = E_0^2 (1 + 2\cos\varphi)^2 = E_0^2(1 + 4\cos\varphi + 4\cos^2\varphi)$$

$$I_0 \propto 9E_0^2 \quad (\text{for } \varphi = 0)$$

So
$$\boxed{I = \frac{I_0}{9}(1 + 4\cos\varphi + 4\cos^2\varphi)}$$

(7)



$$\theta_R = \theta_B + 20°$$

For constructive interference, $\frac{d \sin\theta}{\lambda}(2\pi) = 2\pi$

$$\text{or } \sin\theta = \frac{\lambda}{d}$$

for Red:

$$\sin\theta_R = \frac{\lambda_R}{d}$$

for Blue

$$\sin\theta_B = \frac{\lambda_B}{d}$$

$$\sin(\theta_B + 20°) = \frac{\lambda_R}{d} \qquad \sin\theta_B = \frac{\lambda_B}{d}$$

divide both sides

$$\sin(\theta_B)\cos 20 + \cos(\theta_B)\sin 20 = \frac{\lambda_R}{d}$$

$$\cos(20) + \cot\theta_B \sin 20 = \frac{\lambda_R}{\lambda_B} = \frac{680}{430}$$

$$\theta_B = 28.06°$$

So $d = \frac{\lambda_B}{\sin\theta_B} = \frac{430\text{ nm}}{\sin(28.06)} = \boxed{914\text{ nm}}$

$$\#\text{ of rulings per mm} = \frac{1\text{ mm}}{914\text{ nm}} \simeq \boxed{1094\ \frac{\text{rulings}}{\text{mm}}}$$