Solutions to Homework 4 (PHY234)

(1)

$$\frac{\sin 40^\circ}{\sin 29^\circ} \approx 1.33$$

$$\frac{\sin 50^\circ}{\sin 35^\circ} \approx 1.34$$

$$\frac{\sin 80^\circ}{\sin 50^\circ} \approx 1.29$$

Yes, the data are consistent with

$$\frac{n_{water}}{n_{air}} \simeq 1.3$$

Since $n_{air} \simeq 1.00$

$$n_{water} \simeq 1.3$$

(2)

$$\frac{1}{o} + \frac{1}{i} = \frac{1}{f}$$

multiply both sides by oif

$$if + of = oi$$

$$\phi = oi - of - if$$

add f^2 to both sides

$$f^2 = oi - of - if + f^2$$

$$f^2 = (o-f)(i-f)$$

$$\boxed{f^2 = xx'}$$

where $x = o - f$
$x' = i - f$

③ a) $\frac{1}{20} + \frac{1}{i} = \frac{1}{10}$

$\frac{1}{i} = \frac{1}{10} - \frac{1}{20} = \frac{1}{20}$

$i = +20$

$m = -\frac{20}{20} = -1$

$i = 20$ cm $m = -1$ real image

b) $\frac{1}{5} + \frac{1}{i} = \frac{1}{10}$

$\frac{1}{i} = \frac{1}{10} - \frac{1}{5} = -\frac{1}{10}$

$i = -10$ cm

$m = -\frac{(-10)}{5} = 2$

$i = -10$ cm $m = +2$ virtual image

c) ~~Since~~ $m > 1$, the image is virtual
$i < 0$

$\frac{1}{5} + \frac{1}{i} = \frac{1}{10}$

$\frac{1}{i} = \frac{1}{10} - \frac{1}{5} = -\frac{1}{10}$

$i = -10$

$m = -\frac{(-10)}{5} = +2$

$i = -10$ cm $m = +2$ virtual image

also, since $m > 1$, ~~the~~ lens cannot be diverging. f must be positive $f = +10$

3 d) $m<1$. Since $|o|<|f|$ the image must be virtual, $i<0$. Since $m<1$ the lens must be diverging $f<0 \Rightarrow f=-10$ cm

$$\frac{1}{5}+\frac{1}{i}=-\frac{1}{10}$$

$$\frac{1}{i}=-\frac{1}{10}-\frac{1}{5}=-\frac{3}{10}$$

$$i=-\frac{10}{3}$$

$$m=-\frac{(-10/3)}{5}=\frac{2}{3}$$

$f=-10$ cm $i=-10/3$ $m=2/3$

e) $$+.5=\frac{-i}{10}$$

$$i=-5\text{ cm}$$

$$\frac{1}{10}-\frac{1}{5}=\frac{1}{f}$$

$$f=-10\text{ cm}$$

$i=-5$ cm $f=-10$ cm virtual image, diverging lens

f) $$-.5=\frac{-i}{10}$$

$$i=5\text{ cm}$$

$$\frac{1}{10}+\frac{1}{5}=\frac{1}{f}$$

$$f=\frac{10}{3}\text{ cm}$$

$i=5$ cm $f=\frac{10}{3}$ cm real image

4) $\frac{1}{o} + \frac{1}{i} = \frac{1}{f}$

$$\frac{1}{i} = \frac{1}{f} - \frac{1}{o} = \frac{o-f}{of}$$

$$i = \frac{of}{o-f}$$

$$y = o + i = o + \frac{of}{o-f}$$

$$\frac{dy}{do} = 1 + \frac{f(o-f) - of(1)}{(o-f)^2} = \phi$$

$$(o-f)^2 + f(o-f) - of = \phi$$

$$o^2 - 2of + f^2 + of - f^2 - of = \phi$$

$$o(o-2f) = \phi$$

$\boxed{o = 2f}$ for $o+i$ to be a minimum

The minimum distance between o and i is

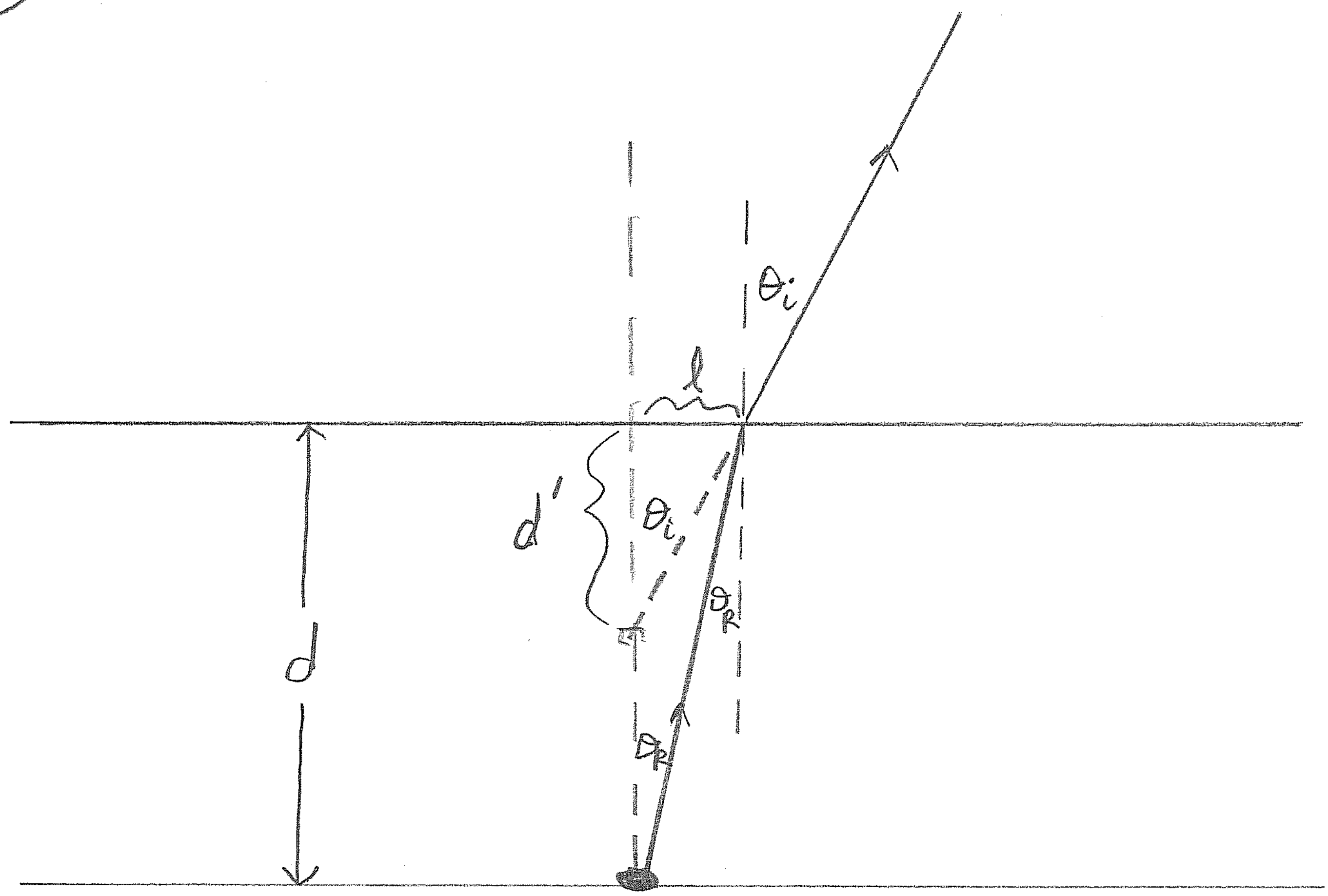
$$y = o + i = 2f + \frac{(2f)f}{2f-f}$$

$$= 2f + \frac{2f^2}{f}$$

$$= 2f + 2f$$

$$= \boxed{4f}$$

(5)



$$\tan\theta_R = \frac{l}{d}$$
$$\tan\theta_i = \frac{l}{d'}$$
$$\left.\right\} \quad d'\tan\theta_i = d\tan\theta_R$$

But, if θ_i and $\theta_R \ll 1$, $\tan\theta_i \simeq \sin\theta_i$, $\tan\theta_R \simeq \sin\theta_R$

So

$$d'\sin\theta_i \simeq d\sin\theta_R$$

$$\frac{d'}{d} \simeq \frac{\sin\theta_R}{\sin\theta_i} = \frac{n_{air}}{n_{water}} = \frac{1}{n}$$

$$d' = \frac{d}{n}$$