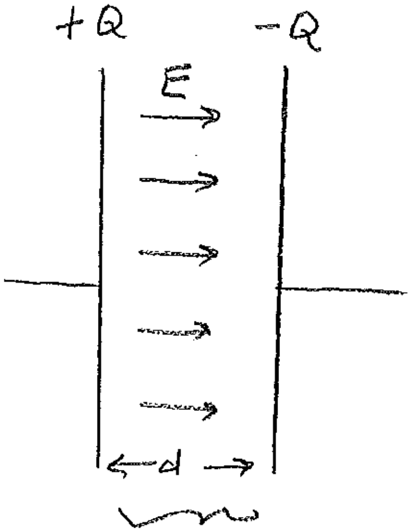
Solutions to Homework 2
PHY 234

(1)



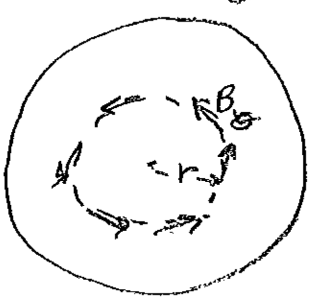
$V(t) = V_0 \sin \omega t$

The electric field between the plates is

$$E = \frac{V}{d}$$

$$E = \frac{V_0}{d} \sin \omega t$$

Due to axial symmetry of the capacitor, $|\vec{B}|$ will only depend on r and be circular.



Taking a circular path, we have

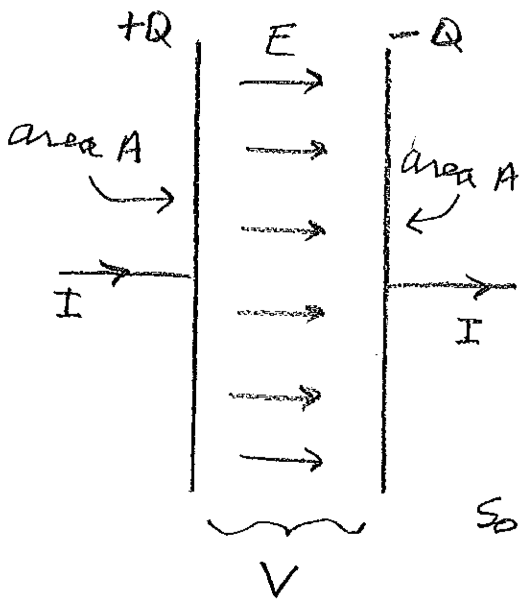
$$\oint \vec{B} \cdot dr = \mu_0 I_{\text{through}} + \mu_0 \epsilon_0 \frac{d}{dt} \Phi_E$$

$$2\pi r B_\theta = 0 + \mu_0 \epsilon_0 \frac{d}{dt}(\pi r^2 E)$$

$$2\pi r B_\theta = \mu_0 \epsilon_0 \pi r^2 \frac{V_0}{d} \frac{d}{dt}(\sin \omega t)$$

$$\boxed{B_\theta = \frac{\mu_0 \epsilon_0 V_0 \omega}{2d} r \cos \omega t}$$

(2)



$$I_d = \epsilon_0 \frac{d\Phi_E}{dt}$$

$$I_d = \epsilon_0 \frac{d}{dt}(EA)$$

But $E = \frac{Q}{\epsilon_0 A} \Rightarrow EA = \frac{Q}{\epsilon_0}$

So $I_d = \epsilon_0 \frac{d}{dt}\left(\frac{Q}{\epsilon_0}\right) = \frac{dQ}{dt}$

But $Q = CV$ $\quad \boxed{I_d = C\frac{dV}{dt}}$

(3) a) $\frac{2\pi}{\lambda} = 500 \Rightarrow \lambda = \frac{2\pi}{500}\ m = \boxed{1.26\ cm}$

$2\pi f = 1.5\times10^{11} \Rightarrow \boxed{f = 23.9\ GHz}$

b) $|\vec{E}| = c|\vec{B}| = (3\times10^{8})(2\times10^{-7}) = 60\ N/m$

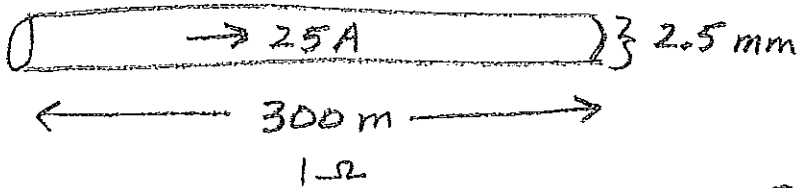
$$\boxed{\vec{E}(x,t) = 60 \sin(500x + (1.5\times10^{11})t)\,\hat{z}\ \frac{N}{m}}$$

since the wave travels in the $-x$ direction

c) $\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B} = \frac{60}{\mu_0}\sin^2(500x + (1.5\times10^{11})t)\,\underbrace{(\hat{z}\times\hat{y})}_{-\hat{x}}$

$$\boxed{\vec{S} = -\frac{60}{\mu}\sin^2(500x + (1.5\times10^{11})t)\,\hat{x}}$$

(4)



$V = IR = 25(1) = 25V$

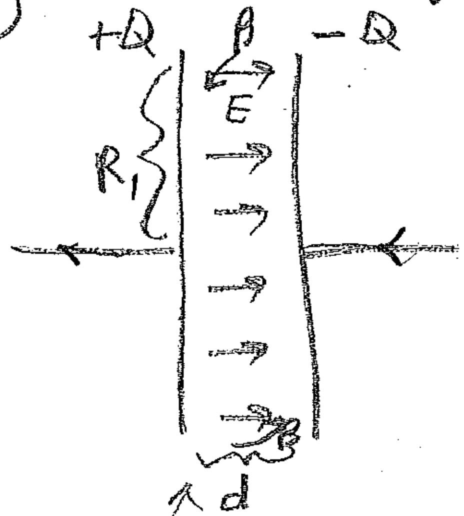
$E = \frac{25V}{300m} = \boxed{.083 \frac{V}{m}}$

$B = \frac{\mu_0 I}{2\pi r} = \frac{(4\pi \times 10^{-7})25}{2\pi(.0025/2)} = \boxed{4\times 10^{-3}\ T} = 4mT$

$|\vec{S}| = \frac{1}{\mu_0}|E||B| = \left(\frac{1}{4\pi\times 10^{-7}}\right)(.083)(4\times 10^{-3}) = \boxed{264 \frac{W}{M^2}}$

5)

$Q = Q_0 e^{-t/RC}$



$E = \frac{Q/\pi R^2}{\epsilon_0} = \frac{Q}{\epsilon_0 \pi R^2} \Rightarrow E\pi R^2 = Q/\epsilon_0$

To find B:

$\oint \vec{B}\cdot d\vec{l} = \mu_0 I + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$

$2\pi R_1 B = 0 + \mu_0 \epsilon_0 \frac{d}{dt}(E\pi R_1^2)$

$2\pi R_1 B = \mu_0 \epsilon_0 \frac{d}{dt}(Q/\epsilon_0) = \mu_0 \frac{dQ}{dt}$

at the side of the capacitor

$B = \frac{\mu_0}{2\pi R_1}\frac{dQ}{dt}$

So $|\vec{S}| = \frac{1}{\mu_0} E B$ since $E \perp B$ at the capacitor's side

(5 cont.)

$$|\vec{S}| = \frac{1}{\mu_0}\left(\frac{Q}{\epsilon_0 \pi R_1^2}\right)\frac{\mu_0}{2\pi R_1}\frac{dQ}{dt}$$

$$\boxed{|\vec{S}| = \frac{1}{\epsilon_0 \pi R_1^2}\left(\frac{1}{2\pi R_1}\right) Q\frac{dQ}{dt}}$$

b) Integrating $\vec{S}$ over the surface of the capacitor gives

$$\oiint \vec{S}\cdot\vec{dA} = \frac{1}{\epsilon_0 \pi R_1^2}\left(\frac{1}{2\pi R_1}\right) Q\frac{dQ}{dt}\left[2\pi R_1 d\right]$$

surface area of side

$$= \frac{d}{\epsilon_0(\pi R_1^2)} Q\frac{dQ}{dt}$$

$$\oiint \vec{S}\cdot\vec{dA} = \frac{1}{C} Q\frac{dQ}{dt}$$

since $C = \epsilon_0 \frac{A}{d}$

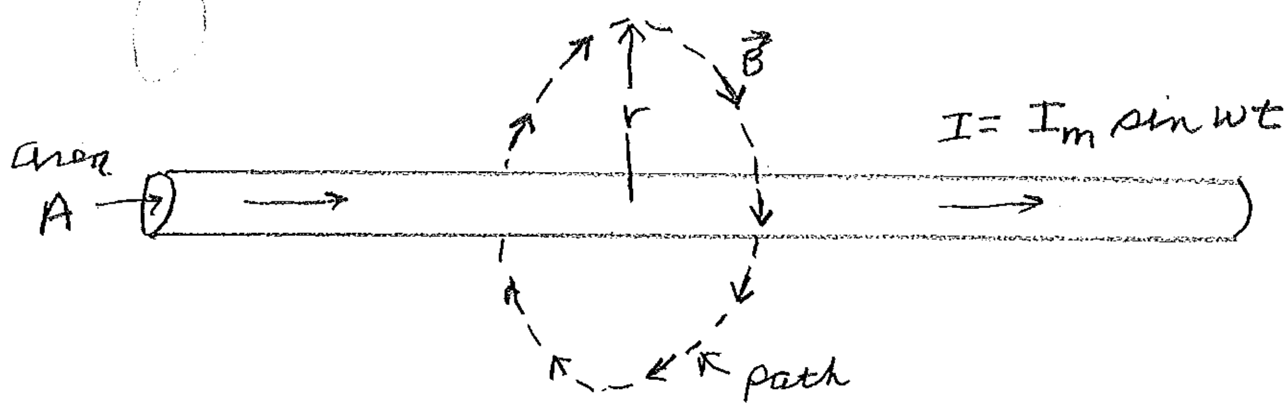
The energy of a capacitor is

$$U_c = \frac{Q^2}{2C}$$

$$\frac{dU_c}{dt} = \frac{d}{dt}\left(\frac{Q^2}{2C}\right) = \frac{Q}{C}\frac{dQ}{dt}$$

the rate at which energy leaves the capacitor equals $\oiint \vec{S}\cdot\vec{dA}$

⑥



$$\oint \vec{B}\cdot d\vec{l} = \underbrace{\mu_0 I}_{\vec{B}\text{ from current}} + \underbrace{\mu_0 \epsilon_0 \frac{d\Phi_E}{dt}}_{\text{Induced }\vec{B}\text{ from displacement current}}$$

$$\mu_0 I = \mu_0 I_m \sin\omega t$$

$$\mu_0 \epsilon_0 \frac{d\Phi_E}{dt} = \mu_0 \epsilon_0 \frac{d}{dt}(EA)$$

but $J = \dfrac{E}{\rho_e}$

$E = J\rho_e$

$EA = JA\rho_e = I\rho_e$

$$\mu_0 \epsilon_0 \frac{d\Phi_E}{dt} = \mu_0 \epsilon_0 \frac{d}{dt}(\rho_e I)$$

$$= \rho_e \mu_0 \epsilon_0 I_m \omega \cos\omega t$$

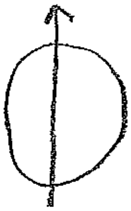
$$\oint \vec{B}\cdot d\vec{l} = 2\pi r B_\theta = \mu_0 I_m \sin\omega t + \rho_e \mu_0 \epsilon_0 I_m \omega \cos\omega t$$

$$\boxed{B_\theta = \underbrace{\frac{\mu_0 I_m}{2\pi r}\sin\omega t}_{\text{from current}} + \underbrace{\frac{\mu_0 I_m \rho_e \epsilon_0 \omega}{2\pi r}\cos\omega t}_{\text{Induced from displacement current}}}$$

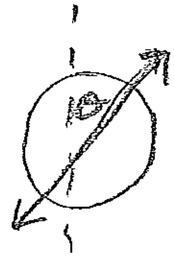
$$\text{Ratio} = \rho_e \epsilon_0 \omega \simeq (1.7\times10^{-8}\,\Omega\text{m})(8.85\times10^{-12})(2\pi\times10^{9}) \simeq 9.5\times10^{-10}$$

(7)

I_0 unpolarized



$\frac{I_0}{2}$ ↑ Polarized



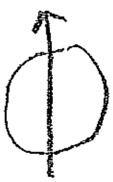
$\frac{I_0}{2}\cos^2\theta$ ↗ Polarized

$$\frac{I_0}{2}\cos^2\theta = \frac{I_0}{3}$$

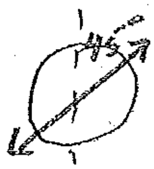
$$\cos^2\theta = \frac{2}{3} \Longrightarrow \boxed{\theta \approx 35.3^\circ}$$

(8)

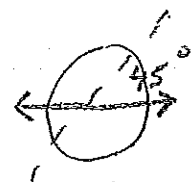
I_0 unpolarized



$\frac{I_0}{2}$ ↑ polarized



$\frac{I_0}{2}\cos^2 45$ ↗ Polarized

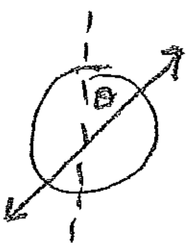


→ $\frac{I_0}{2}\cos^2 45 \cos^2 4$

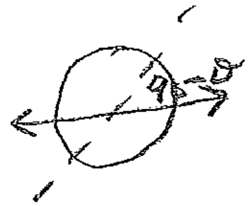
$$I = \frac{I_0}{2}\cos^2 45 \cos^2 45 = \boxed{\frac{I_0}{8}}$$

(9)

↑ E_0



$E_0\cos\theta$ ↗



→ $E_0\cos\theta\cos(90-\theta)$

$$I = E_0^2\cos^2\theta\cos^2(90-\theta) = E_0^2(.1)$$

$$\cos^2\theta\sin^2\theta = .1$$

$$\left(\frac{\sin 2\theta}{2}\right)^2 = .1$$

$$\sin(2\theta) = 2\sqrt{.1} \Rightarrow \boxed{\theta \approx 19.6^\circ}$$