

**Physics 234**  
**Homework 2 (Due Monday, January 25)**  
Maxwell's Equations and Radiation

**Problem 1.**

A parallel plate capacitor has a sinusoidal voltage of the form  $V(t) = V_0 \sin(\omega t)$  across its plates. The plates are circular, with a radius  $R$ , and are separated by a distance  $d$ . Find an expression for the magnetic field at positions between the plates that are a distance  $r < R$  from the axis. Express your answer in terms of  $\omega$ ,  $\epsilon_0$ ,  $\mu_0$ ,  $d$ ,  $V_0$ , and an appropriate sinusoidal function of time.

**Problem 2.**

Show that the displacement current  $I_d$  through the area of a parallel-plate capacitor can be written as

$$I_d = C \frac{dV}{dt} \quad (1)$$

**Problem 3.**

The magnetic field of a traveling electromagnetic wave is given by

$$\vec{B} = (2 \times 10^{-7}) \sin(500x + (1.5 \times 10^{11})t) \hat{j} T \quad (2)$$

where  $x$  is in meters and  $t$  in seconds. Find:

- a) The wavelength and frequency of the wave.
- b) The electric field of the traveling electromagnetic wave.
- c) The Poynting Vector of the traveling electromagnetic wave.

**Problem 4.**

A copper wire, diameter 2.5 mm and resistance  $1.0 \Omega$  per 300 meters, carries a current of 25 amp. Calculate  $\vec{E}$ ,  $\vec{B}$  and  $\vec{S}$  for a point on the surface of the wire.

**Problem 5.**

Consider a parallel plate capacitor that has circular plates. Let the radius of the plates be  $R$ , and the distance between the plates to be  $d$ . Suppose that the capacitor is discharging through a resistor of resistance  $R$ .

- a) Find an expression for the Poynting vector  $\vec{S}$  on the side of the capacitor. This can be done by determining  $\vec{E}$  and  $\vec{B}$  at the side surface of the capacitor.
- b) Integrate  $S$  over the cylindrical surface to find the rate at which energy is leaving the capacitor.

**Problem 6.**

A sinusoidal current flows in an infinitely long straight wire, and is of the form  $I = I_m \sin(\omega t)$ . The resistivity of the wire is  $\rho_e$ , and the frequency is  $1 \text{ GHz} = 10^9 \text{ Hz}$ . Find:

- a) An expression for the magnetic field produced by the wire at a distance  $r$  from its center due to the current  $I$ .
- b) An expression for the induced magnetic field produced by the displacement current of the wire at a distance  $r$  from its center. Here, assume that the electric field producing the current is uniform and lies entirely within the wire.
- c) Estimate the ratio of the two magnetic fields if the wire is made of copper.

**Problem 7.**

Unpolarized light falls on two polarizing sheets placed one on top of the other. What must be the angle between the characteristic directions of the sheets if the intensity of the transmitted light is one-third the intensity of the incident beam? Assume that each polarizing sheet is ideal, that is, that it reduces the intensity of unpolarized light by exactly 50%.

**Problem 8.**

Three polarizing plates are stacked. The first and third are crossed: the one between has its axis at  $45^\circ$  to the axes of the other two. What fraction of the intensity of an incident unpolarized beam is transmitted by the stack?

**Problem 9.**

A beam of polarized light strikes two polarizing sheets. The characteristic direction of the second is  $90^\circ$  with respect to the incident beam. The characteristic direction of the first is at an angle  $\theta$  with respect to the incident light. Find the angle  $\theta$  for a transmitted beam intensity that is 0.10 times the incident beam intensity.