Solutions to Hwk 1 PHY234

1) $f = \frac{\omega_0}{2\pi} = \frac{1}{2\pi\sqrt{LC}}$

$$\sqrt{LC} = \frac{1}{2\pi f}$$

$$L = \left(\frac{1}{2\pi f}\right)^2 \frac{1}{C} = \left(\frac{1}{2\pi(10000)}\right)^2 \frac{1}{6.7\times 10^{-6}}$$

$$\boxed{L \approx 37.8\,\mu H}$$

2) With S_1 closed, we have an RC circuit

$$\tau_c = RC \quad \Rightarrow \quad C = \frac{\tau_c}{R}$$

With S_2 closed, we have an RL circuit

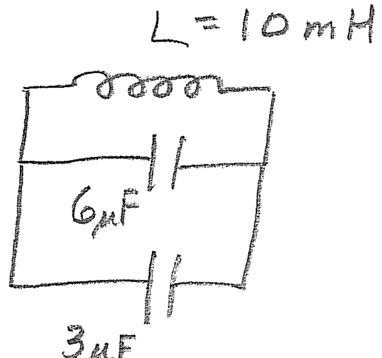
$$\tau_L = L/R \qquad L = \tau_L R$$

With S_3 closed, we have an LC circuit

$$T = \frac{2\pi}{\omega_0} = \frac{2\pi}{1/\sqrt{LC}} = 2\pi\sqrt{LC}$$

$$T = 2\pi\sqrt{(\tau_L R)\left(\frac{\tau_c}{R}\right)} = \boxed{2\pi\sqrt{\tau_L \tau_c}}$$

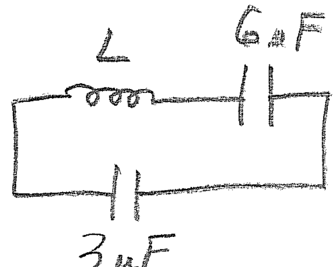
3) There are two combinations

a) 

Here $C_{TOTAL} = 9\,\mu F$

$$f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi\sqrt{LC}}$$

$$f_0 = \frac{1}{2\pi\sqrt{(10^{-2})(9\times10^{-6})}} \approx \boxed{530\ Hz}$$

b) 

Here $\frac{1}{C_{TOTAL}} = \frac{1}{3} + \frac{1}{6} = \frac{1}{2}$

$C_{TOTAL} = 2\,\mu F$

$$f_0 = \frac{1}{2\pi\sqrt{(10^{-2})(2\times10^{-6})}} \approx \boxed{1125\ Hz}$$

4) The charge on the capacitor in a LRC circuit is

$$Q(t) = A_0\, e^{-\frac{R}{2L}t} \cos(\omega t + \alpha)$$

If at $t=0$, the capacitor has its maximum charge, Q_0, then

$$Q(t) = Q_0\, e^{-\frac{R}{2L}t} \cos(\omega t)$$

after one cycle, $t=T$, and the charge is

$$Q(T) = Q_0\, e^{-\frac{RT}{2L}} \qquad \text{since } \cos(\omega T) = 1$$

after 2 cycles, the charge on the capacitor is

$$Q(2T) = Q_0 e^{-\frac{R}{2L}(2T)} \qquad \text{since } \cos(\omega 2T) = 1$$

after n cycles,

$$Q(nT) = Q_0 e^{-\frac{R}{2L}(nT)} \qquad \text{since } \cos(\omega n T) = 1$$

The energy stored in a capacitor is

$$U = \frac{Q^2}{2C}$$

So from cycle n to $n+1$ we have

$$\frac{\Delta U}{U} = \frac{\frac{Q_0^2}{2C} e^{-\frac{R}{L}nT} - \frac{Q_0^2}{2C} e^{-\frac{R}{L}(n+1)T}}{\frac{Q_0^2}{2C} e^{-\frac{R}{L}nT}}$$

$$= 1 - e^{-\frac{R}{L}T}$$

If $\frac{RT}{L} \ll 1$ $\qquad e^{-\frac{RT}{L}} \approx 1 - \frac{RT}{L}$

So $\frac{\Delta U}{U} \approx \frac{RT}{L} = \frac{R}{L}\left(\frac{2\pi}{\omega}\right) = \frac{2\pi R}{\omega L}$

(5) (a) at 1000 Hz, $X_L = L\omega = L 2\pi f$

$$X_L = (.05\,H) 2\pi (1000) \cong 314\,\Omega$$

$$I_m = \frac{V}{X_L} = \frac{30\,V}{314\,\Omega} = \boxed{.095\ A}$$

(b) at 8000 Hz $\quad X_L = (.05\,H) 2\pi (8000) = 2513\,\Omega$

$$I_m = \frac{30\,V}{2513\,\Omega} \simeq \boxed{.012\ A}$$

(6) $X_L = X_C$ when $\omega_0 = \dfrac{1}{\sqrt{LC}}$

so $f = \dfrac{\omega_0}{2\pi} = \dfrac{1}{2\pi\sqrt{LC}} = \dfrac{1}{2\pi\sqrt{(6\times10^{-3})(10^{-5})}}$

$$\boxed{f \simeq 650\ Hz}$$

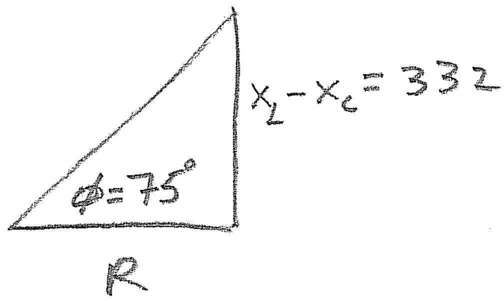
(7)

$$X_L = \omega_d L = 930(2\pi)(.088) \simeq 514\,\Omega$$

$$X_C = \frac{1}{\omega_d C} = \frac{1}{930(2\pi)(.94\times10^{-6})} \simeq 182\,\Omega$$

$$X_L - X_C \simeq 332\,\Omega$$

$$\tan 75 = \frac{332}{R}$$



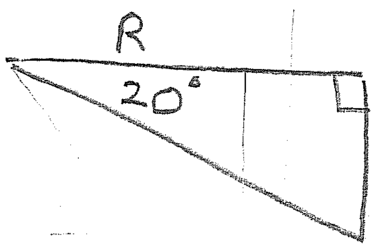
$$R = \frac{332}{\tan 75} \simeq \boxed{90\,\Omega}$$

(8) $\omega_d = 60(2\pi) = 377 \ 1/s$

let $X_c = \frac{1}{\omega_d C}$

In position 2, $|(X_L - X_c)| = \frac{120}{2} = 60\,\Omega$

With the switch open, we have

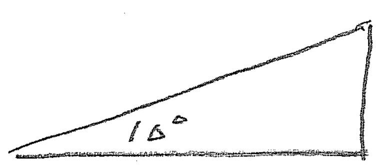


So, $\frac{60}{R} = \tan 20°$ $\Rightarrow$ $R = \frac{60}{\tan 20°} \approx \boxed{165\,\Omega}$

and $X_c > X_L$ so $\boxed{X_c = 60 + X_L}$

In position 1, the capacitive reactance is $\frac{X_c}{2}$

So



$\tan 10° = \frac{X_L - \frac{X_c}{2}}{165}$

$X_L - \frac{X_c}{2} = 165 \tan 10°$

$\boxed{X_L - \frac{X_c}{2} = 29\,\Omega}$

Solving for X_c + X_L gives

$\frac{X_c}{2} = 60 + 29$

$\boxed{X_c = 178\,\Omega \quad X_L = 118\,\Omega}$ $\Rightarrow$

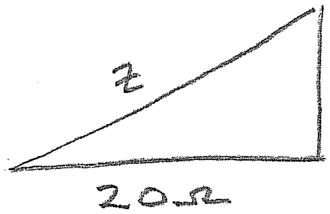
$C = \frac{1}{178(377)} \approx \boxed{14.9\,\mu F}$

$L = \frac{118}{377} \approx \boxed{313\,mH}$

(9) $X_c = \frac{1}{\omega C} = \frac{1}{2\pi(550)(4.7\times10^{-6})} \approx 61.6\,\Omega$

$X_L = \omega L = 2\pi(550)(25\times10^{-3}) \approx 86.4\,\Omega$

(a)



$86.4 - 61.6 = 24.8$

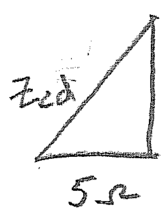
$z = \sqrt{20^2 + 24.8^2} \approx 31.9\,\Omega$

$I_{Rms} \approx \frac{75\,V}{31.9\,\Omega} \approx \boxed{2.35\,A}$

(b) $V_{ab} = R\,I_{Rms} = 15(2.35) \approx \boxed{35.3\,V}$

(c) $V_{bc} = X_c\,I_{Rms} = 61.6\,(2.35) \approx \boxed{145\,V}$

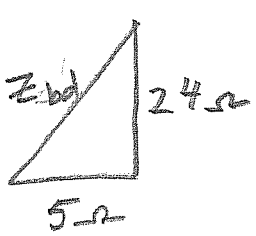
V_{cd}:



$Z_{cd} = \sqrt{5^2 + 86.4^2} \approx 86.5\,\Omega$

$V_{cd} = 86.5(2.35) \approx \boxed{203\,V}$

V_{bd}:

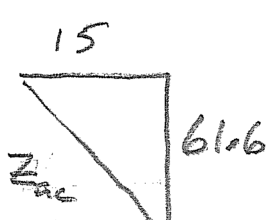


$Z_{bd} = \sqrt{5^2 + 24^2}$

$Z_{bd} = 24.5\,\Omega$

$V_{bd} = 24.5\,(2.35) \approx \boxed{57.6\,V}$

V_{ac}:



$Z_{ac} = \sqrt{15^2 + 61.6^2} = 63.4\,\Omega$

$V_{ac} = 63.4(2.35) \approx \boxed{149\,V}$

(10) For a parallel plate capacitor,

$$C = \frac{\epsilon_0 A}{d}$$

For a ~~solenoid~~ $L = \frac{\mu_0 N^2}{L}(\pi r^2) = \frac{\mu_0 \pi r^2 N^2}{L}$

So $f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi\sqrt{LC}}$

$$f_0 = \frac{1}{2\pi\sqrt{\frac{\mu_0 \pi r^2 N^2}{L}\frac{\epsilon_0 A}{d}}} = \boxed{\frac{1}{2\pi}\sqrt{\frac{L d}{(\epsilon_0 \mu_0)\pi r^2 N^2 A}}}$$