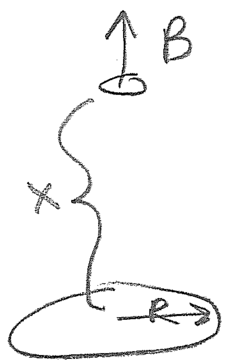
Solutions to Problem Set 7
Phy 133

① ⓐ $|\mathcal{E}| = \left|\frac{d\Phi_m}{dt}\right| = \boxed{16t + 6 \text{ millivolts}}$

ⓑ the current is clockwise

②



$$B = \frac{\mu_0 I R^2}{2(x^2+R^2)^{3/2}}$$

$$\Phi_m = B\pi r^2$$

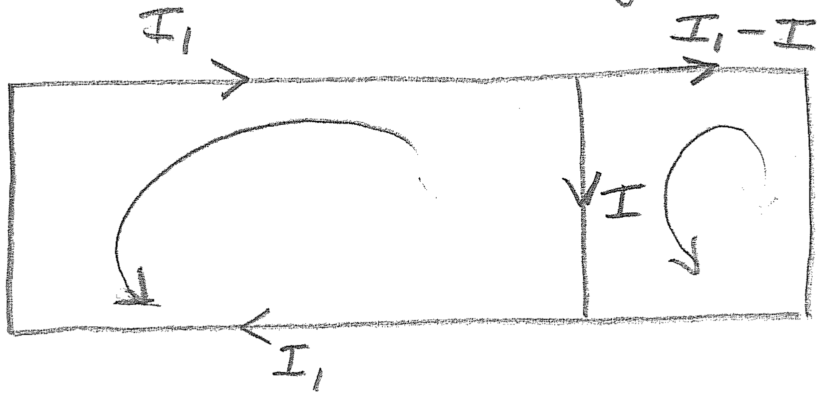
$$\Phi_m = \frac{\mu_0 I R^2 \pi r^2}{2(x^2+R^2)^{3/2}}$$

$$|\mathcal{E}| = \left|\frac{d\Phi_m}{dt}\right| = \frac{d\Phi_m}{dx}\frac{dx}{dt}$$

$$|\mathcal{E}| = \left|\frac{\mu_0 I R^2 \pi r^2}{2(x^2+R^2)^{5/2}}\left(-\frac{3}{2}\right)(2x)\, v\right|$$

$$\boxed{|\mathcal{E}| = \frac{3\mu_0 I R^2 \pi r^2 x v}{2(x^2+R^2)^{5/2}}}$$

③ Let $r =$ resistance/length



Left Loop:

$$+(7ar)I_1 + (ar)I = \frac{d\phi}{dt} = \frac{d}{dt}(3a^2ct)$$

$$\boxed{7arI_1 + arI = 3a^2c} \quad (1)$$

Right loop:

$$-arI + 3ar(I_1 - I) = \frac{d\phi}{dt} = \frac{d}{dt}(a^2ct)$$

$$\boxed{3arI_1 - 4arI = a^2c} \quad (2)$$

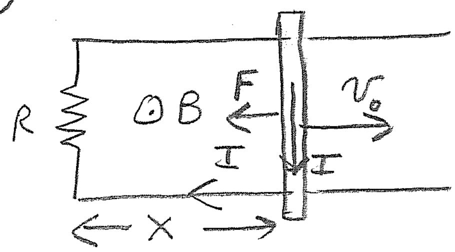
$3\times(1) - 7\times(2)$:

$$3arI - 7(-4arI) = 9a^2c - 7a^2c$$

$$31arI = 2a^2c$$

$$\boxed{I = \frac{2ac}{31r}}$$

④



$$\Phi_m = BLx$$

ⓐ $$\mathcal{E} = \frac{d\Phi_m}{dt} = BL\frac{dx}{dt}$$

$$\boxed{\mathcal{E} = BLv_0}$$

ⓑ $$I = \frac{\mathcal{E}}{R} = \boxed{\frac{BLv_0}{R}}$$

ⓒ $$\vec{F} = I\vec{L}\times\vec{B}$$

$$|\vec{F}| = ILB = \boxed{\frac{B^2L^2}{R}v_0}$$ opposite to v

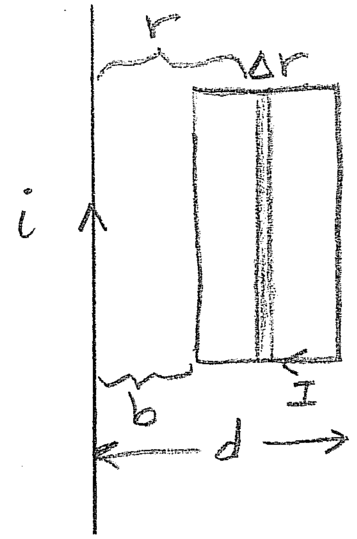
ⓓ $$m\frac{dv}{dt} = -\frac{B^2L^2}{R}v$$

$$\frac{dv}{v} = -\frac{B^2L^2}{mR}dt$$

$$\int_{v_0}^{v}\frac{dv}{v} = -\int_0^t\frac{B^2L^2}{mR}dt$$

$$\ln\left(\frac{v}{v_0}\right) = -\frac{B^2L^2}{mR}t \Rightarrow \boxed{v = v_0 e^{-\frac{B^2L^2}{mR}t}}$$

5



$$\Delta\Phi_m = \frac{\mu_0 i}{2\pi r}\, l\, \Delta r$$

$$\Phi_m = \int_b^d \frac{\mu_0 i l}{2\pi r}\, dr$$

$$\Phi_m = \frac{\mu_0 i l}{2\pi} \ln\left(\frac{d}{b}\right)$$

but $i = i_0(1-at)$

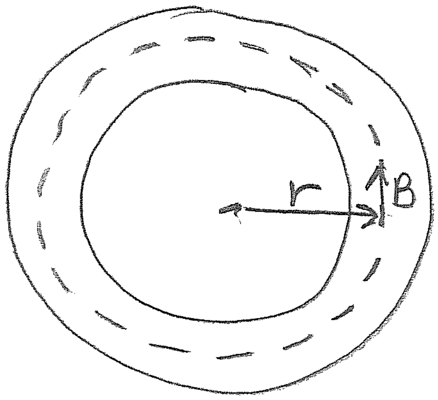
$$|\varepsilon| = \left|\frac{d\Phi_m}{dt}\right| = \frac{\mu_0 L}{2\pi} \ln\left(\frac{d}{b}\right) \left|\frac{di}{dt}\right|$$

$$|\varepsilon| = \frac{\mu_0 L \ln\left(\frac{d}{b}\right)}{2\pi}\, a$$

$$I = \frac{|\varepsilon|}{R} = \boxed{\frac{\mu_0 L \ln\left(\frac{d}{b}\right) a}{2\pi R}} \quad \text{for } 0 \le t \le \frac{1}{a}$$

b $\quad Q = \int I\, dt = I\left(\frac{1}{a}\right) = \boxed{\frac{\mu_0 L \ln\left(\frac{d}{b}\right)}{2\pi R}}$

(6)

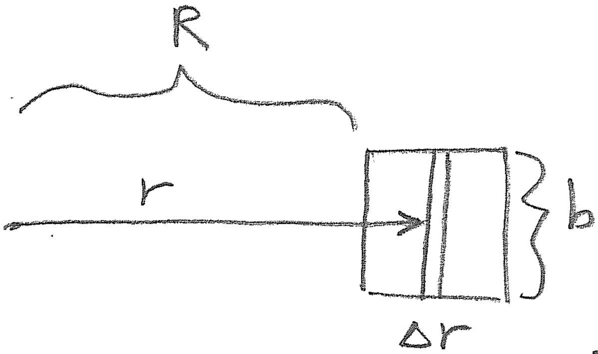


Using Ampere's law to find the magnetic field:

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{through}$$

$$2\pi r B = \mu_0 N I$$

$$B = \frac{\mu_0 N}{2\pi r} I$$



Finding the magnetic flux

$$\Delta\Phi_m = NB\, b\Delta r$$

$$\Delta\Phi_m = \frac{\mu_0 N^2 I}{2\pi r} b\, \Delta r$$

$$\Phi_m = \int_R^{R+a} \frac{\mu_0 N^2 I b}{2\pi r} dr = \frac{\mu_0 N^2 b \ln\left(\frac{R+a}{R}\right)}{2\pi} I$$

so $\boxed{L = \frac{\mu_0 N^2 b}{2\pi} \ln\left(\frac{R+a}{R}\right)}$