

**Physics 133 Homework 7**  
**Faraday's Law**  
**Due Friday, November 30**

**Problem 1.**

A closed loop of wire has a resistance  $R$ . A magnetic field points out of the page. The magnetic flux through the loop is given by:

$$\Phi_m = 8t^2 + 6t \tag{1}$$

where  $\Phi_m$  is in units of milliwebers and  $t$  is in seconds. See the figure on the figures page.

Find:

- a) The magnitude of the Voltage induced in the loop.
- b) The direction of the current through the resistor  $R$ .

**Problem 2.**

Consider the two circular loops of wire, which have a common axis. The smaller loop has a radius  $r$ , and the larger loop a radius  $R$ . They are separated by a distance  $x$ , which is large compared to  $R$ . (See the figure on the figures page.)

A constant current of magnitude  $I$  flows in the larger loop as shown. If the smaller loop is moving away from the larger loop with a speed  $v = dx/dt$ , what is the magnitude of the Voltage induced in the smaller loop? Assume that the smaller loop is far enough away so that the magnetic field is approximately a constant through the loop.

**Problem 3.**

Consider the circuit (shown on the figures page) in which there is a constant magnetic field pointing out of the page. The wires are connected so as to have a rectangle of sides  $a$  by  $3a$  next to a square of sides  $a$ . The resistance of the wires per unit length is  $r$ . The magnitude of the magnetic field increases linearly everywhere out of the page in time by the expression  $B = ct$ , where  $c$  is a constant.

Find an expression for the current  $I$  shown in the figure. Express your answer in terms of  $a$ ,  $c$ ,  $r$  and any other constants you deem necessary.

**Problem 4.**

Consider the circuit shown on the figures page. There is a magnetic field  $\vec{B}$  pointing out of the page. A bar of length  $L$  slides along the rails with a velocity  $v_0$ :

- a) What is the Voltage induced in the circuit?
- b) If the total resistance of the circuit is  $R$ , how much current flows in the circuit?
- c) What force is necessary to keep the bar moving with a velocity  $v_0$ ?
- d) At  $t = 0$  the velocity of the bar is  $v_0$ , and no force is applied to the bar. What is the velocity of the bar for times  $t > 0$ ?

**Problem 5.**

An infinite straight wire initially has a current  $i_0$ . Next to the infinite wire is a rectangular wire, with one side a distance  $b$  and the other side a distance  $d$  from the infinite wire. The length of the rectangle is  $l$ . See the figure on the figures page.

At time  $t = 0$ , the current in the infinite wire is reduced to zero linearly:

$$I(t) = i_0(1 - at) \tag{2}$$

for times  $0 < t < 1/a$ . After the time  $t = 1/a$ , the current in the infinite wire remains zero. The resistance of the rectangular loop is  $R$ .

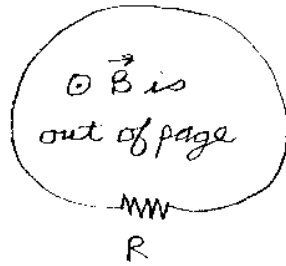
- a) Find an expression for the current in the rectangular loop as a function of time. Express your answer in terms of  $i_0$ ,  $b$ ,  $d$ ,  $l$ ,  $a$ , and any other constants from the laws of electricity and magnetism.
- b) Find an expression for the total charge that flows in the rectangular wire. Note: your answer should be independent of  $a$ , the rate at which the current goes to zero in the long infinite wire.

**Problem 6.**

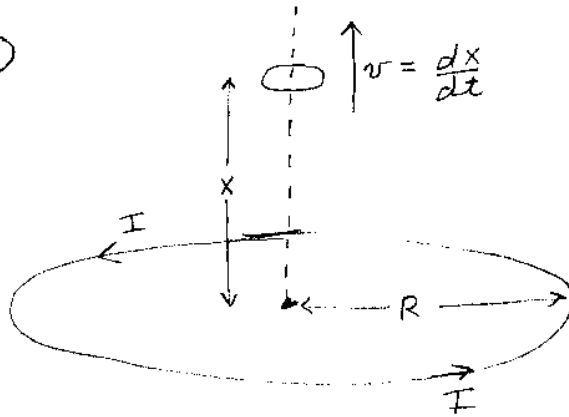
Find an expression for the self-inductance of a rectangular toroid. The toroid has a total of  $N$  turns, the rectangle has dimensions  $a$  by  $b$ , and the inner side is a distance  $R$  from the center of the toroid. See the figure on the figures page.

Figures for HWK 7 Phy 133

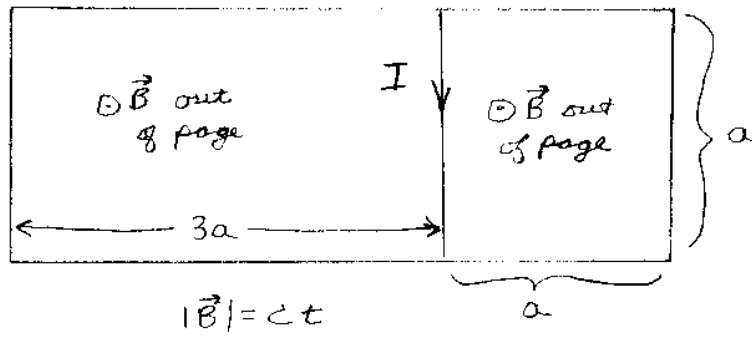
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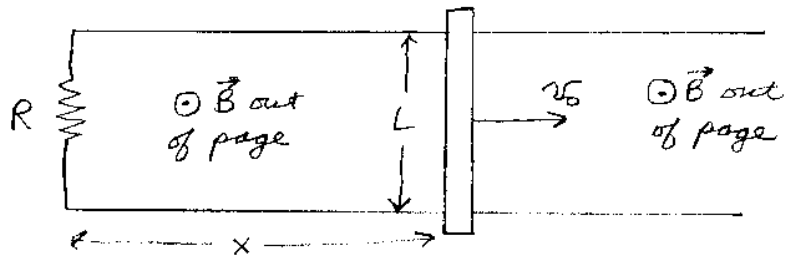
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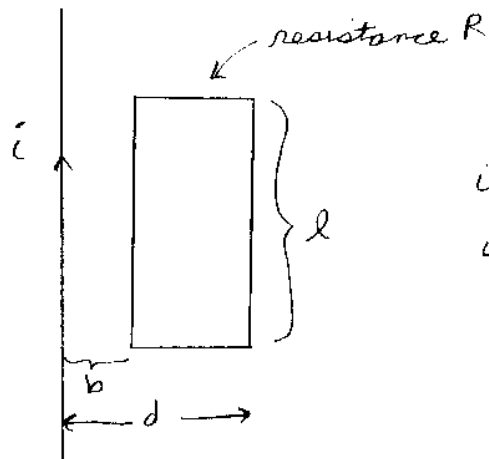
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④



⑤



$$i = i_0(1 - at) \quad 0 \leq t \leq \frac{1}{a}$$
$$i = 0 \quad t > \frac{1}{a}$$

⑥

