Solutions to Sixth Problem Set
PHY 133

(1)

$$\vec{B}_{\text{at center}} = \vec{B}_{\substack{\text{semi-infinite}\\ \text{line}}} + \vec{B}_{\text{semi-circle}} + \vec{B}_{\substack{\text{semi-infinite}\\ \text{line}}}$$

$$= \vec{B}_{\substack{\text{infinite}\\ \text{line}}} + \vec{B}_{\text{semi-circle}}$$

$$|\vec{B}| = \frac{\mu_0 I}{2\pi R} + \frac{1}{2}\left(\frac{\mu_0 I}{2R}\right) = \boxed{\frac{\mu_0 I}{2R}\left(\frac{1}{\pi} + \frac{1}{2}\right)}$$

(2)

charge 1: negative

charge 2: neutral

charge 3: positive

charge 4: positive

(3) $\vec{F} = q\vec{v} \times \vec{B}$

$$2\hat{\imath} + 4\hat{\jmath} + 3\hat{k} = -1.5 \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 6 & -3 & 0 \\ 0 & B_y & B_z \end{vmatrix}$$

$$= -1.5\left(\hat{\imath}(-3B_z) + \hat{\jmath}(-6B_z) + \hat{k}(6B_y)\right)$$

so

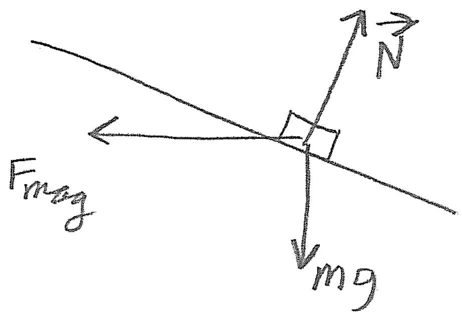
$\underline{2 = 4.5 B_z}$ $\quad$ $\underline{4 = 9B_z}$ $\quad$ $\underline{3 = -9B_y}$

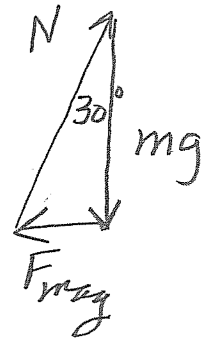
3 cont

So $B_z = \frac{4}{9}$ and $B_y = -\frac{1}{3}$

$$\boxed{\vec{B} = -\frac{1}{3}\hat{j} + \frac{4}{9}\hat{k}}$$ Tesla

4





$$\frac{F_{mag}}{mg} = \tan 30$$

$$F_{mag} = mg\tan\theta$$

but $F_{mag} = I\ell B$

$$I\ell B = mg\tan\theta$$

$$I = \frac{mg}{\ell B}\tan\theta$$

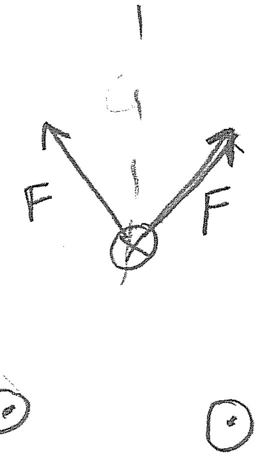
$I = \frac{V}{R}$ so $\frac{V}{R} = \frac{mg}{\ell B}\tan\theta$

$$R = \frac{VB\ell}{mg\tan\theta}$$

$$5(2x) = \frac{20(2)(1)}{1(9.8)\tan 30°}$$

$$x = .707M = \boxed{70.7\,cm}$$

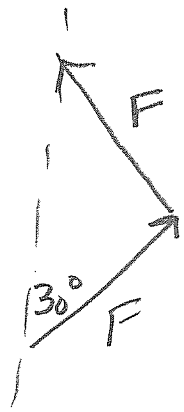
(5)



Since the currents are in the opposite direction, the wires repel each other.

For parallel wires,

$$\frac{F}{\ell} = \frac{\mu_0 I^2}{2\pi L}$$

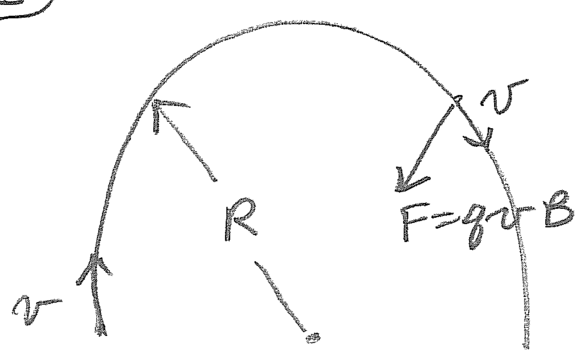


If the vectors are added,

$$F_{net} = 2\cos 30^\circ F$$

$$\frac{F_{net}}{\ell} = \frac{2\cos 30^\circ \mu_0 I^2}{2\pi L} = \boxed{\frac{\sqrt{3}\mu_0 I^2}{2\pi L}}$$

(6)



$$\frac{mv^2}{R} = qvB$$

$$v = \frac{qRB}{m}$$

$$v^2 = \frac{q^2}{m^2} R^2 B^2$$

$$\frac{mv^2}{2} = qV$$

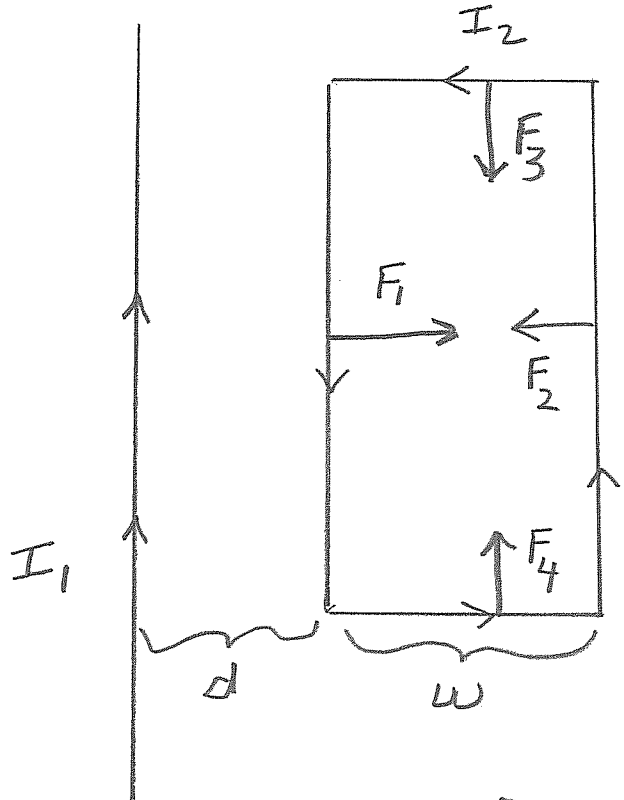
$$v^2 = \frac{2qV}{m}$$

$$\frac{2qV}{m} = \frac{q^2}{m^2} R^2 B^2$$

$$R = \frac{1}{B}\sqrt{\frac{2mV}{q}}$$

$$x = 2R = \boxed{\frac{2}{B}\sqrt{\frac{2mV}{q}}} \text{ or } \boxed{m = \frac{x^2 B^2 q}{8V}}$$

(7)



$\vec{F}_3 + \vec{F}_4$ cancel

$\vec{F}_3 + \vec{F}_4 = \vec{0}$

$$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4$$

$$|\vec{F}_1| = \frac{\mu_0 I_1 I_2 l}{2\pi d}$$ to the right

$$|\vec{F}_2| = \frac{\mu_0 I_1 I_2 l}{2\pi (d+w)}$$

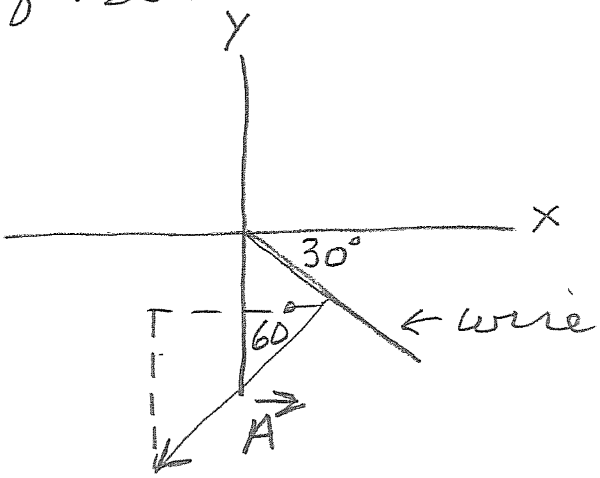
$$\vec{F}_{Net} = \frac{\mu_0 I_1 I_2 l}{2\pi}\left(\frac{1}{d} - \frac{1}{d+w}\right)$$ to the right

$$\boxed{\vec{F}_{Net} = \frac{\mu_0 I_1 I_2 l w}{2\pi d (d+w)} \text{ to the right}}$$

(8) $\vec{\tau} = \vec{\mu} \times \vec{B}$

$\vec{B} = .5\,\hat{i}\ T$ $\qquad$ $\vec{\mu} = I\vec{A}$

To find the area vector:



$\vec{A} = -A\cos 60\,\hat{i} - A\sin 60\,\hat{j}$

$\vec{A} = -\frac{A}{2}\hat{i} - \frac{A\sqrt{3}}{2}\hat{j}$

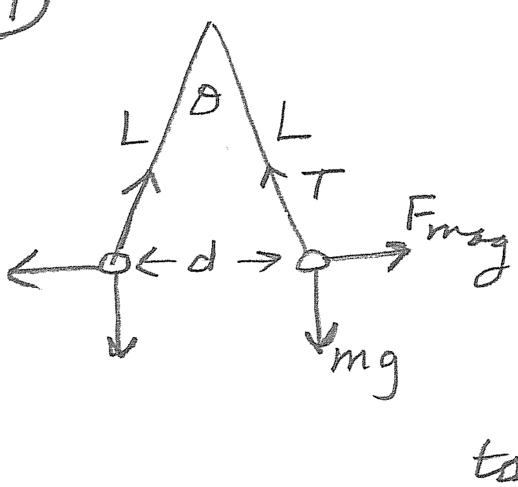
$I\vec{A} = -A\hat{i} - \sqrt{3}A\hat{j}$

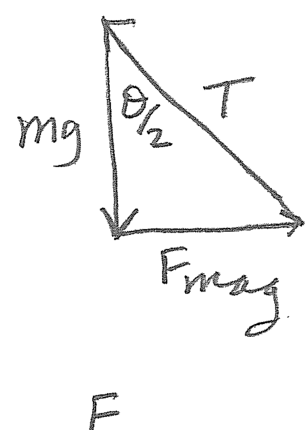
$$\vec{\tau} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -A & -\sqrt{3}A & 0 \\ .5 & 0 & 0 \end{vmatrix} = .5\sqrt{3}\,A\,\hat{k}$$

$$= \frac{\sqrt{3}}{2}(.1)(.05)\,\hat{k}\ TM^2$$

$$= \boxed{4.33 \times 10^{-3}\ TM^2}$$

(9)





$$\tan\theta/2 = \frac{F_{mag}}{mg}$$

(9 cont)

$$F_{mag} = \frac{\mu_0 (I) I \, l}{2\pi d}$$ where l is the length of the wire

and $m = \mu l$

so $$\tan\theta/2 = \frac{\dfrac{\mu_0 I^2 l}{2\pi d}}{\mu l g} = \frac{\mu_0 I^2}{2\pi d \mu g}$$

also, $$\sin\theta/2 = \frac{d/2}{L} \Rightarrow d = 2L \sin\theta/2$$

giving $$I^2 = \frac{2\pi d \mu g \tan\theta/2}{\mu_0}$$

$$\boxed{I = \sqrt{\frac{4\pi L \mu g \sin\theta/2 \tan\theta/2}{\mu_0}}}$$