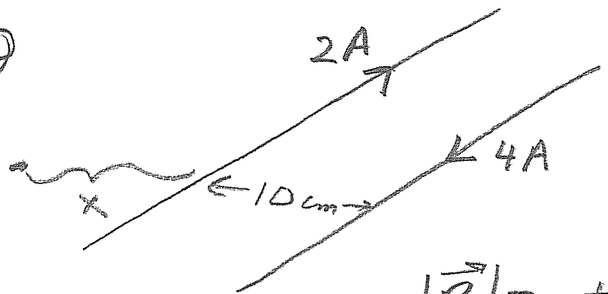
Solutions to Homework 5

PHY133

(1)

(a)



The magnetic field a distance x from the left wire is

$$|\vec{B}| = +\frac{\mu_0 2}{2\pi x} - \frac{\mu_0 4}{2\pi(x+.1)} = 0$$

$$\frac{1}{x} - \frac{2}{x+.1} = 0$$

$$x + .1 = 2x$$

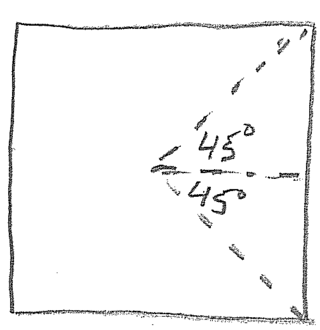
$$x = .1\,M = \boxed{10\text{ cm}}$$

(b) at the point (a), $\boxed{\vec{B} = 0}$

at the point (b), $|\vec{B}| = \frac{\mu_0 4}{2\pi(.1)} - \frac{\mu_0 2}{2\pi(.2)}$

$$|\vec{B}| = \frac{\mu_0}{2\pi}(30) = \frac{(4\pi \times 10^{-7})}{2\pi}(30) = \boxed{\begin{array}{c}6 \times 10^{-6}\text{ Tesla}\\ \text{upward}\end{array}}$$

(2)



$\sin 45° = \sqrt{2}/2$

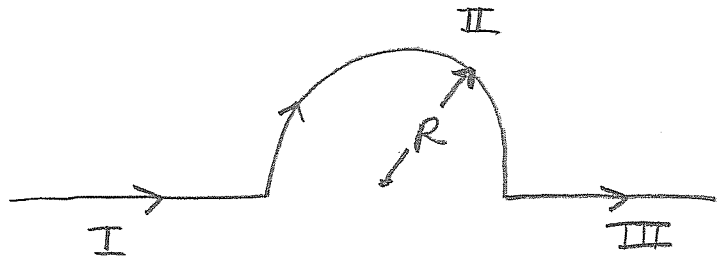
$\vec{B}_{net}$ = sum of $\vec{B}$ due to the 4 sides

$|\vec{B}_{net}| = 4(|\vec{B}|$ from 1 side$)$

$$|\vec{B}_{net}| = 4\frac{\mu_0 I}{4\pi(L/2)}(\sin 45° + \sin 45°)$$

$$|\vec{B}_{net}| = \boxed{\frac{\mu_0 I\, 2\sqrt{2}}{\pi L}}$$

(3)



$$\vec{B} = \vec{B}_{\text{from part I}} + \vec{B}_{\text{from part II}} + \vec{B}_{\text{from part III}}$$

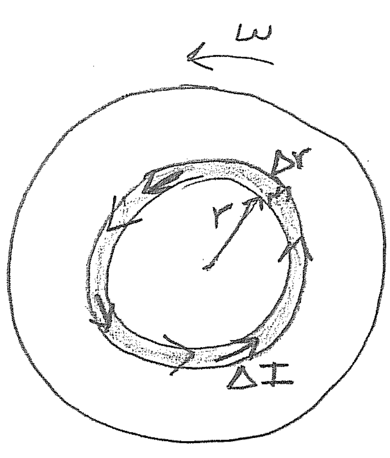
But $\vec{B}_{\text{Part I}} = B_{\text{Part III}} = 0$ since the current flows toward (away from) the point.

$$|\vec{B}| = \frac{1}{2}\,|\vec{B}_{\text{from a complete circular loop}}| = \frac{1}{2}\left(\frac{\mu_0 I}{2R}\right) = \boxed{\frac{\mu_0 I}{4R}}$$

(4) $\vec{B} = \vec{B}_{\text{from circular loop}} + \vec{B}_{\text{straight wire}}$

$$|\vec{B}| = \frac{\mu_0 I}{2R} + \frac{\mu_0 I}{2\pi R} = \boxed{\frac{\mu_0 I}{2R}\left(1 + \frac{1}{\pi}\right)}$$

(5)



The disk can be divided up into rings, which are circular loops of current ΔI. The magnetic field at the center of the rings is

$$\Delta B = \frac{\mu_0 \Delta I}{2r}$$

but $\Delta I = \frac{\text{charge}}{\text{sec}} = \frac{\Delta q}{\text{Time for 1 rev}} = (\Delta q)\frac{\omega}{2\pi}$

where Δq = charge in ring

(5 cont.)

$$\Delta q = \sigma (2\pi r \, \Delta r)$$

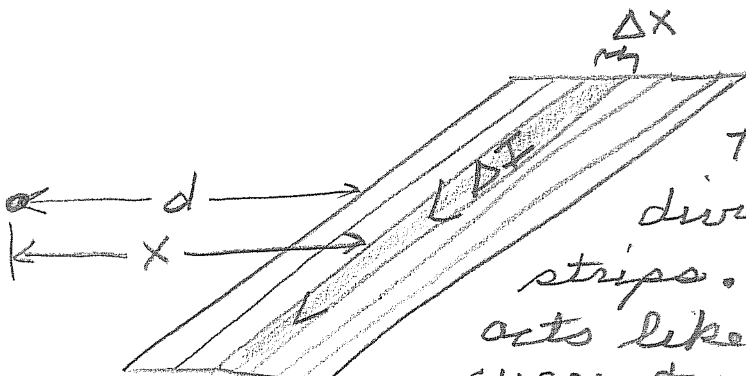
So $\Delta I = \sigma 2\pi r \, \Delta r \, \frac{\omega}{2\pi} = \omega \sigma r \, \Delta r$

$$\Delta B = \frac{\mu_0}{2r}(\omega \sigma r)\Delta r = \frac{\mu_0 \omega \sigma}{2} \Delta r$$

Integrating over the disk,

$$B = \int_0^R \frac{\mu_0 \omega \sigma}{2} dr = \boxed{\frac{\mu_0 \omega \sigma R}{2}}$$

(6)



The strip can be divided into thin strips. Each strip acts like a "thin wire" of current.

The magnetic field at the point from each thin strip is

$$\Delta B = \frac{\mu_0 \Delta I}{2\pi x}$$

but $\Delta I = I \frac{\Delta x}{w}$, so $\Delta B = \frac{\mu_0 I \Delta x}{w \, 2\pi x}$

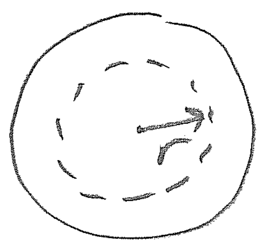
adding up the magnetic fields from the strips:

$$|\vec{B}| = \int_d^{d+w} \frac{\mu_0 I}{2\pi w} \frac{dx}{x} = \frac{\mu_0 I}{2\pi w} \ln x \Big|_d^{d+w}$$

$$|\vec{B}| = \boxed{\frac{\mu_0 I}{2\pi w} \ln\left(\frac{d+w}{d}\right)}$$

7 From the symmetry ideas discussed in class, and $\oiint \vec{B}\cdot d\vec{A} = 0$, the magnetic field must be circular and $|\vec{B}|$ only depends on r.

a) for $r < a$

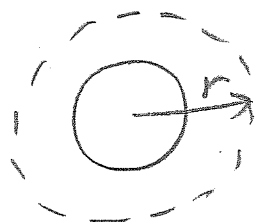


$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{through}$$

$$2\pi r\, B = \mu_0 \left(\frac{\pi r^2}{\pi a^2}\right) I$$

$$B = \boxed{\frac{\mu_0 I r}{2\pi a^2}}$$

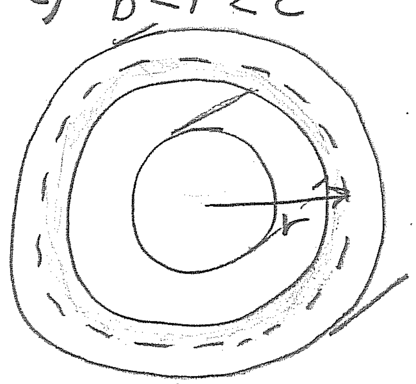
b) $a < r < b$



$$2\pi r B = \mu_0 I$$

$$\boxed{B = \frac{\mu_0 I}{2\pi r}}$$

c) $b < r < c$



$$2\pi r B = \mu_0 \left(I - \left(\text{current between } r \text{ and } b\right)\right)$$

$$2\pi r B = \mu_0 \left(I - \frac{I\left(\pi r^2 - \pi b^2\right)}{\left(\pi c^2 - \pi b^2\right)}\right)$$

$$\boxed{B = \frac{\mu_0 I}{2\pi r}\left(\frac{c^2 - r^2}{c^2 - b^2}\right)}$$

d) $r > c$

$$2\pi r B = \mu_0 (I - I) = 0$$

$$\boxed{B = 0}$$