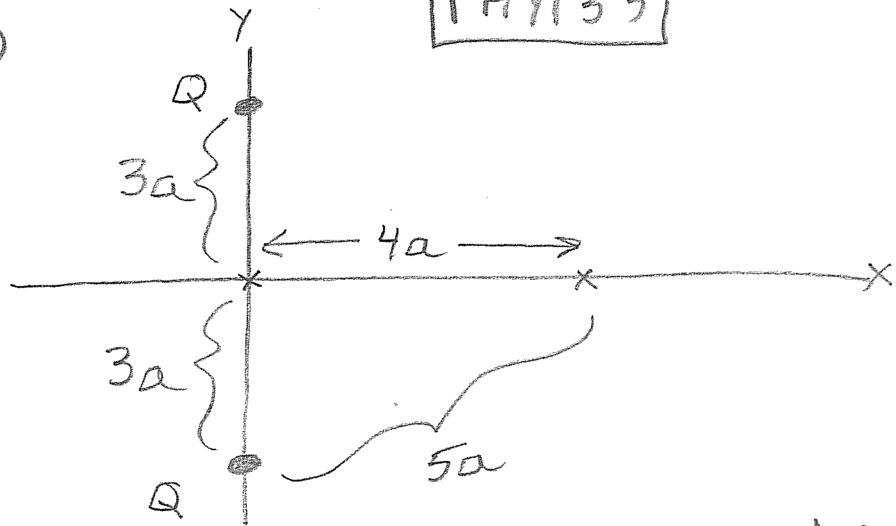
Solutions to Third Homework

PHY133

(1)



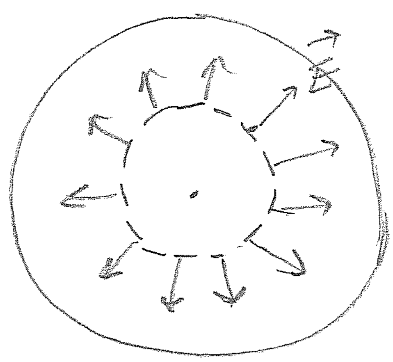
The electric potential at a position which is a distance r from a point source Q is $\frac{kQ}{r}$

$$\Delta V = \frac{kQ}{3a} + \frac{kQ}{3a} - \frac{kQ}{5a} - \frac{kQ}{5a}$$

$$\Delta V = \frac{kQ}{a}\left(\frac{2}{3} - \frac{2}{5}\right) = \boxed{\frac{4kQ}{15a}}$$

(−) is also ok

(2)



First use Gauss' Law to find $\vec{E}$ inside the sphere. Since there is spherical symmetry, $\vec{E}$ is radial and $|\vec{E}|$ only depends on r

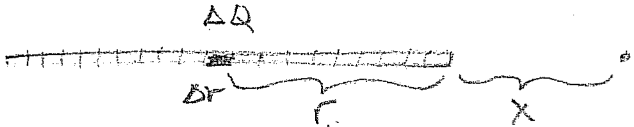
$$\oiint \vec{E} \cdot d\vec{A} = \frac{Q_{inside}}{\epsilon_0}$$

$$4\pi r^2 E = \tfrac{4}{3}\pi r^3 \rho / \epsilon_0$$

$$E = \frac{\rho r}{3\epsilon_0}$$

$$\Delta V = \int_0^R \frac{\rho r}{3\epsilon_0}\, dr = \frac{\rho r^2}{6\epsilon_0}\Big|_0^R = \boxed{\frac{\rho R^2}{6\epsilon_0}}$$

(3)



$$V_i = \frac{k\Delta Q}{r_i + x} = \frac{kQ\,\Delta r}{L(r_i + x)}$$

$$\frac{\Delta Q}{Q} = \frac{\Delta r}{L}$$

$$V = \sum V_i \longrightarrow \int_0^L \frac{kQ}{L}\frac{dr}{(r+x)}$$

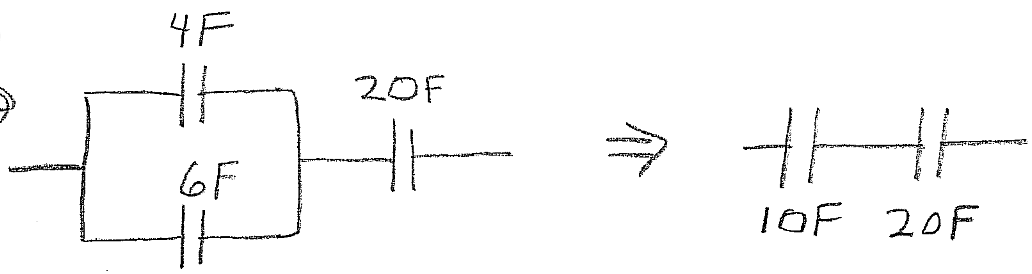
$$\boxed{V = \frac{kQ}{L}\left(\ln(L+x) - \ln x\right)}$$

$$E_x = -\frac{\partial V}{\partial x} = -\frac{kQ}{L}\left(\frac{1}{L+x} - \frac{1}{x}\right) = \boxed{\frac{kQ}{x(L+x)}}$$

In agreement with the result from lecture.

(4)

(a)



$$\frac{1}{C_{eq}} = \frac{1}{10} + \frac{1}{20} = \frac{3}{20}$$

$$\boxed{C_{eq} = \frac{20}{3}F}$$

(b) $Q = CV = \left(\frac{20}{3}\right)(12V) =$ $\boxed{\text{80 Coulombs on the 20F capacitor}}$

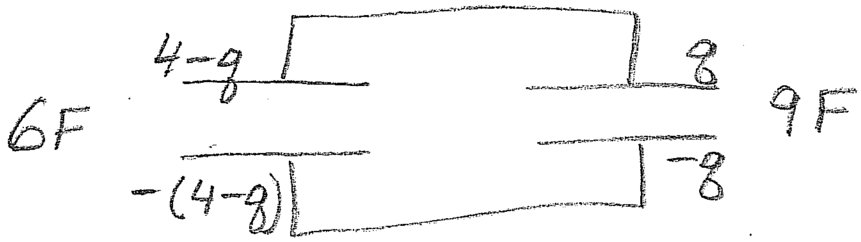
$V = \frac{Q}{C} = \frac{80}{20} = 4$ Volts across the 20F capacitor

So there are $12 - 4 = 8$ volts across both the 4F and 6F capacitors:

$Q_4 = 8(4) =$ $\boxed{\text{32 coulombs}}$ across the 4F capacitor

$Q_6 = 8(6) =$ $\boxed{\text{48 Coulombs}}$ across the 6F capacitor

(5) after the switch is closed, and the charges have come to equilibrium:



where q is charge that flows on the 9F capacitor.

Since the voltages are the same across the capacitors:

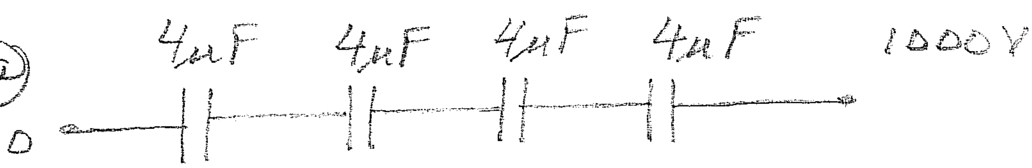
$$\frac{4-q}{6} = \frac{q}{9}$$

$$36 - 9q = 6q \Rightarrow \boxed{q = \frac{36}{15}}$$ coulombs on the 9F capacitor

$$4 - q = \boxed{\frac{24}{15}}$$ coulombs on the 6F capacitor

(6)

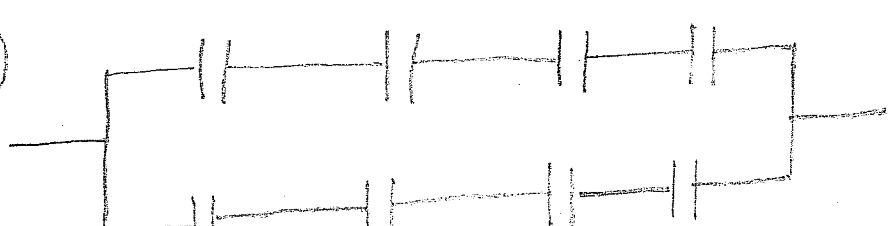
(a)



250V across each capacitor

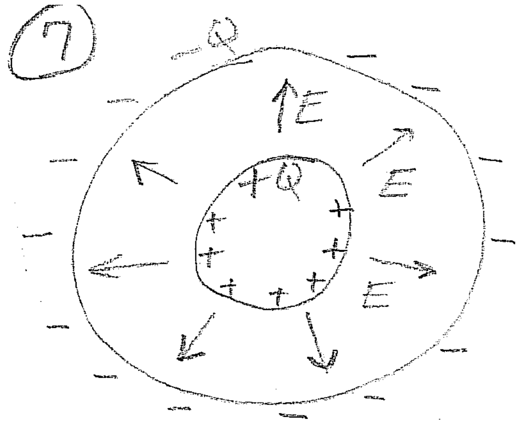
$$\frac{1}{C_{equiv}} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = 1 \Rightarrow C_{equiv} = 1\mu F$$

(b)



$$C_{equiv} = 1 + 1$$
$$= 2\mu F$$

(7)



The electric field between the conductors is

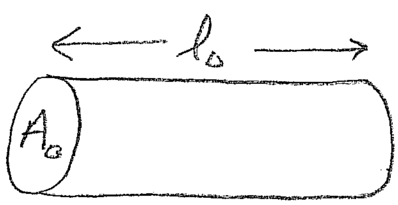
$|\vec{E}| = \frac{kQ}{r^2}$ and is radial.

$$V = \int E\,dr = \int_a^b \frac{kQ}{r^2}\,dr$$

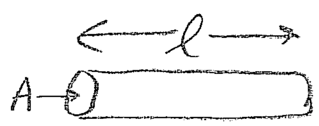
$$V = kQ\left(-\frac{1}{r}\right)\Big|_a^b = kQ\left(\frac{1}{a} - \frac{1}{b}\right) = \frac{kQ(b-a)}{ab}$$

So $Q = \frac{ab}{k(b-a)} V \Rightarrow C = \frac{ab}{k(b-a)} = \boxed{\frac{4\pi\epsilon_0 ab}{b-a}}$

(8)



$$R = \frac{\rho l_0}{A_0}$$



$$l = l_0/2$$

$$A = \pi\left(\frac{r_0}{2}\right)^2 = \frac{\pi r_0^2}{4} = \frac{A_0}{4}$$

So New Resistance $= \frac{\rho l}{A} = \frac{\rho\, l_0/2}{(A_0/4)} = 2\frac{\rho l_0}{A}$

$= \boxed{2R}$