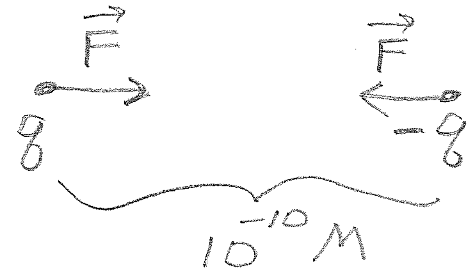
Solution to First Homework
PHY 133

① ⓐ



q for an electron and proton is $q = 1.6 \times 10^{-19}$ C

$$|\vec{F}| = \frac{k(q)(q)}{r^2} = \frac{(9\times10^{9})(1.6\times10^{-19})^2}{(10^{-10})^2}$$

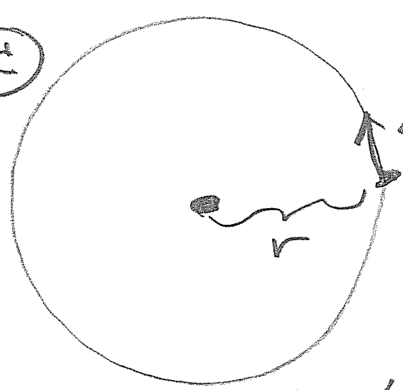
$$\boxed{|\vec{F}| = 2.3\times10^{-8}\,\text{N} \quad \text{towards proton}}$$

ⓑ From Newton's third law: $\vec{F}_{12} = -\vec{F}_{21}$

so

$$\boxed{|\vec{F}| = 2.3\times10^{-8}\,\text{N} \quad \text{towards electron}}$$

ⓒ



Since the electron's motion is assumed to be circular, the net force must be

$$|\vec{F}_{NET}| = \frac{m_e v^2}{r}$$

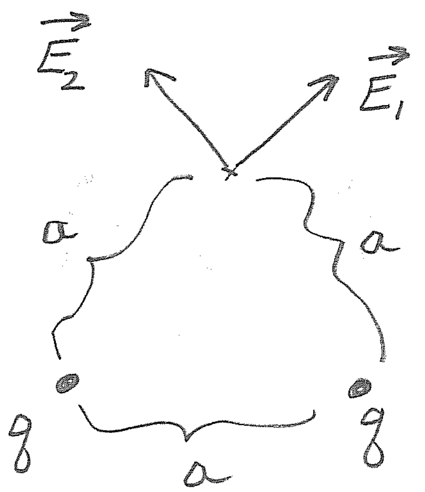
This force is due to the electrical force:

$$\boxed{\frac{kq^2}{r^2} = \frac{m_e v^2}{r}}$$

so

$$v = \sqrt{\frac{kq^2}{m_e r}} = \sqrt{\frac{(9\times10^{9})(1.6\times10^{-19})^2}{(9.1\times10^{-31})(10^{-10})}} = \boxed{1.59\times10^{6}\,\text{M/s}}$$

(2)

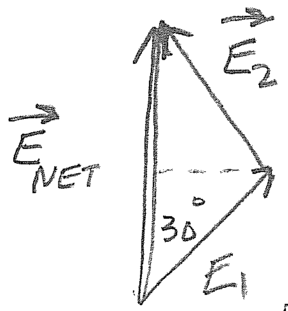


$$|\vec{E}_1| = \frac{kq}{a^2}$$

$$|\vec{E}_2| = \frac{kq}{a^2}$$

$$\vec{E}_{NET} = \vec{E}_1 + \vec{E}_2$$

From symmetry, $\vec{E}_{NET}$ points upward.



$$|\vec{E}_{NET}| = |\vec{E}_1| \cos 30 + |\vec{E}_2| \cos 30$$

$$= \frac{kq}{a^2}\left(\frac{\sqrt{3}}{2}\right) + \frac{kq}{a^2}\left(\frac{\sqrt{3}}{2}\right)$$

$$\boxed{|\vec{E}_{NET}| = \frac{\sqrt{3}\,kq}{a^2}} \quad \boxed{\text{upward}}$$

or in terms of unit vectors

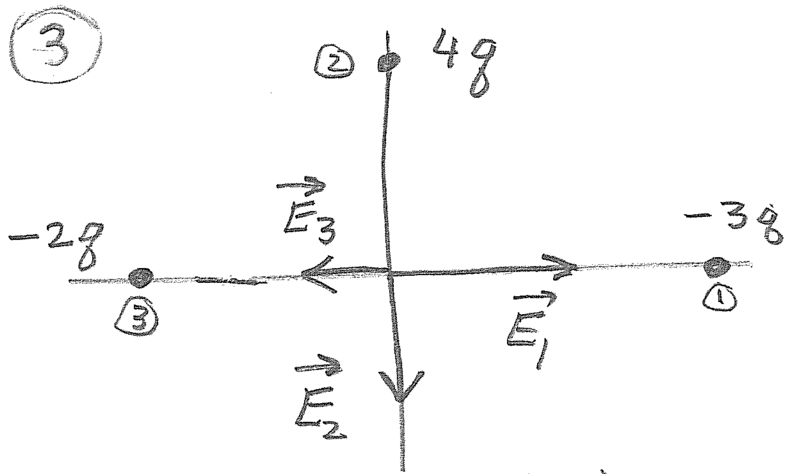
$$\vec{E}_1 = \frac{kq}{a^2}(\sin 30\ \hat{\imath} + \cos 30\ \hat{\jmath})$$

$$\vec{E}_2 = \frac{kq}{a^2}(-\sin 30\ \hat{\imath} + \cos 30\ \hat{\jmath})$$

$$\vec{E}_1 + \vec{E}_2 = \frac{kq}{a^2}\ (2 \cos 30\ \hat{\jmath})$$

$$\vec{E}_1 + \vec{E}_2 = \boxed{\frac{\sqrt{3}\,kq}{a^2}\ \hat{\jmath}}$$

③



$$\vec{E}_1 = \frac{k(3q)}{(3a)^2}(+\hat{\imath}) = \frac{kq}{3a^2}\hat{\imath}$$

$$\vec{E}_3 = \frac{k(2q)}{(2a)^2}(-\hat{\imath}) = -\frac{kq}{2}\hat{\imath}$$

$$\vec{E}_2 = \frac{k(4q)}{(2a)^2}(-\hat{\jmath}) = -\frac{kq}{a^2}\hat{\jmath}$$

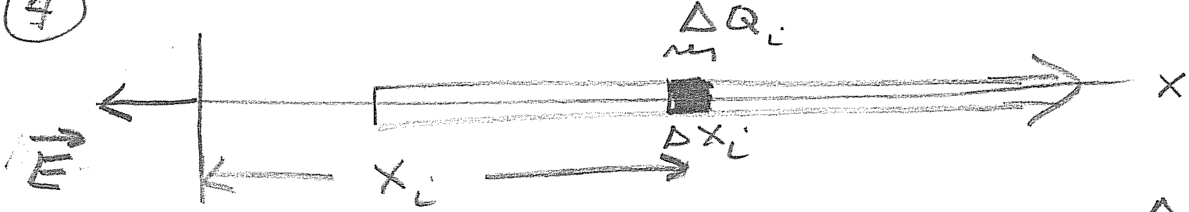
$$\vec{E}_{NET} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$$

$$= \frac{kq}{a^2}\left[\left(\frac{1}{3}-\frac{1}{2}\right)\hat{\imath} - \hat{\jmath}\right]$$

$$\vec{E}_{NET} = \frac{kq}{a^2}\left(-\frac{\hat{\imath}}{6} - \hat{\jmath}\right)$$

$$\boxed{\vec{E}_{NET} = -\frac{kq}{a^2}\left(\frac{\hat{\imath}}{6} + \hat{\jmath}\right)}$$

(4)



but $\lambda = \dfrac{\Delta Q_i}{\Delta x_i}$

$$|\Delta\vec{E}_i| = \frac{k\,\Delta Q_i}{x_i^2}$$

$$\Delta Q_i = \lambda\,\Delta x_i$$

$$\Delta Q_i = \frac{c}{x_i}\,\Delta x_i$$

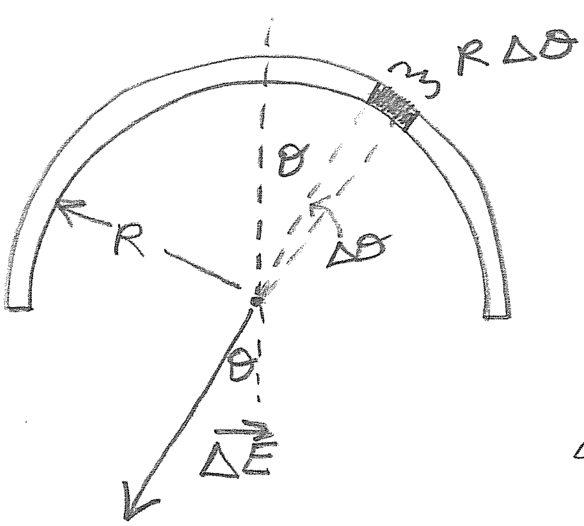
$$|\Delta\vec{E}_i| = \frac{k\,c\,\Delta x_i}{x_i^3}$$

$$\vec{E} = \sum \Delta E_i \longrightarrow \int_{x_0}^{\infty} \frac{k c}{x^3}\,dx \quad (-\hat{\imath})$$

$$\vec{E} = -\frac{kc}{2x^2}\Bigg|_{x_0}^{\infty} (-\hat{\imath}) = -\frac{kc}{2}\left(\frac{1}{\infty^2} - \frac{1}{x_0^2}\right)(-\hat{\imath})$$

$$\boxed{\vec{E} = -\frac{kc}{2x_0^2}\,\hat{\imath}}$$

(5)



$$|\Delta\vec{E}| = \frac{k\,\lambda(R\Delta\theta)}{R^2}$$

upon integrating around the semi-circle, the "x" components cancel. Only the downward component is non-zero.

5 cont.

$$\Delta E_y = |\Delta \vec{E}| \cos\theta$$

$$\Delta E_y = \frac{k\lambda R \Delta\theta}{R^2} \cos\theta \qquad \text{(downward)}$$

Integrating over the semi-circle gives

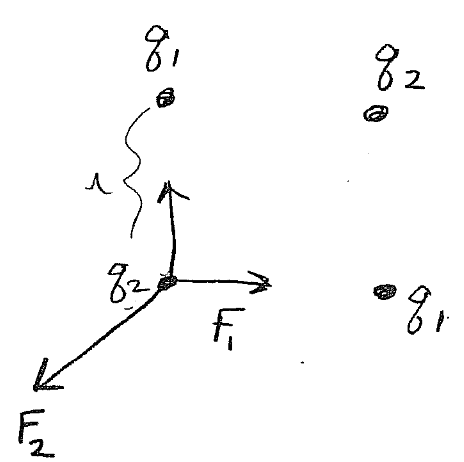
$$E_y = \int_{-\pi/2}^{\pi/2} \frac{k\lambda}{R} \cos\theta \, d\theta$$

$$E_y = \frac{k\lambda}{R} \sin\theta \Big|_{-\pi/2}^{\pi/2} = \frac{k\lambda}{R}(1-(-1)) = \frac{2k\lambda}{R}$$

but $\lambda = \frac{Q}{\pi R}$ so

$$\boxed{E_y = \frac{2kQ}{\pi R^2}} \quad \boxed{\text{downward}}$$

6

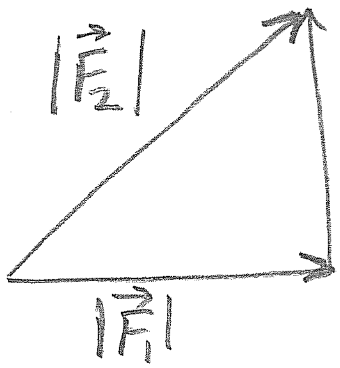


$$|\vec{F}_1| = \frac{k q_1 q_2}{a^2}$$

$$|\vec{F}_2| = \frac{k q_2^2}{(\sqrt{2}a)^2} = \frac{k q_2^2}{2a^2}$$

Note: q_1 and q_2 are opposite in sign, since the sum of the forces on $q_2 = 0$

(6 cont.)



Since the forces must add to zero,

$$\sqrt{2}\,|F_1| = |F_2|$$

$$\sqrt{2}\,\frac{k q_1 q_2}{a^2} = \frac{k q_2^2}{2a^2}$$

so

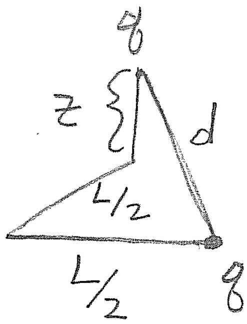
$$\sqrt{2}\, q_1 = \frac{q_2}{2}$$

$$|q_2| = 2\sqrt{2}\,|q_1|$$

Since the signs are opposite, we have

$$\boxed{q_2 = -2\sqrt{2}\, q_1}$$

(7)



The distance between the charge on the z-axis and a charge on the corner is

$$d = \sqrt{(L/2)^2 + (L/2)^2 + Z^2}$$

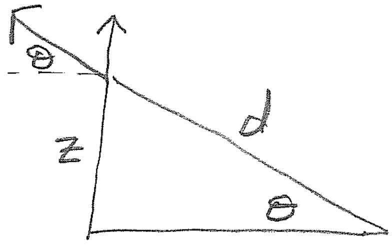
$$d = \sqrt{\frac{L^2}{2} + Z^2}$$

The <u>magnitude</u> of the force between the charge on the z-axis and a corner charge is:

7 cont

$$|\vec{F}_1| = \frac{kq^2}{d^2} = \frac{kq^2}{\left(\frac{L^2}{2}+z^2\right)}$$

By symmetry, the sum of the forces due to the 4 corners is in the $+z$ direction



$$\vec{F}_{net} = 4|F_1|\sin\theta\ \hat{k}$$

$$\vec{F}_{Net} = \frac{4kq^2}{\left(\frac{L^2}{2}+z^2\right)}\frac{z}{\sqrt{\frac{L^2}{2}+z^2}}\ \hat{k}$$

so $$\boxed{|\vec{F}_{Net}| = \frac{4kq^2 z}{\left(\frac{L^2}{2}+z^2\right)^{3/2}}}$$

F is maximized when

$$\frac{d|\vec{F}_{net}|}{dz} = 0$$

$$\frac{d|\vec{F}_{NET}|}{dz} = 4kq^2\left[\frac{1}{\left(\frac{L^2}{2}+z^2\right)^{3/2}} - \frac{3z^2}{\left(\frac{L^2}{2}+z^2\right)^{5/2}}\right] = 0$$

or at $\boxed{z = \frac{L}{2}}$

(b) The value of $|\vec{F}_{NET}|$ for this value of z is

$$|\vec{F}_{NET}| = \frac{4kq^2\left(\frac{L}{2}\right)}{\left(\frac{L^2}{2}+\frac{L^2}{4}\right)^{3/2}} = \frac{16kq^2}{\sqrt{27}\,L^2}$$

So the largest mass is

$$\boxed{m = \frac{16kq^2}{\sqrt{27}\,L^2 g}}$$