Solutions to Seventh Problem Set
PHY132

(1) $Q_{in} = 1500\,J \qquad Q_{out} = 1000\,J$

(a) $\eta = 1 - \frac{1000}{1500} = \boxed{\frac{1}{3}}$

(b) $W = Q_{in} - Q_{out} = \boxed{500\,J}$

(c) $\text{Power} = \frac{\text{Work}}{\text{time}} = \frac{500\,J}{.4\,\text{sec}} = \boxed{1250\ \text{Watts}}$

(2) Energy taken from the ocean (and transfered to ice)

$$= 10g\left(.5\frac{cal}{g\,°c}\right)(10°c) + 10g\left(80\frac{cal}{g}\right) + 10g\left(1\frac{cal}{g}\right)15°c$$

$$= 50 + 800 + 150$$

$$|Q| = 1000\ cal$$

$$\Delta S_{ocean} = \frac{-|Q|}{T_{ocean}}$$

because the ocean's temp didn't change during the transfer

$$\Delta S_{ocean} = \frac{-1000\ cal}{288\ °K} = -3.47\ \frac{cal}{°K}$$

2 cont

$$\Delta S_{ice} = m_{ice} C_{ice} \int_{263}^{273} \frac{dT}{T} + \frac{m_{ice}\, 80 \frac{cal}{g}}{273°K} + m_{water} C_{water} \int_{273}^{288} \frac{dT}{T}$$

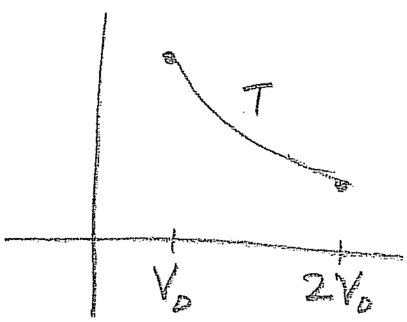
$$= 10g\left(.5 \frac{cal}{g°C}\right) \ln\left(\frac{273}{263}\right) + \frac{800}{273} + 10(1)\ln\left(\frac{288}{273}\right)$$

$$\Delta S_{ice} = (+.187 + 2.93 + .535)\ cal/°K$$

$$\Delta S_{ice} = +3.65\ cal/°K$$

$$\Delta S_{NET} = 3.65 - 3.47 \simeq \boxed{.18 \frac{cal}{°K}}$$

3



We can use any path to calculate ΔS. Since $\Delta U = 0$, T does not change, so we can choose an isothermal.

$\Delta S = \frac{Q}{T}$ but $Q = W$,

$$W = \int_{V_0}^{2V_0} nRT \frac{dV}{V} = nRT\ln 2$$

$$\text{so } \Delta S = \frac{nRT\ln 2}{T} = nR\ln 2 = R\ln 2 = \boxed{5.76 \frac{J}{°K}}$$

b) $\boxed{\Delta U = 0}$

c) $T = T_0 = \boxed{300°K}$

(4)

a) $E = \int C_v \, dT = \int_4^8 A T^3 \, dT$

$$E = \frac{A}{4} T^4 \Big|_4^8 = \frac{7.53 \times 10^{-6} (2 \text{ moles})}{4} (8^4 - 4^4)$$

$$\boxed{E = .0145 \text{ cal}}$$

(b) $\Delta S = \int \frac{\Delta Q}{T} = \int \frac{C_v}{T} dT = \int_4^8 \frac{A T^3}{T} dT$

$$\Delta S = \frac{A}{3} T^3 \Big|_4^8 = \frac{7.53 \times 10^{-6} (2 \text{ moles})}{3} (8^3 - 4^3)$$

$$\boxed{\Delta S = .00225 \ \frac{\text{cal}}{°K}}$$

(5)

$W_{a \to b} = W_{c \to d} = 0$

$W_{b \to c} = 2P_0(2V_0 - V_0) = 2P_0V_0$

$W_{d \to a} = P_0(V_0 - 2V_0) = -P_0V_0$

$\Delta U = \frac{3}{2} nR\Delta T$ for a monatomic ideal gas

$\Delta U = \frac{3}{2}\Delta(nRT) = \frac{3}{2}\Delta(PV)$

So $\Delta U_{a \to b} = \frac{3}{2}[2P_0V_0 - P_0V_0] = \frac{3}{2} P_0V_0$

5 cont

$$\Delta U_{b\to c} = \frac{3}{2}\left[2P_0(2V_0) - 2P_0V_0\right] = 3P_0V_0$$

$$\Delta U_{c\to d} = \frac{3}{2}\left[P_0(2V_0) - 2P_0(2V_0)\right] = -3P_0V_0$$

$$\Delta U_{d\to a} = \frac{3}{2}\left[P_0V_0 - P_0(2V_0)\right] = -\frac{3}{2}P_0V_0$$

Q is found using $Q = \Delta U + W_{BY}$

Process	Q	W	ΔU	ΔS
$a \to b$	$\frac{3}{2}P_0V_0$	0	$\frac{3}{2}P_0V_0$	$\frac{3}{2}nR\ln 2$
$b \to c$	$5P_0V_0$	$2P_0V_0$	$3P_0V_0$	$\frac{5}{2}nR\ln 2$
$c \to d$	$-3P_0V_0$	0	$-3P_0V_0$	$-\frac{3}{2}nR\ln 2$
$d \to a$	$-\frac{5}{2}P_0V_0$	$-P_0V_0$	$-\frac{3}{2}P_0V_0$	$-\frac{5}{2}nR\ln 2$
complete cycle	P_0V_0	P_0V_0	0	0

$$\Delta S_{a\to b} = \int \frac{dQ}{T} = \int nC_v\frac{dT}{T} = \frac{3}{2}nR\int_{T_i}^{T_f}\frac{dT}{T} = \frac{3}{2}nR\ln 2$$

$$\Delta S_{b\to c} = \int nC_p\frac{dT}{T} = \frac{5}{2}nR\int_{T_i}^{T_f}\frac{dT}{T} = \frac{5}{2}nR\ln 2$$

$$\Delta S_{c\to d} = \int nC_v\frac{dT}{T} = \frac{3}{2}nR\int\frac{dT}{T} = \frac{3}{2}nR\ln\left(\frac{1}{2}\right) = -\frac{3}{2}nR\ln 2$$

$$\Delta S_{d\to a} = \int nC_p\frac{dT}{T} = \frac{5}{2}nR\int_{T_i}^{T_f}\frac{dT}{T} = \frac{5}{2}nR\ln\left(\frac{1}{2}\right) = -\frac{5}{2}nR\ln 2$$

5 cont

a) $\epsilon = \frac{W_{NET}}{Q_{in}}$

$W_{NET} = P_0 V_0$

$Q_{in} = \frac{3}{2} P_0 V_0 + 5 P_0 V_0 = \frac{13}{2} P_0 V_0$

So, $\epsilon = \frac{P_0 V_0}{\frac{13}{2} P_0 V_0} = \boxed{\frac{2}{13}}$

$\epsilon_{carnot} = 1 - \frac{T_C}{T_H}$

Since $T = \frac{PV}{nR}$,

$\epsilon_{carnot} = 1 - \frac{\left(\frac{P_0 V_0}{nR}\right)}{\left(\frac{2P_0(2V_0)}{nR}\right)} = 1 - \frac{1}{4} = \boxed{\frac{3}{4}}$

$\frac{2}{13}$ is much less than $3/4$, not such a good engine