Solutions to Problem Set 4
PHY 132

① $d_1 + d_2 = 4\sin\omega t + 4\cos\omega t$

$= 4\sin\omega t + 4\sin(\pi/2 - \omega t)$

$= 4(2)\sin\left(\frac{\omega t + \frac{\pi}{2} - \omega t}{2}\right)\cos\left(\frac{\omega t - (\frac{\pi}{2} - \omega t)}{2}\right)$

$d_1 + d_2 = 8\sin\left(\frac{\pi}{4}\right)\cos(\omega t - \pi/4)$

$d_1 + d_2 = \boxed{4\sqrt{2}\cos(\omega t - \pi/4)}$ since $\sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$

② ⓐ Intensity $= \frac{10 \times 10^{-3}\,W}{4\pi\,(4)^2\,M^2} = 4.97 \times 10^{-5}\,\frac{W}{M^2}$

$dB = 10\log\left(\frac{4.97 \times 10^{-5}}{10^{-12}}\right) \simeq \boxed{77\,dB}$

ⓑ Intensity $= \frac{20 \times 10^{-3}\,W}{4\pi\,(5)^2\,M^2} = 6.37 \times 10^{-5}\,\frac{W}{M^2}$

$dB = 10\log\left(\frac{6.37 \times 10^{-5}}{10^{-12}}\right) = \boxed{78\,dB}$

ⓒ Since the sources are incoherent, one adds the intensities.

$I = (4.97 + 6.37) \times 10^{-5} = 11.34 \times 10^{-5}\,W/M^2$

$dB = 10\log\left(\frac{11.34 \times 10^{-5}}{10^{-12}}\right) = \boxed{80.5\,dB}$

③ $560 = f_0 \left(\frac{340}{340-v}\right)$ $480 = f_0\left(\frac{340}{340+v}\right)$

$$(340-v)\frac{560}{340} = (340+v)\frac{480}{340}$$

$$340(560) - 560v = 340(480) + 480v$$

$$v = \frac{340(560-480)}{560+480}$$

$$\boxed{v = 26\ m/s}$$

④ ⓐ $$2000 = 1500\left(\frac{340}{340-v}\right)$$

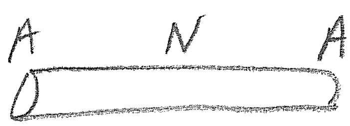
$$(340-v) = \frac{1500}{2000}(340)$$

$$v = 340 - \frac{1500}{2000}(340) = \boxed{85\ m/s}$$

ⓑ $$f = 2000\left(\frac{340+85}{340}\right)$$ (observer moving)

$$\boxed{f = 2500\ Hz}$$

⑤ ⓐ

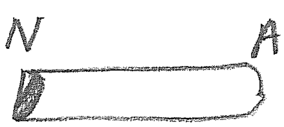


$$\lambda = 2L$$

$$f = \frac{v}{\lambda} = \frac{v}{2L}$$

$$L = \frac{v}{2f} = \frac{340}{2(55)} = \boxed{3.09\ m}$$

ⓑ

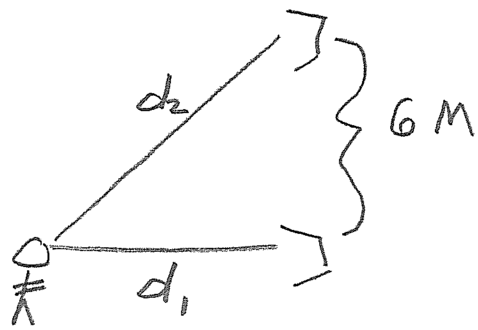


$$\lambda = 4L$$

$$f = \frac{v}{\lambda} = \frac{v}{4L}$$

$$L = \frac{v}{4f} = \frac{340}{4(55)} = \boxed{1.55\ m}$$

⑥ The wavelength is $\lambda = \frac{v}{f} = \frac{340}{200} = \underline{\underline{1.7M}}$



ⓐ a minimum in intensity (destructive interference) occurs when $d_2 - d_1 = \frac{1.7}{2}, \frac{3}{2}(1.7), \frac{5}{2}(1.7), \cdots$

$= .85M, 2.55M, 4.25M, 5.95M, 7.65M$

Since the largest value $d_2 - d_1$ can be is 6M, there can only be $\boxed{4 \text{ minima}}$

ⓑ $d_2 - d_1 = \sqrt{6^2 + d_1^2} - d_1 = (\Delta d)$

$\sqrt{6^2 + d_1^2} = \Delta d + d_1$

$6^2 + \cancel{d_1^2} = (\Delta d)^2 + 2(\Delta d)d_1 + \cancel{d_1^2}$

$$\boxed{d_1 = \frac{36 - (\Delta d)^2}{2(\Delta d)}}$$

Δd	d_1
.85	20.75 M
2.55	5.78 M
4.25	2.11 M
5.95	5 cm

(7) assuming the pipes are open at both ends, $f = \frac{v}{2L}$

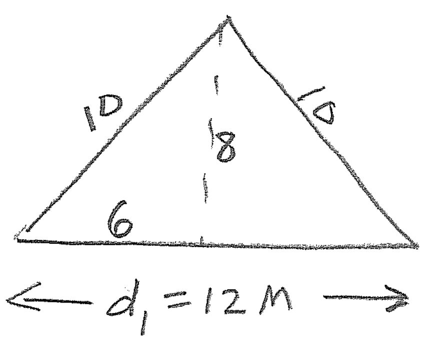
$L = \frac{v}{2f}$:

$L_1 = \frac{340}{2(256)} = .664\,M$

$L_2 = \frac{340}{2(440)} = .386\,M$

Total length = $\boxed{1.05\,M}$

(8)



$d_2 = 20\,M$

$d_2 - d_1 = 20 - 12 = 8\,M$

For constructive interference

$d_2 - d_1 = \lambda$

so $\lambda = 8\,M$

$f = \frac{v}{\lambda} = \frac{340}{8} = \boxed{42.5\ Hz}$

a very low frequency