Solutions to Third Homework
PHY132

① Let A_0 = original amplitude

a) $E_{TOT} = \frac{k}{2}A^2$ if $A = 2A_0$, $E = \frac{k}{2}(2A_0)^2 = 4\left(\frac{k}{2}A_0^2\right)$

$E = 4E_0$ $\boxed{\text{The energy increases by a factor of 4}}$

b) $v_{max} = \omega A$

$v_{max} = \omega(2A_0) = 2(\omega A_0)$ $\boxed{v_{max} \text{ doubles}}$

c) $a_{max} = \omega^2 A$

$a_{max} = \omega^2(2A_0) = 2(\omega^2 A_0)$ $\boxed{a_{max} \text{ doubles}}$

d) $T = 2\pi\sqrt{\frac{m}{k}}$ T does not depend on A

$\boxed{\text{NO CHANGE}}$

② $T = 2\pi\sqrt{\frac{l}{g}}$

a) $l \to 2l$ $\Rightarrow$ $\boxed{T \text{ increases by a factor of } \sqrt{2}}$

b) T does not depend on m $\Rightarrow$ $\boxed{\text{NO CHANGE}}$

c) $T = 2\pi\sqrt{\frac{l}{g/6}} = \sqrt{6}\left(2\pi\sqrt{\frac{l}{g}}\right) = \sqrt{6}\,T_0$

$\boxed{T \text{ increases by a factor of } \sqrt{6}}$

(3)

$$k = \frac{\Delta F}{\Delta x} = \frac{100(9.8)\,N}{.03\,M} = 32667\ N/M$$

$$\omega = \sqrt{\frac{k}{m}}$$

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{32667}{2100}} = \boxed{.628\ Hz}$$

or $T = \frac{1}{.628} = 1.59$ sec for one oscillation of the car

(4)

a) $A = \boxed{6 \text{ units}}$

b) $3\pi T = 2\pi \Rightarrow \boxed{T = \frac{2}{3}\text{ sec}}$ (if t is in sec)

c) $f = \frac{1}{T} = \boxed{\frac{3}{2}\ Hz}$

(5) $T = 6$ sec $\Rightarrow \omega = \frac{2\pi}{6} = \frac{\pi}{3}$ sec^{-1}

$x(t) = 5\sin\left(\frac{\pi}{3}t + \varphi\right)$ since $A = 5$ cm

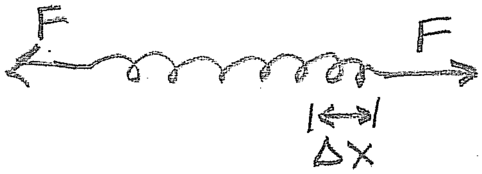
at $t = 0$, $x(0) = 3$, so

$3 = 5\sin(0 + \varphi)$

$\sin\varphi = 3/5 \Rightarrow \varphi = .643$ radians (or 2.498 radians)

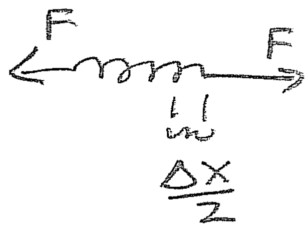
If the initial velocity is in the + direction, then $\varphi = .643$ radians, so $\boxed{x(t) = 5\sin\left(\frac{\pi}{3}t + .643\right)}$

(6)



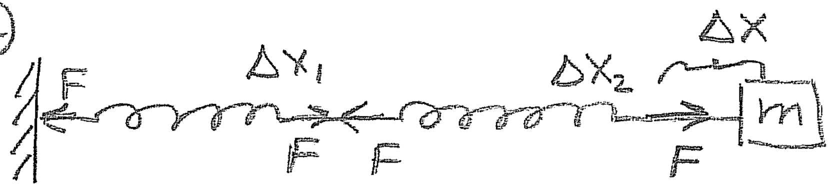
$\Delta x = \frac{F}{k} \Rightarrow k = \frac{F}{\Delta x}$

when the spring is cut in half, the same force streches the spring $\frac{\Delta x}{2}$



$$\text{spring constant} = \frac{F}{\Delta x/2} = 2\,\frac{F}{\Delta x} = \boxed{2k}$$

(7) a)



$\Delta x = \Delta x_1 + \Delta x_2$

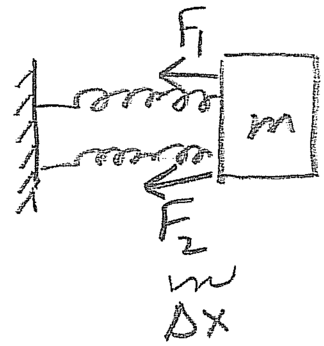
The force is the same for both springs

$\Delta x_1 = \frac{F}{k_1} \qquad \Delta x_2 = \frac{F}{k_2}$

$$\Delta x = \frac{F}{k_1} + \frac{F}{k_2} = \left(\frac{1}{k_1} + \frac{1}{k_2}\right) F$$

$$k = \frac{F}{\Delta x} = \frac{1}{\frac{1}{k_1} + \frac{1}{k_2}} = \boxed{\frac{k_1 k_2}{k_1 + k_2}} \qquad T = 2\pi\sqrt{\frac{m(k_1+k_2)}{k_1 k_2}}$$

(b)

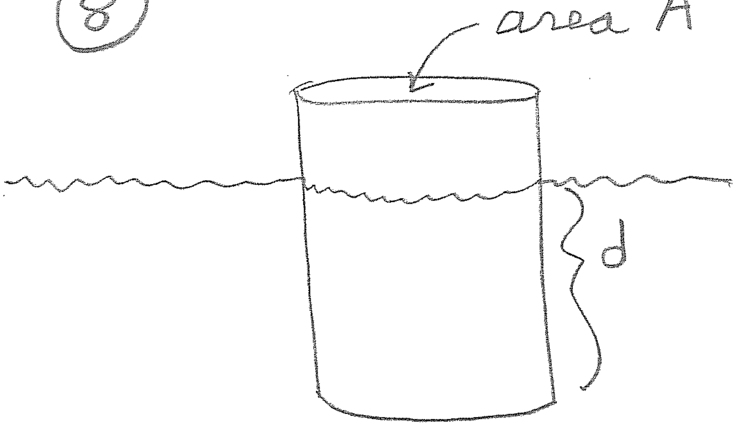


$$F = F_1 + F_2 = k_1 \Delta x + k_2 \Delta x$$

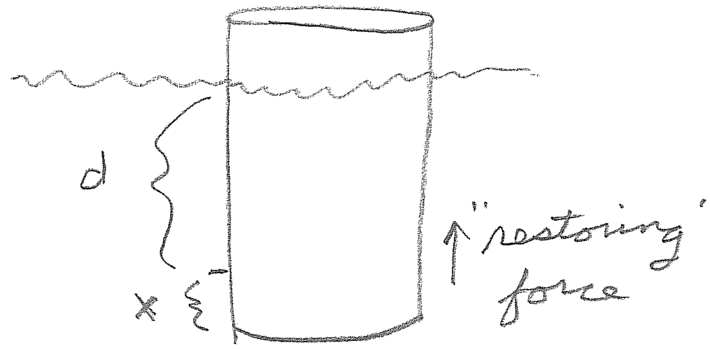
$$F = (k_1 + k_2)\,\Delta x$$

$$k = \frac{F}{\Delta x} = \boxed{k_1 + k_2} \qquad T = 2\pi\sqrt{\frac{m}{k_1 + k_2}}$$

8



at equilibrium, the cylinder is a length d under water



If the cylinder is pushed down a distance x, it will be pushed upward due to the "extra" Bouyant force.

$F_{restoring}$ = weight of additional fluid displaced when the cylinder is pushed down a distance x

$$F_x = -\rho(Ax)g$$ The Force is in the opposite direction to x

$$\boxed{F_x = -(\rho A g)\, x}$$

Since the restoring force is proportional to x (i.e. linear) the motion is sinusoidal in time.

The restoring constant, $c = \rho A g$, so

$$T = 2\pi\sqrt{\frac{m}{c}} = \boxed{2\pi\sqrt{\frac{m}{\rho A g}}}$$ where m is the mass of the disk.

9 (a) $$Y_1(x,t) = \frac{5}{(3x-4t)^2+2}$$ travels to $\boxed{\text{right}}$

$$Y_2(x,t) = \frac{-5}{(3x+4t-6)^2+2}$$ travels to $\boxed{\text{left}}$

(b) The two pulses cancel everywhere when

$$(3x-4t)^2 = (3x+4t-6)^2$$

So when $\cancel{3x}-4t = \cancel{3x}+4t-6$

$\boxed{t = \frac{3}{4}\text{ sec}}$ the pulses cancel everywhere

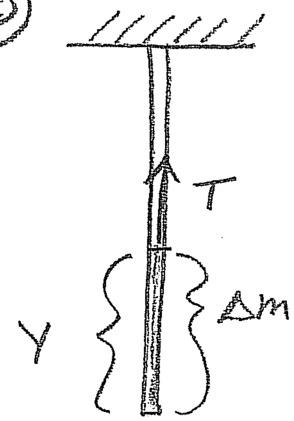
or

(c) $$3x-4t = -(3x+4t-6)$$

$$3x = -3x+6$$

at $\boxed{x=1}$ the pulses cancel at all times

(10)



(a) $T =$ weight of rope between zero and Y

$$T = (\Delta m) g = \left(\frac{Y}{L} m\right) g$$

Since $$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{\left(\frac{Y}{L}\right) m g}{\left(\frac{m}{L}\right)}}$$

$$\boxed{v = \sqrt{gY}}$$

(b) $$\frac{dY}{dt} = v$$

$$\frac{dy}{v} = dt \Rightarrow \int_0^L \frac{dy}{v} = \int_0^t dt = t$$

$$t = \int_0^L \frac{dy}{v} = \int_0^L \frac{dy}{\sqrt{gY}} = \frac{1}{\sqrt{g}} 2 Y^{\frac{1}{2}} \Big|_0^L$$

$$\boxed{t = 2\sqrt{\frac{L}{g}}}$$