Solutions to Second Homework
PHY132

1) $|\vec{F}| = \frac{GmM}{r^2}$

a) $m = 70\,Kg$ $\quad r = 6.37 \times 10^6\,M$

$M = 5.98 \times 10^{24}\,Kg$

$$F = \frac{(6.67\times10^{-11})(70)(5.98\times10^{24})}{(6.37\times10^{6})^2} = \boxed{688\ N}$$

b) $m = 70\,Kg$ $\quad r = 3.82 \times 10^8\,M$

$M = 7.36 \times 10^{22}\,Kg$

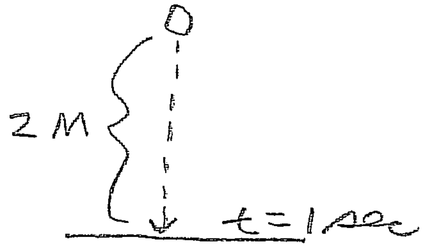
$$F = \frac{(6.67\times10^{-11})(70)(7.36\times10^{22})}{(3.82\times10^{8})^2} = \boxed{.00238\ N}$$

c) $m = 70\,Kg$ $\quad r = 1.5 \times 10^{11}\,M$

$M = 1.99 \times 10^{30}\,Kg$

$$F = \frac{(6.67\times10^{-11})(70)(1.99\times10^{30})}{(1.5\times10^{11})^2} = \boxed{.413N}$$

So, the largest gravitational force we feel is due to the earth.

② 

Since $h = \frac{g}{2} t^2$

$$g = \frac{2h}{t^2} = \frac{2(2)}{1^2} = 4\ M/s^2$$

$\rho = 6000 \frac{Kg}{M^3}$

$$mg = \frac{GMm}{R^2}$$

$$g = \frac{GM}{R^2} = \frac{G\left(\frac{4}{3}\pi R^3 \rho\right)}{R^2} = \frac{4}{3}\pi G \rho R$$

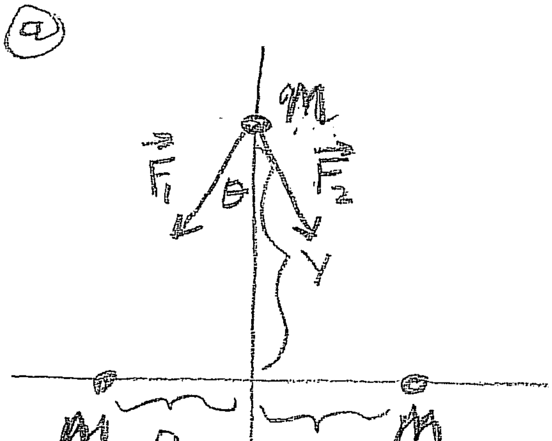
$$R = \frac{3g}{4\pi G \rho} = \frac{3(4)}{4\pi(6.67\times10^{-11})(6000\,Kg/M^3)}$$

$$\boxed{R = 2.39\times10^{6}\ M}$$

$$M = \frac{4}{3}\pi R^3 \rho = \frac{4}{3}\pi(2.39\times10^{6})^3\left(6000\ \frac{Kg}{M^3}\right)$$

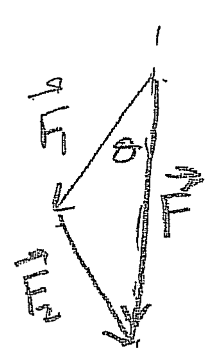
$$\boxed{M = 3.41\times10^{23}\ Kg}$$

③ ⓐ



$$|\vec{F}_1| = |\vec{F}_2| = \frac{GmM}{\left(\sqrt{y^2+a^2}\right)^2}$$

$$|\vec{F}_1| = |\vec{F}_2| = \frac{GmM}{y^2+a^2}$$



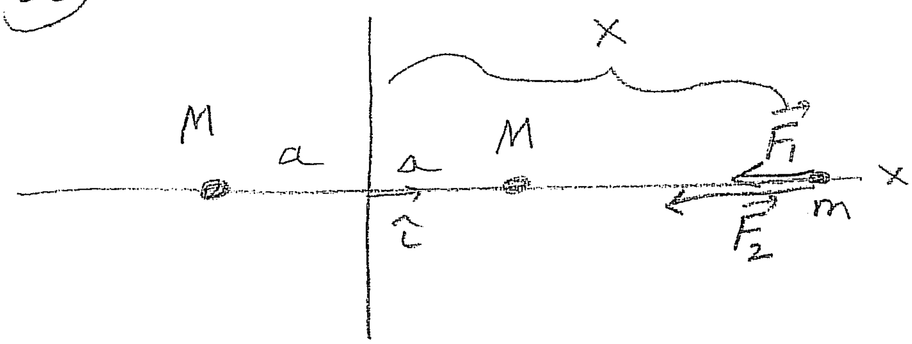
$$|\vec{F}| = 2|\vec{F}_1|\cos\theta$$

3 cont. $|\vec{F}| = \frac{2GmM}{y^2+a^2}\cos\theta$

but $\cos\theta = \frac{y}{\sqrt{y^2+a^2}}$ so $|\vec{F}| = \frac{2GmMy}{(y^2+a^2)^{3/2}}$

Since the direction is always towards the origin so $\boxed{\vec{F} = -\frac{2GmMy}{(y^2+a^2)^{3/2}}\hat{j}}$

3b)



The marble of mass m is attracted to each of the other 2 marbles.

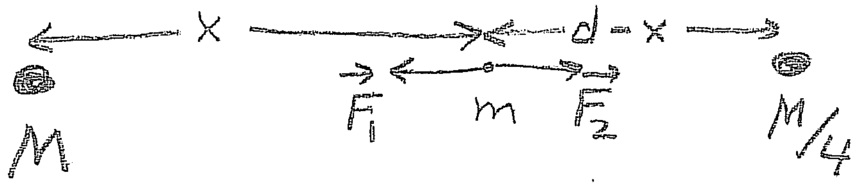
$|\vec{F}_1| = \frac{GMm}{(x+a)^2}$ in the $(-\hat{i})$ direction

$|\vec{F}_2| = \frac{GMm}{(x-a)^2}$ in the $(-\hat{i})$ direction

$$\vec{F}_{NET} = \frac{GMm}{(x+a)^2}(-\hat{i}) + \frac{GMm}{(x-a)^2}(-\hat{i})$$

$$\boxed{\vec{F}_{NET} = GMm\left(\frac{1}{(x+a)^2} + \frac{1}{(x-a)^2}\right)(-\hat{i})}$$

(4)



$$|\vec{F}_1| = \frac{GmM}{x^2} \qquad |\vec{F}_2| = \frac{Gm\left(\frac{M}{4}\right)}{(d-x)^2}$$

for $|\vec{F}_1| = |\vec{F}_2|$ we have

$$\frac{GmM}{x^2} = \frac{Gm\left(\frac{M}{4}\right)}{(d-x)^2}$$

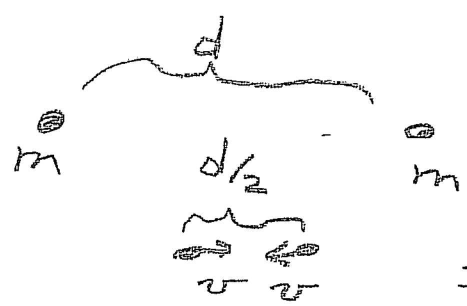
$$\frac{1}{x^2} = \frac{1/4}{(d-x)^2} \Rightarrow \frac{4}{x^2} = \frac{1}{(d-x)^2}$$

Taking the square root of both sides, gives

$$\frac{2}{x} = \frac{1}{d-x} \Rightarrow 2(d-x) = x$$

$$\boxed{x = \frac{2d}{3}}$$

(5)



$$\text{Initial energy} = -\frac{Gm^2}{d}$$

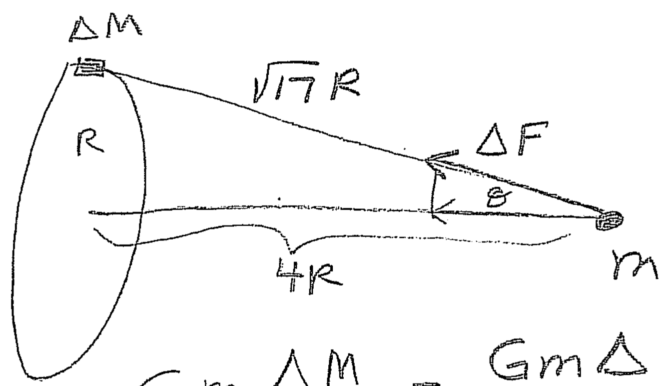
$$\text{Final energy} = -\frac{Gm^2}{d/2} + \left(\frac{m}{2}v^2\right)2$$

Since mechanical energy is conserved,

$$-\frac{Gm^2}{d} = -\frac{2Gm^2}{d} + mv^2$$

$$\boxed{v = \sqrt{\frac{Gm}{d}}}$$

(6) (a)



$$|\vec{\Delta F}| = \frac{G m \Delta M}{r^2} = \frac{G m \Delta M}{(\sqrt{17}R)^2} = \frac{G m \Delta M}{17 R^2}$$

after integrating around the ring, only the component that survives is along the axis

$$\Delta F_x = |\vec{\Delta F}| \cos\theta = \frac{G m \Delta m}{17 R^2} \frac{4R}{\sqrt{17} R} = \frac{G m \Delta M \, 4}{(17)^{3/2} R^2}$$

$$F_x = \Sigma \Delta F_x = \int_{\text{around ring}} \frac{4 G m}{(17)^{3/2} R^2} \, dM = \boxed{\frac{4 G m M}{(17)^{3/2} R^2}}$$

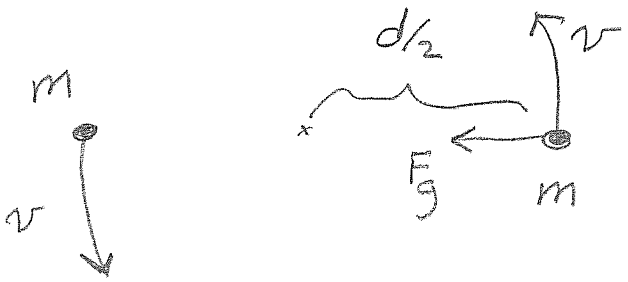
(b) $(K.E.)_{final} + (P.E.)_{final} = (K.E.)_{initial} + (P.E.)_{initial}$

$$\frac{m}{2} v^2 - \frac{G M m}{R} = 0 - \frac{G M m}{\sqrt{17} R}$$

$$v^2 = \frac{2 G M}{R}\left(1 - \frac{1}{\sqrt{17}}\right)$$

$$\boxed{v = \sqrt{\frac{2 G M}{R}\left(\frac{\sqrt{17} - 1}{\sqrt{17}}\right)}}$$

⑦



For circular motion $F = \dfrac{mv^2}{(d/2)} = \dfrac{2mv^2}{d}$

The force that causes the circular motion is the gravitational force: $\dfrac{Gm^2}{d^2}$

$$\frac{2mv^2}{d} = \frac{Gm^2}{d^2}$$

$$v = \sqrt{\frac{Gm}{2d}}$$

The period $T = \dfrac{2\pi\left(\frac{d}{2}\right)}{v} = \dfrac{\pi d}{\sqrt{\frac{Gm}{2d}}} = \pi\sqrt{\dfrac{2}{Gm}}\, d^{3/2}$

or

$$\boxed{T^2 = \frac{2\pi^2}{Gm} d^3}$$