Solutions to First Homework
PHY 132

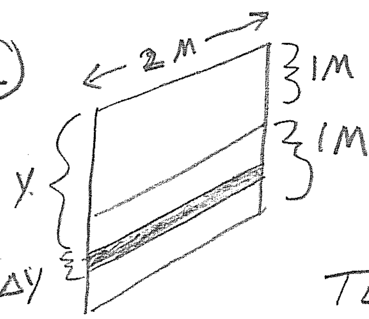
(1)

$$P = P_{ATM} + \rho g h \qquad P_{ATM} = 1.013\times10^{5}\ \text{Pa}$$

$$P = 1.013\times10^{5} + \left(1000\ \frac{\text{kg}}{\text{m}^3}\right)\left(9.8\ \frac{\text{m}}{\text{s}^2}\right)\left(100\ \text{m}\right)$$

$$\boxed{P = 1.081\times10^{6}\ \text{Pa}} = \boxed{10.7\ \text{atm}}$$

(2)
(a)



$$\Delta F = P\Delta A$$

$$\Delta F = (\rho g y)(2\Delta y) = 2\rho g y\,\Delta y$$

To find the total force, we need to integrate over the door from $y=1$ to $y=2$:

$$F = \int_1^2 2\rho g y\,dy = \rho g y^2\Big|_1^2 = \rho g(4-1)$$

$$F = \left(1000\ \frac{\text{kg}}{\text{m}^3}\right)\left(9.8\ \frac{\text{m}}{\text{s}^2}\right)(3\text{m}) = \boxed{29400\ \text{N}}$$

(b)

$$\Delta\tau = (y-1)\Delta F = (y-1)(2\rho g y\,\Delta y)$$

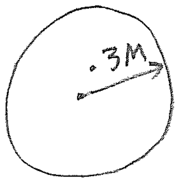
Integrating over the door gives:

$$\tau = \int_1^2 (y-1)(2\rho g y)\,dy = \left(\frac{2\rho g y^3}{3} - \rho g y^2\right)\Big|_1^2$$

$$\tau = \rho g\left(\frac{2}{3}(7) - 3\right) = \boxed{\frac{5}{3}\rho g} = \frac{5}{3}(1000)(9.8)$$

$$\boxed{\tau = 16333\ \text{N-M}}$$

(3) Consider 1 balloon:



$$\text{wt of 1 balloon} = \left(\frac{4}{3}\pi r^3 \rho_{He}\right) g$$

$$= \left(\frac{4}{3}\pi (.3)^3 \, .18\right) 9.8$$

$$= (.0204)\ \text{Kg}\ (9.8\ \text{m/s})$$

wt of one balloon $\approx .2$ N

$$F_{\text{Bouyant for one balloon}} = \rho_{air} V g = \left(1.29 \frac{\text{Kg}}{\text{M}^3}\right)\left(\frac{4}{3}\pi r^3\right)(9.8)$$

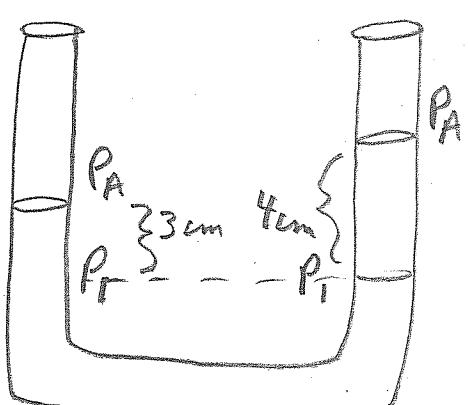
$$= 1.29\left(\frac{4}{3}\pi (.3)^3\right)(9.8) = 1.43\ \text{N}$$

Let n = number of balloons.

$$n(1.43\ \text{N}) = 70(9.8) + n(.2)$$

$$n = \frac{70(9.8)}{1.43 - .2} = \boxed{558 \text{ balloons}}$$

(4)



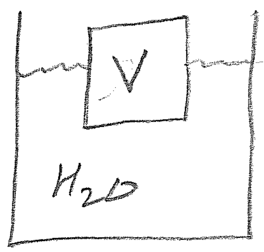
$$P_1 = P_A + \rho_{H_2O}\, g\,(.03\text{M})$$

$$P_1 = P_A + \rho_{oil}\, g\,(.04\text{M})$$

so $\rho_{oil}\, g(.04) = \rho_{H_2O}\, g(.03)$

$$\rho_{oil} = \frac{3}{4}\rho_{H_2O} = \boxed{750 \frac{\text{Kg}}{\text{M}^3}}$$

(5) (a)

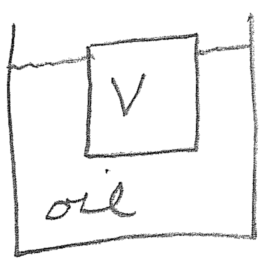


$$mg = \frac{2}{3} V \rho_{H_2O}\, g$$

$$\rho V g = \frac{2}{3} V \rho_{H_2O}\, g$$

$$\rho = \boxed{\frac{2}{3} \rho_{H_2O}} = \boxed{667 \frac{\text{Kg}}{\text{M}^3}}$$

(b)



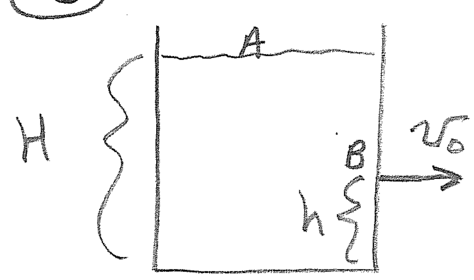
$$mg = \frac{9}{10} V \rho_{oil}\, g$$

$$\rho V g = \frac{9}{10} V \rho_{oil}\, g$$

$$\rho_{oil} = \frac{10}{9} \rho = \frac{10}{9}\left(\frac{2}{3} \rho_{H_2O}\right) = \boxed{\frac{20}{27} \rho_{H_2O}}$$

$$\boxed{\rho_{oil} = 741 \text{ Kg/M}^3}$$

(6)



$$P_A + \frac{\rho}{2} v_A^2 + \rho g H = P_B + \frac{\rho}{2} V_B^2 + \rho g h$$

but $P_A = P_B = P_{atm}$, so

$$\frac{\rho}{2} v_A^2 + \rho g H = \frac{\rho}{2} v_0^2 + \rho g h$$

if we make the approximation that $\frac{\rho}{2} v_A^2 \ll \rho g (H-h)$ then

$$\frac{\rho}{2} v_0^2 \simeq \rho g (H-h)$$

(6 cont)

$v_0 = \sqrt{2g(H-h)}$

If t = time of flight, $x = v_0 t$

but $h = \frac{g}{2}t^2 \Rightarrow t = \sqrt{\frac{2h}{g}}$

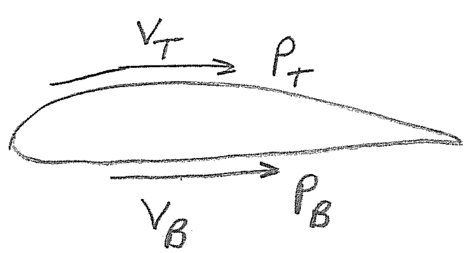
So $x = \sqrt{2g(H-h)}\sqrt{\frac{2h}{g}} = \boxed{2\sqrt{h(H-h)}}$

(b) x is maximized when $\frac{dx}{dh} = 0$

$$\frac{dx}{dh} = 2\left(\frac{1}{2}\right)\frac{(H-2h)}{\sqrt{h(H-h)}} = 0$$

$$\boxed{h = H/2}$$

(7)



Since the heights are approximatley the same,

$P_T + \frac{\rho}{2}v_T^2 \simeq P_B + \frac{\rho}{2}v_B^2$

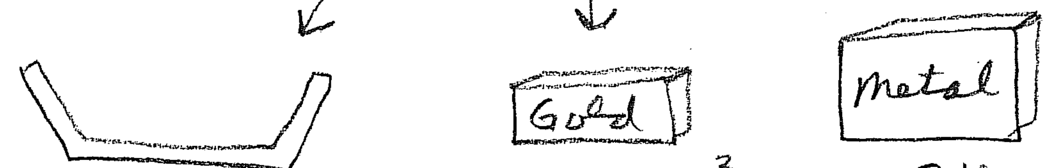
$P_B - P_T = \frac{\rho}{2}(v_T^2 - v_B^2)$

$F = (P_B - P_T)A$

$$\boxed{F = \frac{\rho A}{2}(v_T^2 - v_B^2)}$$

⑧

Same mass $\equiv m$



$V = 120\,cm^3$ $\quad V = 100\,cm^3$ $\quad V = 210\,cm^3$

Let x = volume of gold in crown
y = volume of metal in crown

$$x + y = 120$$

also $$m_{crown} = \rho_{gold}\, x + \rho_{metal}\, y$$

$$m = \frac{m}{100} x + \frac{m}{210} y$$

$$\boxed{\begin{aligned} 1 &= \frac{x}{100} + \frac{y}{210} \\ x + y &= 120 \end{aligned}}$$

Solving for x:

$$1 = \frac{x}{100} + \frac{120 - x}{210}$$

$$(100)(210) = 210x + 100(120) - 100x$$

$$100(90) = 110x$$

$$\boxed{x = \frac{900}{11}\ cm^3} = \boxed{81.8\ cm^3}$$

So the goldsmith stole $\boxed{18.2\ cm^3}$ of gold.