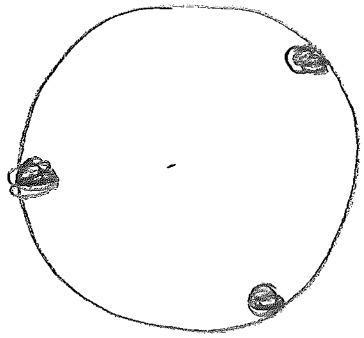
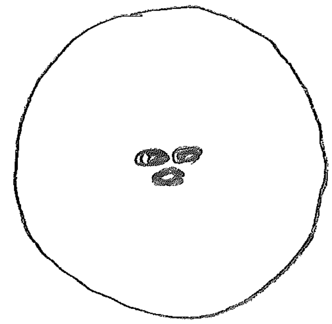
Solution to Seventh Homework

(1)





$$I_i = \frac{240}{2}(4)^2 + 3(40)4^2$$

$$I_i = 3840 \ \text{Kg M}^2$$

$$I_f = \frac{240}{2}(4)^2$$

$$I_f = 1920 \ \text{Kg M}^2$$

Since all the forces are internal, angular momentum is conserved:

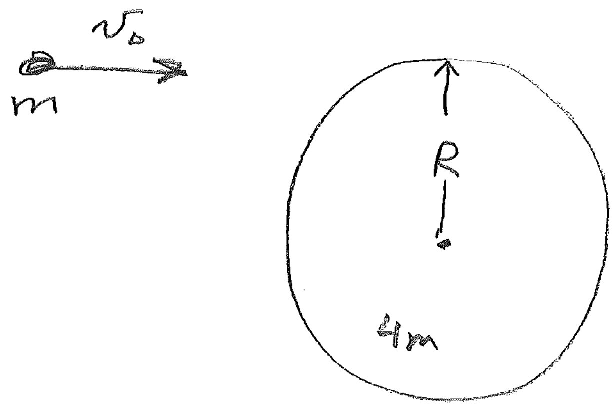
$$L_i = L_f$$

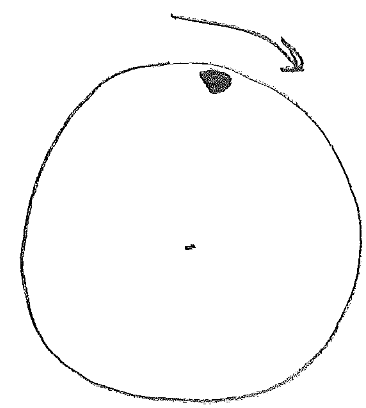
$$3840 \ \omega_i = 1920 \ \omega_f$$

$$\omega_f = 2\omega_i \quad \Rightarrow \quad T_f = \frac{T_i}{2}$$

$$T_f = \frac{2 \text{ sec}}{2} = \boxed{1 \text{ sec}}$$

(2) Since all forces are internal, angular momentum is conserved





$L_i = m v_0 R$

$L_f = I \omega_f$

$L_f = \left(\frac{4m}{2} R^2 + m R^2 \right) \omega_f$

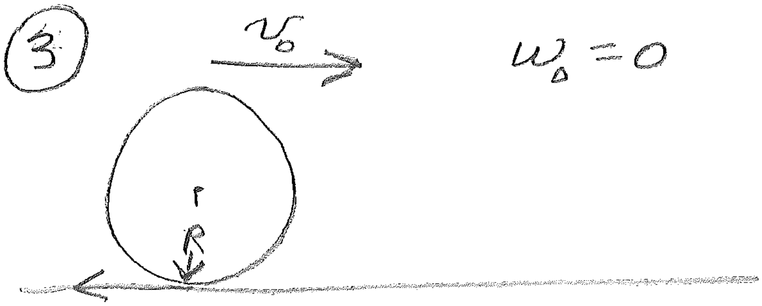
$L_f = 3 m R^2 \omega_f$

$L_f = L_i$

$3 m R^2 \omega_f = m v_0 R$

$$\boxed{\omega_f = \frac{v_0}{3R}}$$

(3)



$\omega_0 = 0$

$f = \mu mg$

$$\vec{F}_{NET} = -f = ma$$
$$-\mu mg = ma$$
$$a = -\mu g$$

As long as the ball "skids", its speed is

$$\boxed{v = v_0 - \mu g t}$$

$$\tau_{NET} = fR = I\alpha$$
$$\mu mgR = I\alpha \quad \Rightarrow \quad \alpha = \frac{\mu mgR}{I}$$

As long as the ball "skids", its angular velocity is $\boxed{\omega = \frac{\mu mgR}{I} t}$

The ball will "skid" until $\underline{\omega R = v}$:

$$\frac{\mu mgR^2}{I} t = v_0 - \mu g t \qquad \left\{ I_{sphere} = \frac{2}{5} MR^2 \right\}$$

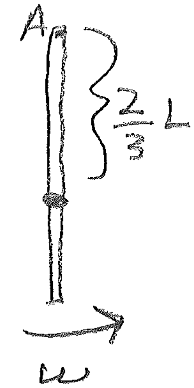
So $\mu g t \left(\frac{5}{2}\right) = v_0 - \mu g t$

$\mu g t = \frac{2}{7} v_0$ or at $\boxed{t = \frac{2}{7} \frac{v_0}{\mu g}}$ the ball starts to roll without slipping

$v = v_0 - \frac{2}{7} v_0 = \boxed{\frac{5}{7} v_0}$ is the final speed of the ball

④ To find the initial angular velocity of the stick, angular momentum is conserved in the collision:

$$\boxed{m v_0 \left(\frac{2}{3}L\right) = I\omega}$$



$$I = \frac{ML^2}{3} + \frac{4}{9}mL^2$$

$$I = \frac{L^2}{9}(3M + 4m)$$

$$\frac{2}{3} m v_0 L = \left(\frac{ML^2}{3} + m\left(\frac{2}{3}L\right)^2\right)\omega$$

$$\omega = \left(\frac{6}{4 + 3M/m}\right)\frac{v_0}{L} \qquad \text{or} \qquad \omega = \frac{2}{3}\frac{L v_0 m}{I}$$

As the stick swings upward, mechanical energy is conserved:

$$\boxed{W_{grav} = \frac{I}{2}\omega^2 - 0}$$

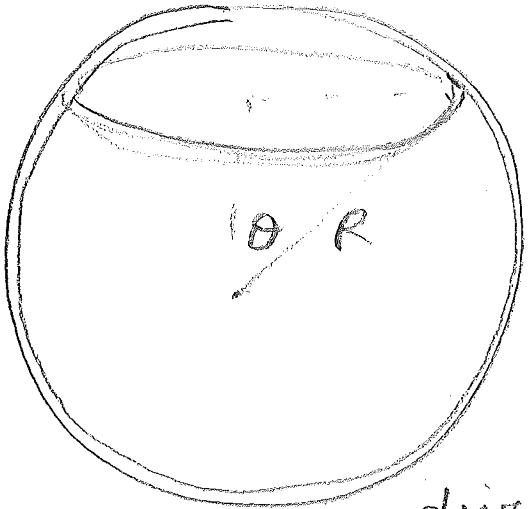
The work done by gravity is $mg\left(\frac{2}{3}L\right) + Mg\frac{L}{2}$

$$\frac{2}{3}mgL + \frac{MgL}{2} = \frac{I}{2}\left(\frac{2}{3}L v_0 \frac{m}{I}\right)^2$$

solving for v_0 we have:

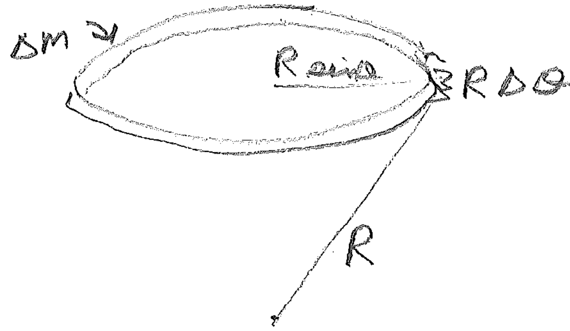
$$\boxed{v_0^2 = \frac{gL}{2}\left(6 + \frac{9x}{2}\right)(4 + 3x)} \qquad \text{where } x = \frac{M}{m}$$

(5)



shell

divide the shell into rings



ring

$$\frac{\Delta m}{m} = \frac{2\pi (R \sin\theta)(R\Delta\theta)}{4\pi R^2}$$

$$I_{Ring} = \Delta m (R \sin\theta)^2$$

$$\frac{\Delta m}{m} = \frac{\sin\theta \; \Delta\theta}{2}$$

$$I_{Ring} = \frac{m}{2} \sin\theta \,(R \sin\theta)^2 \Delta\theta$$

$$I_{shell} = \int_0^{\pi} \frac{m}{2} \sin\theta \,(R^2 \sin^2\theta)\, d\theta$$

$$I_{shell} = \frac{mR^2}{2} \int_0^{\pi} \sin^3\theta \, d\theta$$

let $u = \cos\theta$ $\quad du = -\sin\theta \, d\theta \quad \sin^2\theta = 1-u^2$

$$I_{shell} = \frac{mR^2}{2} \int_1^{-1} (1-u^2)(-du) = \frac{mR^2}{2} \int_{-1}^{1} (1-u^2)\, du = \boxed{\frac{2}{3} m R^2}$$