Solutions to Sixth Homework
PHY 131

(1) Time for ball to reach home plate:

$$v = 90\,\text{mph}\left(\frac{88\ \text{ft/s}}{60\ \text{mph}}\right) = 132\ \text{ft/sec}$$

$$t = \frac{d}{v} = \frac{60\ \text{ft}}{132\ \text{ft/s}} = .454\ \text{sec}$$

$$\#\text{ of Revolutions} = 30\,\frac{\text{rev}}{\text{sec}}(.454\ \text{sec}) = \boxed{13.6\ \text{revolution}}$$

(2)

(a) $\omega_f = \omega_0 + \alpha t$

$0 = 30\ \text{r/s} - .5\,t$

$\boxed{t = 60\ \text{sec}}$

(b) To find the total number of revolutions,

$\omega_f^2 = \omega_0^2 + 2\alpha(\Delta\theta)$

$0 = (30)^2 - 2(.5)(\Delta\theta)$

$\Delta\theta = 900$ radians

$\#\text{ of revolutions} = \frac{900}{2\pi} = 143.24$

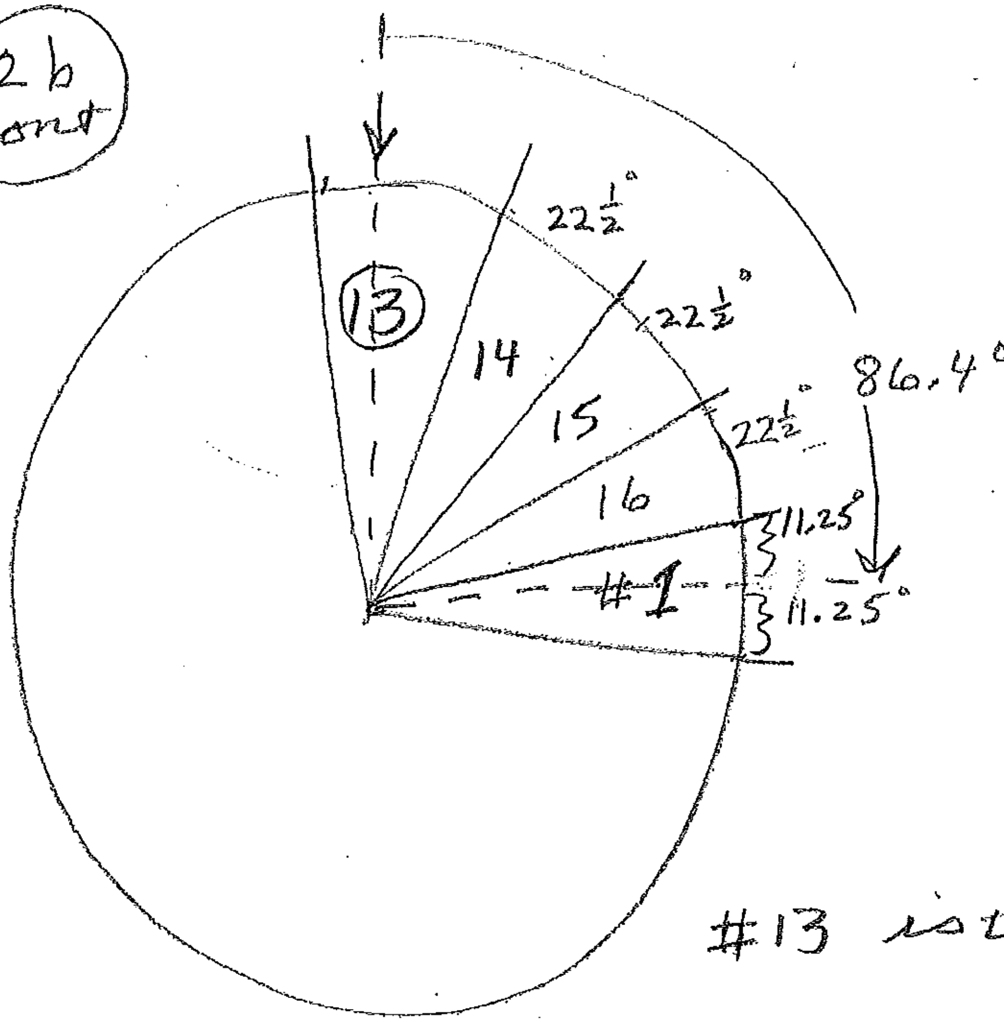
after 143 revolutions the wheel rotates

.24 revolutions or $.24(360^\circ) = \underline{86.4^\circ}$

Each number spans an angle of $\frac{360}{16} = 22.5^\circ$

The orientation of the wheel is

2b cont



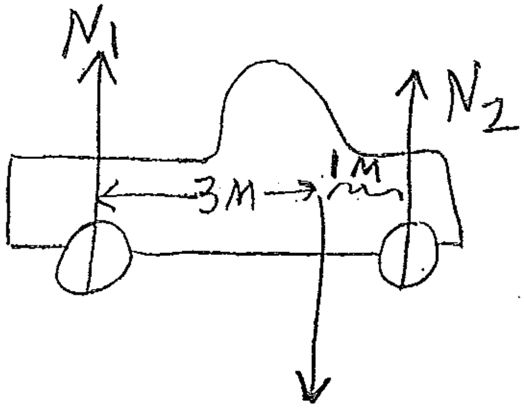
$$\begin{array}{r} 86.4^\circ \\ -\ 11.25^\circ \\ \hline 75.15^\circ \end{array}$$

$$\frac{75.15}{22.5} = 3.34$$

$$\begin{array}{r} 16 \\ -\ 3 \\ \hline \boxed{13} \end{array}$$

#13 is the winning number

(3)



$mg = 1000(9.8)$
$= 9800\,N$

$$\boxed{N_1 + N_2 = 9800N}$$

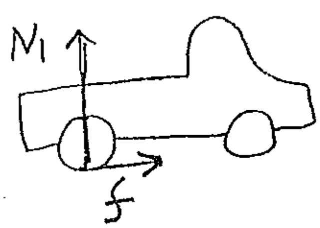
$$\tau_{\substack{\text{about}\\ \text{front}\\ \text{wheel}}} = 1(mg) - N_1\, 4 = 0$$

$$N_1 = \frac{mg}{4}$$

so $N_1 = \frac{9800}{4} = \boxed{2450N}$

(b) $N_2 = N - N_1 = 9800 - 2450 = \boxed{7350N}$

(c)



$$f = \mu N_1 = ma$$

$$a = \frac{\mu N_1}{m} = \frac{\mu \frac{mg}{4}}{m} = \boxed{\frac{\mu g}{4}}$$

$$a = \frac{(.6)(9.8)}{4} = \boxed{1.47\ M/s^2}$$

(4) The energy of a rotating sphere is $\frac{I}{2}\omega^2$

The energy lost from the earth if it were to spin 1 sec slower is

$$\Delta E = \frac{I}{2}\omega_i^2 - \frac{I}{2}\omega_f^2 = \frac{I}{2}\left(\omega_i^2 - \omega_f^2\right) \qquad I_{sphere} = \frac{2}{5}mR^2$$

Since $\omega = \frac{2\pi}{T}$,

$$\Delta E = \frac{\left(\frac{2}{5}mR^2\right)}{2}\left(\left(\frac{2\pi}{T_i}\right)^2 - \left(\frac{2\pi}{T_f}\right)^2\right) = \frac{mR^2}{5}(2\pi)^2\left(\frac{1}{T_i^2} - \frac{1}{T_f^2}\right)$$

1 sec

$$\Delta E = \frac{mR^2}{5}(2\pi)^2\left(\frac{T_f^2 - T_i^2}{T_i^2 T_f^2}\right) = \frac{mR^2}{5}(2\pi)^2\left[\frac{(T_f + T_i)(T_f - T_i)}{T_i^2 T_f^2}\right]$$

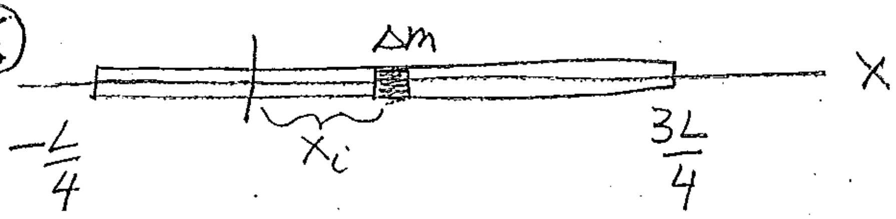
with $T_i = 86400$ sec and $T_f = 86401$ sec

$$\Delta E = \frac{(5.98\times10^{24})(6.37\times10^6)^2}{5}(2\pi)^2\left[\frac{(172801)(1)}{(86400)^2(86401)^2}\right]$$

$$\boxed{\Delta E = 5.96\times10^{24}\text{ Joules}}$$

Wow, and I get to sleep one second longer

(5)



as discussed in lecture, $I = \sum x_i^2 \Delta m$

$$\frac{\Delta m}{m} = \frac{\Delta X}{L} \qquad\qquad \Longrightarrow \qquad I = \sum x_i^2 \frac{m}{L} \Delta X$$

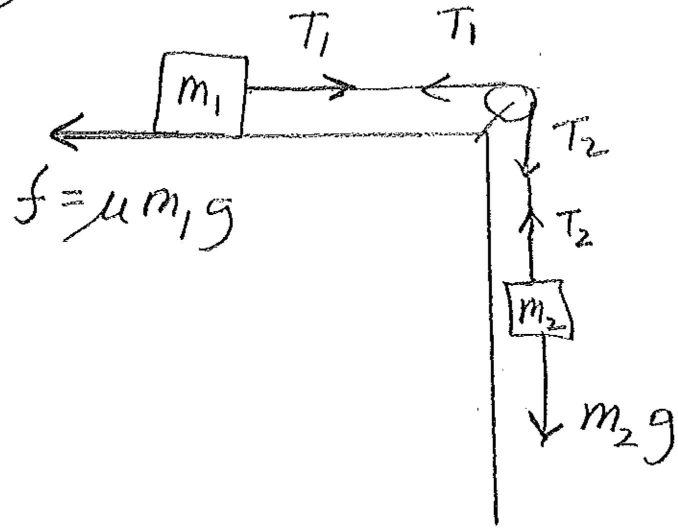
$$\Delta m = \frac{m}{L} \Delta X \qquad\qquad I = \int_{-L/4}^{3L/4} \frac{m}{L} x^2 \, dx$$

$$I = \frac{m}{L} \left. \frac{X^3}{3} \right|_{-L/4}^{\frac{3L}{4}} = \frac{m}{3L} \left(\frac{27}{64} L^3 + \frac{L^3}{64} \right) = \boxed{\frac{7mL^2}{48}}$$

or,
Using the parallel axis theorem:

$$I = m\left(\frac{L}{4}\right)^2 + \frac{mL^2}{12} = mL^2\left(\frac{1}{16} + \frac{1}{12}\right) = \boxed{\frac{7mL^2}{48}}$$

(6)



$$T_1 - \mu m_1 g = m_1 a$$

$$m_2 g - T_2 = m_2 a$$

$$T_2 - T_1 = \frac{m}{2} a$$

$$T_2 R - T_1 R = I\alpha$$

$$(T_2 - T_1) R = \frac{m}{2} R^2 \frac{a}{R}$$

$$T_2 - T_1 = \frac{m}{2} a$$

$$m_2 g - \mu m_1 g = \left(m_1 + m_2 + \frac{m}{2}\right) a$$

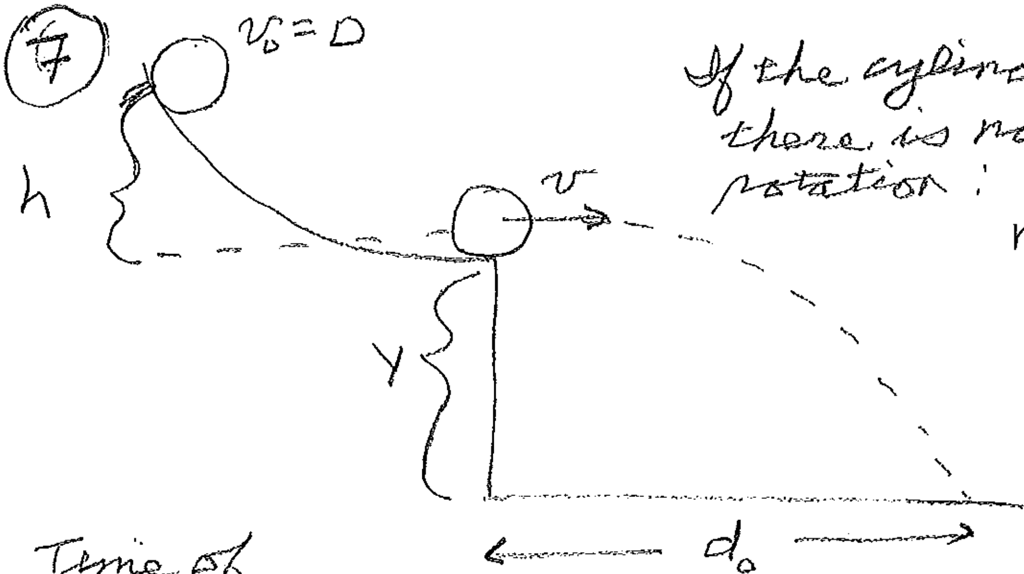
$$\boxed{a = g \frac{(m_2 - \mu m_1)}{m_1 + m_2 + \frac{m}{2}}}$$

$$T_1 = m_1 a + \mu m_1 g$$

$$T_2 = m_2 g - m_2 a$$

These are a bit "messy". I'll only grade your answer for a.

(7)



If the cylinder slides, there is no energy of rotation:

$$mgh = \frac{m}{2}v^2$$

$$v = \sqrt{2gh}$$

Time of flight:

$$y = \frac{g}{2}t^2$$

$$t = \sqrt{\frac{2y}{g}}$$

$$d_0 = vt$$

$$d_0 = \sqrt{2gh}\sqrt{\frac{2y}{g}} = \underline{2\sqrt{hy}}$$

If the cylinder <u>rolls</u>: $mgh = \frac{m}{2}v^2 + \frac{I}{2}\omega^2$

$$\left.\begin{array}{l} \text{but } I = \frac{m}{2}R^2 \\ \text{and } \omega = v/R \end{array}\right\}$$

$$mgh = \frac{m}{2}v^2 + \frac{\frac{m}{2}R^2}{2}\left(\frac{v}{R}\right)^2$$

$$mgh = \frac{m}{2}v^2 + \frac{m}{4}v^2 = \frac{3}{4}mv^2$$

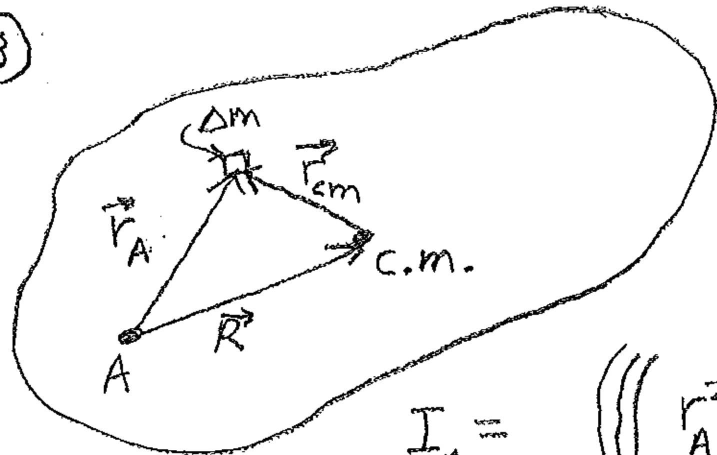
$$v = \sqrt{\frac{4}{3}gh}$$

In this case, $d = vt$

$$d = \sqrt{\frac{4}{3}gh}\sqrt{\frac{2y}{g}} = 2\sqrt{\frac{2}{3}}\sqrt{hy}$$

or $\boxed{d = \sqrt{\frac{2}{3}}\, d_0}$

(8)



$\Delta m = \rho \Delta V$

$dm = \rho \, dV$

$$I_A = \iiint r_A^2 \rho \, dV$$

but $\vec{r}_A = \vec{R} + \vec{r}_{cm}$

$$r_A^2 = R^2 + r_{cm}^2 + 2\vec{R}\cdot\vec{r}_{cm}$$

$$I_A = \iiint R^2 \rho \, dV + \iiint r_{cm}^2 \rho \, dV + 2\vec{R}\cdot \underbrace{\iiint \vec{r}_{cm} \rho \, dV}_{= \vec{0}}$$

$$I_A = R^2 \iiint \rho \, dV + I_{cm} + 2\vec{R}\cdot\vec{0}$$

$$\boxed{I_A = R^2 M + I_{cm}}$$

M = total mass $= \iiint \rho \, dV$