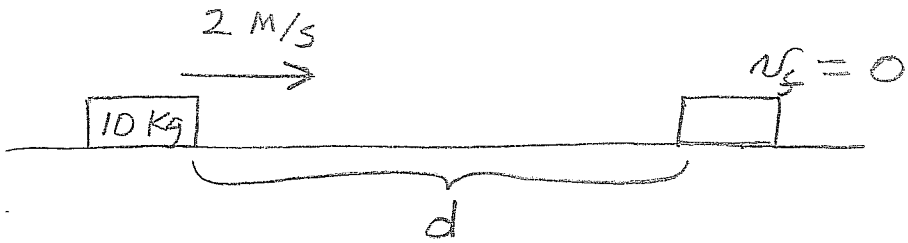
Solutions to Problem Set 5
PHY 131

(1)



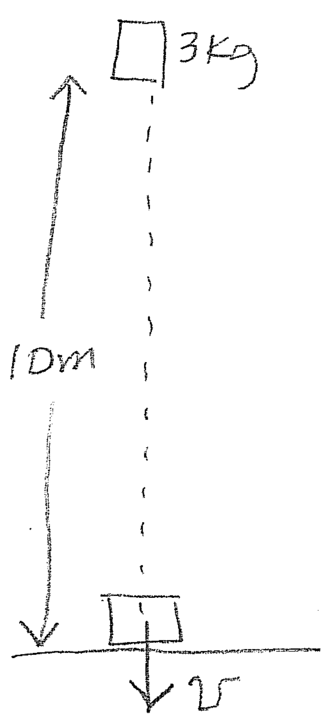
Net Work = change in Kinetic Energy

$$W_{friction} = 0 - \frac{m}{2} v_0^2$$

$$-mg\mu d = -\frac{m}{2} v_0^2$$

$$d = \frac{v_0^2}{2g\mu} = \frac{2^2}{2(9.8)(.1)} = \boxed{2.04\text{ M}}$$

(2)



NET WORK = change in K.E.

$$W_{grav} + W_{FRICTION} = \frac{m}{2}v^2 - 0$$

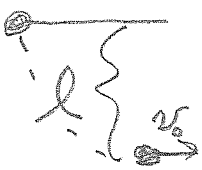
$$mgh - 20\text{ J} = \frac{m}{2}v^2$$

$$v^2 = 2gh - 2\frac{(20)}{m}$$

$$v = \sqrt{2(9.8)(10) - \frac{20(2)}{3}}$$

$$\boxed{v = 13.5\text{ m/s}}$$

③ before the collision:

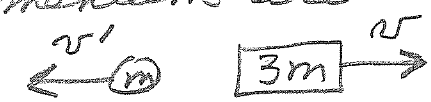


$mg\ell = \frac{m}{2} v_0^2$

$v_0 = \sqrt{2g\ell}$

$v_0 = \sqrt{2g\ell}$

Since kinetic energy and momentum are conserved in the collision:

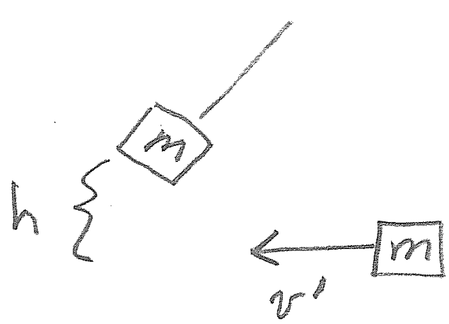


$$\left.\begin{aligned} m v_0 &= m v' + 3 m v \\ \frac{m}{2} v_0^2 &= \frac{m}{2} v'^2 + \frac{3m}{2} v^2 \end{aligned}\right\} \qquad \begin{aligned} v &= v_0/2 \\ v' &= -v_0/2 \end{aligned}$$

combining these equations gives:

ⓐ $$v = \frac{v_0}{2} = \frac{\sqrt{2g\ell}}{2} = \boxed{\sqrt{\frac{g\ell}{2}}}$$

ⓑ and $v' = -\frac{1}{2} v_0 = -\sqrt{\frac{g\ell}{2}}$



Energy is conserved as the block swings up.

$$\frac{m}{2} v'^2 = mgh$$

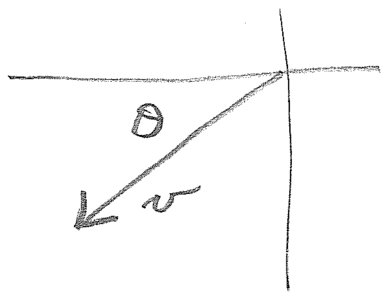
$$h = \frac{v'^2}{2g}$$

$$h = \frac{\left(-\sqrt{\frac{g\ell}{2}}\right)^2}{2g} = \boxed{\frac{\ell}{4}}$$

(4) Since momentum is conserved in the explosion, $\vec{P}_{TOT}(\text{final}) = 0$

(a) $$0 = 2(2)\,\hat{i} + 1(3)\,\hat{j} + 5\vec{v}$$

$$\boxed{\vec{v} = -\tfrac{4}{5}\,\hat{i} - \tfrac{3}{5}\,\hat{j} \quad \text{M/s}}$$



$$|\vec{v}| = \sqrt{\left(\tfrac{4}{5}\right)^2 + \left(\tfrac{3}{5}\right)^2} = \boxed{1 \text{ M/s}}$$

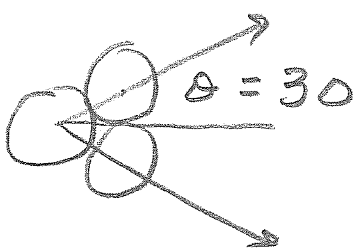
$$\tan\theta = \frac{3/5}{4/5}$$

$$\boxed{\theta = 37^\circ}$$

(b) $$\text{K.E. (final)} = \tfrac{1}{2}(3)^2 + \tfrac{2}{2}(2)^2 + \tfrac{5}{2}(1)^2$$

$$\text{K.E.} = \boxed{11 \text{ Joules}}$$

(5)



Since all the balls are touching upon impact, $\underline{\theta = 30^\circ}$

Since momentum is conserved

$$m\,20 = 2mv\cos 30 - mv'$$

$$20 = 2v\,\frac{\sqrt{3}}{2} - v'$$

5 cont momentum conservation gives

$$\boxed{20 = \sqrt{3}\, v - v'}$$

Kinetic Energy conservation gives

$$\frac{m}{2}(20)^2 = 2\left(\frac{m}{2}v^2\right) + \frac{m}{2}v'^2$$

$$\boxed{400 = 2v^2 + v'^2}$$

combining these two equations gives:

$$\boxed{v = 8\sqrt{3} \text{ M/s}}$$

and $\boxed{v' = 4 \text{ M/s}}$

6 ~~Since~~ momentum is conserved in the collision,

a

$$5(20)\,\hat{i} = 10\,\vec{v} - 5(4)\,\hat{j}$$

$$\boxed{\vec{v} = 10\,\hat{i} + 2\,\hat{j} \text{ M/s}}$$

$$|\vec{v}| = \sqrt{10^2 + 2^2} = \sqrt{104} = \boxed{10.2 \text{ M/s}}$$

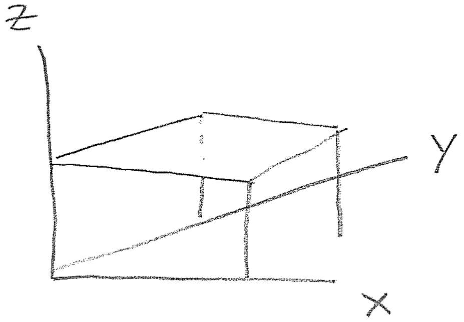
$$\tan\theta = 2/10 \Rightarrow \boxed{\theta = 11.3^\circ}$$

b K.E. (initial) $= \frac{5}{2}(20)^2 = 1000$ J

K.E. (final) $= \frac{5}{2}(4)^2 + \frac{10}{2}\left(\sqrt{104}\right)^2 = 560$ J

So $1000 - 560 = \boxed{440 \text{ J}}$ of Kinetic Energy were transfered in the collision.

(7)



The x-y position of the center-of-mass is in the center of the table

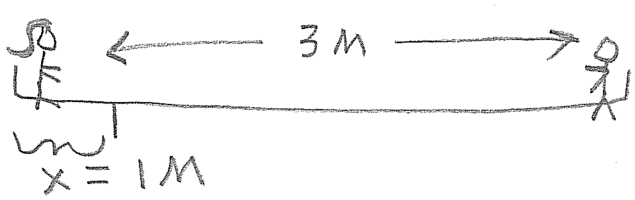
The height of the center-of-mass is found from:

$$z_{cm} = \frac{8m(L) + 4m\left(\frac{L}{2}\right)}{12m}$$

$$\boxed{z_{cm} = \frac{5}{6}L}$$

(8) Since there are no external forces, the center-of-mass does not change while they switch places.

Before $$x_{cm} = \frac{90(0) + m4 + 20(2)}{90 + m + 20}$$



after $$x_{cm} = \frac{m(-1) + 90(3) + 20(1)}{90 + m + 20}$$

Equating the two gives:

$$\frac{90(0) + m4 + 20(2)}{90 + m + 20} = \frac{-m + 90(3) + 20(1)}{90 + m + 20}$$

$$\boxed{m = 50\text{ Kg}}$$