Numerical Methods Übungen

Diagonal Scaling and LU Decomposition

Problem 1

Consider the system of equations $[a]\vec{x} = \vec{b}$, where $[a]$ and $\vec{b}$ are given by

$$[a] = \begin{bmatrix} 4 & 10 \\ 10 & 100 \end{bmatrix} \qquad \vec{b} = \begin{bmatrix} 4 \\ 20 \end{bmatrix} \tag{1}$$

a. Perform "diagonal scaling" of the coefficient matrix $[a]$. What is $[s]$?

b. Examine if the condition number $cond([a]_2)$ has significantly been reduced.

c. Solve the scaled equation $[a_s]\vec{x}_s = \vec{b}_s$ for $\vec{x}_s$.

d. Determine $\vec{x} = [s]\vec{x}_s$. Check if it solves the original equation.

Problem 2
Consider the matrix $[a]$ below:

$$[a] = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & -2 \end{bmatrix} \tag{2}$$

One of its eigenvalues is zero: $\lambda_1 = 0$.

a. What is the eigenvector, normalized to 1, for λ_1?

b. What are the other two eigenvalues?

Problem 3
Factoring matricies into the product of a lower times an upper triangular matrix.

a. First, consider the 4×4 matrix below:

$$[a_4] = \begin{bmatrix} 6 & -2 & 2 & 4 \\ 12 & -8 & 6 & 10 \\ 12 & -12 & 9 & 3 \\ -6 & 4 & 1 & -25 \end{bmatrix} \tag{3}$$

Factor the matrix $[a_4]$ above into the product of lower times an upper trianglar matrix: $[a_4] = [l][u]$ using the Gauss-Doolittle method.