Numerical Methods Übungen

Determinants and Cramer's Rule

Problem 1

Consider the product of matrix elements of matrix $[a]$ below:

$$a_{11}a_{23} \qquad a_{12}a_{21}a_{33} \qquad a_{43}a_{32}a_{21}$$

$$a_{11}a_{23}a_{34}a_{42} \qquad a_{11}a_{22}a_{34}a_{41} \qquad a_{51}a_{32}a_{13}a_{45}a_{24}$$

a. Which of the products above are a term in the expansion of the $det([a])$?

b. What is the sign, + or −, of the valid terms?

c. What is the rank of the matrix $[a]$ below?

$$[a] = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 5 & 2 \\ 2 & 4 & 6 \\ -2 & -10 & -4 \end{bmatrix} \tag{1}$$

Problem 2

a. Find the determinant of the following matrix:

$$[a] = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 2 & 2 & 1 \\ 1 & 3 & 0 & 1 \\ 0 & 2 & 0 & 6 \end{bmatrix} \tag{2}$$

Problem 3
Consider the following 2 matricies

$$[a] = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 2 & 4 \\ 0 & 0 & 3 \end{bmatrix} \qquad [g] = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \tag{3}$$

a. Using Cramer's Rule, find $[a]^{-1}$.

b. Using Cramer's Rule, find $[g]^{-1}$.

c. Using your result from b), what is the most general form for a 2×2 orthogonal matrix?