Numerical Methods Übungen

Matrix Operations and Determinants

Problem 1

Consider the matrix $[a]$ and the two vectors $[\vec{v}]$ and $\vec{w}$:

$$[a] = \begin{bmatrix} \frac{3}{5} & \frac{4}{5} \\ \frac{4}{5} & 1 \\ 0 & \frac{3}{5} \end{bmatrix} \qquad [\vec{v}] = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} \qquad [\vec{w}] = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \tag{1}$$

a. Find the quadratic form of these two vectors with $[a]$.

b. Are these two vectors $[a]$ – *orthogonal*?

Problem 2

Consider the matrix and vector below:

$$[u] = \begin{bmatrix} 2 & 1+i \\ 1-i & 1 \end{bmatrix} \qquad \vec{v} = \begin{bmatrix} \binom{1}{i} \\ 1 \end{bmatrix} \tag{2}$$

a. Is $[u]$ Hermitian?

b. Is $[u]$ Unitary?

c. Find $\vec{v}^*[u]\vec{v}$

Problem 3

$$[a] = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 \end{bmatrix} \tag{3}$$

a. Find the determinant of $[a]$, $det([a])$.

Problem 4

Condider the matrix below:

$$[a] = \begin{bmatrix} 0 & \lambda & -4 \\ \lambda & 1-\lambda & 2 \\ 0 & 4 & 6-\lambda \end{bmatrix} \tag{4}$$

a. For what value(s) of λ is $[a]$ singular?