Numerical Methods Übungen

Newton's Method, Iterations and Vector Operations

Problem 1

Suppose you want to write a computer program that calculates the square root of a number $R > 0$ using Newton's method. You decide to do this by finding the zeros of the function $f(x) = x^2 - R$.

a. Make a sketch of $f(x)$.

b. Obtain the iterative equation for $x_{\nu+1}$ in terms of x_ν and R.

c. There are two roots to this equation. For what interval on the $x - axis$ will the iteration converge to the positive root? to the negative root?

d. Check out your iteration equation for $R = 2$. Pick a starting value for x_0, and calculate the first 4 iterates, i.e. to x_4. You can check your numbers using the program "newton.html" on my home page.

Problem 2

You want to solve for the zeros of the function

$$f(x) = \frac{x^2}{1 + x^2} \tag{1}$$

using Newton's method.

a. Make a sketch of $f(x)$.
b. Obtain the iterative equation for $x_{\nu+1}$ in terms of x_{ν}.
c. For what starting values x_0 will your iterative equation converge?

Problem 3

A famous iterative sequence, called "The Logistic Map", is the following:

$$x_{\nu+1} = rx_\nu(1 - x_\nu) \tag{2}$$

where $0 < r < 4$, and $0 < x_\nu < 1$.

a. Determine one non-zero fixed point. Hint: use the fixed point equation (2.5). The fixed point will depend on r.
b. For what values of r is this fixed point convergent?
c. To see some interesting features of this iteration, run the program logistic.html on my home page for values of $r = 2.9, 3.2, 3.5, 3.55, 3.565, \textit{and}\, 3.6$. Note: the logistic map is used to demonstrate the period doubling route to deterministic chaos, a property that can be seen in some mechanical systems.

Problem 4

Consider the two vectors $\vec{a} = -4\vec{e}_1 + 3\vec{e}_2 + 12\vec{e}_3$ and $\vec{b} = 2\vec{e}_1 + 2\vec{e}_2 - 2\vec{e}_3$. Find:

a. $|\vec{a}| =$

b. $-2\vec{a} =$

c. $\vec{e}_a =$

d. $\vec{a} + \vec{b} =$

e. $|\vec{a} - \vec{b}| =$

f. $\vec{e}_a \cdot \vec{b} =$

g. The angle in degrees between $\vec{b}$ and one of the axes.

h. The angle in degrees between $\vec{a}$ and $\vec{b}$.

Problem 5

The *Levi-Civita* symbol, e_{ijk} can be used to define a very important vector operation, the cross product. The cross product of two vectors can be defined as

$$\vec{a}\, cross\, \vec{b} \equiv \vec{a} \times \vec{b} = \sum_{ijk} e_{ijk} a_i b_j \vec{e}_k \tag{3}$$

where $\vec{a} = a_1\vec{e}_1 + a_2\vec{e}_2 + a_3\vec{e}_3$ and $\vec{b} = b_1\vec{e}_1 + b_2\vec{e}_2 + b_3\vec{e}_3$.

a. Carry out the sum and find the vector $\vec{a} \times \vec{b}$.
b. Let the vector $\vec{c} = c\vec{e}_1$ and $\vec{d} = d\, cos(\theta)\vec{e}_1 + d\, sin(\theta)\vec{e}_2$. Find the vector $\vec{c} \times \vec{d}$ in terms of c, d, and θ.
c. Using the properties of the *Levi-Civita* symbol, show that $\vec{b} \times \vec{a} = -\vec{a} \times \vec{b}$.