

Numerical Methods Übungen

Truncation Errors, Random Numbers, Zeros of a Function

Problem 1

Suppose the floating point representation on your computer is only 4 decimal digits. Assume the power of 10 is not limited, and can be any integer. For example $a.bcd \times 10^n$. Determine the floating point representation for the following numbers:

- a. 0.00378123
- b. 1001.25
- c. 30.2000004
- d. 0.00002
- e. π
- f. $\frac{5}{7}$

Problem 2

Suppose the floating point representation on your computer is only 4 decimal digits. Assume the power of 10 is not limited, and can be any integer. Determine the truncation error and relative truncation error for the following operations:

a. $2 \cdot \pi$

b. $\frac{1}{22} - \frac{1}{23}$

Problem 3

Humans like to work in base 10, but the computer calculates best in base 2. Consider the following two limitations of a 32 bit computer:

- a. The integer data type stores integers in base two. The first bit determines the sign of the integer, and 31 bits are reserved for the value of the integer. What is the largest integer that can be stored?
- b. In standard floating point, 24 bits are reserved for the number: $1.\dots 23 \text{ bits} \dots \times 2^n$ How would the computer store the number one tenth: $\frac{1}{10} = (0.1)_{10}$? What is the truncation error in storing this number?

Problem 4

You want to design a random "dice" to run on a calculator or computer. It should produce integers from 1 to 6 randomly. Your friend came up with a simple way to do this by using the "mod" function:

$$i^{\nu+1} = \text{mod}_7(3 * i^{\nu}) \quad (1)$$

- Determine the first 10 random numbers generated if one starts with $i^0 = 3$.
- Is this a good way to make a random dice for the calculator or computer?
- If not, suggest a better way.

Problem 5

Use the bisection method to solve the following problems:

- Find $\sqrt{2}$ to an accuracy of $\pm 10^{-3}$. Hint: find the zero of the function $f(x) = x^2 - 2$. Write down your values of $f(a)$, a , x , b , and $f(b)$ as in Table 2.1.
- Solve for x for the equation $\sin(x) = \frac{x}{2}$ to an accuracy of $\pm 10^{-3}$, and $x \neq 0$. Write down your values of $f(a)$, a , x , b , and $f(b)$ as in Table 2.1.