

Putting your heart into physics

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Citation: *American Journal of Physics* **72**, 324 (2004); doi: 10.1119/1.1615567

View online: <http://dx.doi.org/10.1119/1.1615567>

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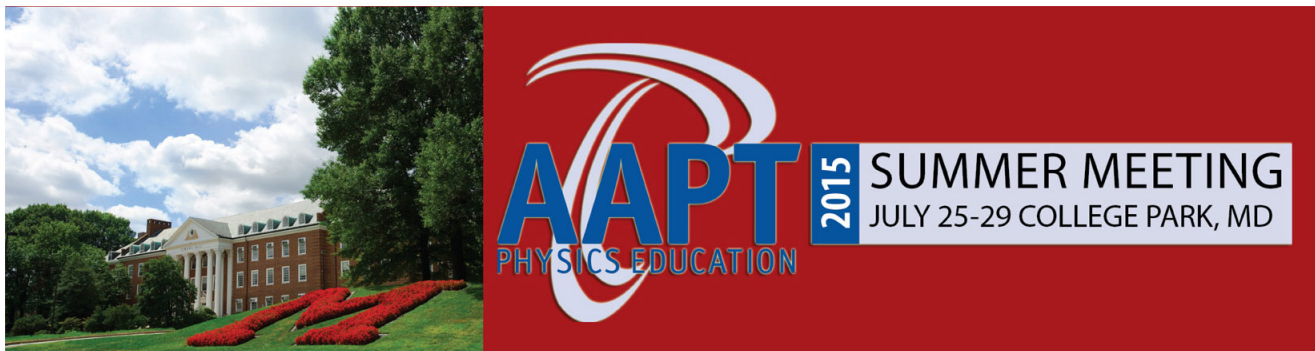
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Putting your heart into physics

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(Received 22 April 2003; accepted 8 August 2003)

We describe techniques for measuring the time interval between successive heartbeats. This time series data can be used in undergraduate physics classes for instruction in resonance phenomena, scaling, and other methods of analysis including Fourier analysis and Poincaré plots. Using methods from physics on data from human physiology are of particular interest to life science students.

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[DOI: 10.1119/1.1615567]

I. INTRODUCTION

Over the past 30 years, research on heart rate variability has studied which properties of heart rate control are important in assessing the health and fitness of the cardiovascular system in humans.¹ The main measurement is the time between successive heartbeats. Measurements are taken over time periods as short as a few minutes to as long as 24 hours, and the resulting data are a series of times usually measured to an accuracy of milliseconds. A number of articles have been published in physics journals which apply methods from nonlinear systems theory to the time series of the heart rate time interval data.^{2–8} In this article we describe a number of experiments on heart rate variability which would make good student projects and laboratory exercises for the undergraduate physics curriculum.

Analyzing data from human physiology is of particular interest to life science students,⁹ who are required to take physics as part of their degree requirement. Although no laws of physics are being investigated by the experiments described here, the phenomena and analysis methods are common to both the physical and biological sciences. The analysis of a driven damped pendulum can be compared to the response of the human heart when driven by controlled breathing. The concepts of frequency, amplitude, phase shift and resonance enter in both applications. The time interval data can be used to teach average values and standard deviations, which in this case have relevance to health and fitness. For more advanced students, the time interval data can be used to introduce spectral analysis, Poincaré plots, and the scaling properties of heart rate control. For physics majors, building the hardware for data collection and writing the software for data analysis make good special projects or upper division laboratory activities.

An advantage of time interval data is that accurate data can be obtained quickly. With the advent of the heart rate monitor for recreational athletes, research quality time interval data can be measured easily and with minimal expense. We start by describing techniques for measuring the heartbeat interval time. We then discuss some experiments that can be used in a physics laboratory class or as a physics project.

II. DATA COLLECTION

An electrocardiogram is a measurement of the voltage between two particular points on the chest which bracket the

heart. The voltage as a function of time takes on the form shown in Fig. 1. A voltage pulse is produced whenever a heartbeat occurs. The large spike is called the *R* peak and is a good reference point to define the time of the heartbeat. The time from one *R* peak to the next *R* peak, an interspike interval, is a good measure of the time between successive heartbeats, and is referred to as the *RR*-interval. We are interested in the *RR*-interval times for many successive heartbeats. Measuring these times to an accuracy of one millisecond is sufficient for all applications in which the subject is at rest. The *RR*-interval times can be measured from an electrocardiogram or by using a heart monitor chest strap. We describe both methods below.

An electrocardiogram can be obtained by amplifying the voltage from electrodes placed across the heart. The amplified signal can be used as input into an analog-to-digital card or sound board.¹⁰ From the digitized signal, the time difference between successive *R* peaks can be measured. Sampling rates greater than 1000 Hz will result in an accuracy of at least one millisecond. If the signal is sampled less than 1000 Hz, parabolic interpolation can be used to determine the time of the *R* peak between sampled data points. A disadvantage of this method is that much memory is used to store the ECG signal. If only the *RR* times are of interest, one could set a trigger in the software to measure the time between the *R* peaks.

A heart rate monitor belt is probably the easiest way to measure *RR*-interval times. The belt is worn around the chest and sends an electromagnetic signal every time an *R* peak is detected. The heart rate monitor (belt plus receiver watch) is used to measure one's heart rate while exercising, and is common gear for runners of all levels. A watch detects the signal and measures the heart rate. There are watches available which measure the *RR* times directly.¹¹ The *RR*-interval times are downloaded from the watch to a computer for data analysis.

The *RR*-interval times also can be obtained from the monitor belt by winding a coil of wire around the belt as shown in Fig. 2. The pick-up coil in Fig. 2 has 80 turns of wire. For the particular monitor we used,¹² the transmission signal is a 5 kHz pulse which lasts for 7 milliseconds. The 80 turns of wire produced a peak-to-peak voltage of 0.8 volts. The *RR*-interval times are obtained by measuring the time between the start of one 5 kHz signal and the next 5 kHz signal. The measurement is most easily done by using a voltage comparator chip. The signal from the pick-up coil is used as input

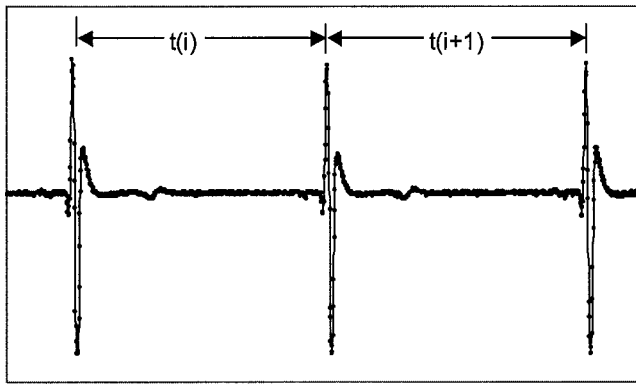


Fig. 1. An electrocardiogram signal, which is a plot of the voltage across the heart as function of time. The large positive peak is referred to as the R peak. The time between heartbeats is measured as the time between R peaks, $t(i)$, and is referred to as the RR interval.

to the comparator chip. Using a 5 volt bias, the output of the comparator serves as a digital input to a TTL port. We have used two different devices to measure the time between signals which can be used as digital input to a micro-controller (HC11) or as input via the parallel port in a personal computer (laptop). The use of the timer in the HC11 or personal computer is described in the Appendix. Of the two methods, the micro-controller has the advantage that it is dedicated, small, and portable. Interfacing with a computer has the advantage that it is very inexpensive, and thus one can build enough setups for an entire laboratory class with little cost.

We have compared the accuracy of measuring with the pickup loop to the heart rate monitor watch and have obtained identical results for both systems. We further tested the accuracy of measuring the same heartbeats with two different chest straps on one subject. The measured times agreed within 0.15%, with the errors being random. Thus, we estimate the accuracy of the RR-interval measurement to be around 0.15% using heart rate monitor straps. The manufacturer claims an accuracy of 1 ms. Because RR times are usually around 1000 ms, this claim implies a percent uncertainty of 0.1%. An uncertainty of 1 ms is the accuracy used by researchers in physiology and sports medicine. Thus, the methods described here also offer the possibility for interdisciplinary projects with biology and kinesiology. As computer chip technology advances, the measurement of RR-interval times will likely become easier and less expensive.

III. DATA ANALYSIS AND APPLICATIONS

Data can be collected while the subject is at rest or exercising. For a physics classroom experiment taking data at rest is more practical. Such data are produced at a rate of



Fig. 2. Coil of wire that is placed around the heart rate monitor to pick up the transmitted signal. With 80 turns of wire, the resulting signal has a peak-to-peak voltage of 0.8 V.

around 60 data points per minute. The data form a series of times, which can be used to introduce students to a variety of analysis methods.

Some basic knowledge of heart rate control helps in the interpretation of the data and experimental design. At rest, either lying or standing, the autonomic nervous system regulates the heart rate. The autonomic nervous system has two different control influences known as sympathetic and parasympathetic. Sympathetic nerve activity increases, while parasympathetic nerve activity decreases heart rate. The relation between the resting heart rate B , the parasympathetic factor n , the sympathetic factor m , and the basic heart rate B_0 is modeled as¹³

$$B = B_0 mn = B_0(1 + S)(1 - P). \quad (1)$$

We have written $m = (1 + S)$ and $n = (1 - P)$, where S refers to the sympathetic activity and P to the parasympathetic activity. If both sympathetic and parasympathetic control is blocked, the basic rate B_0 for most people is between 70 and 110 beats/min. In the lying position, the sympathetic activity is usually small ($0 < S < 0.1$), the parasympathetic activity is large ($0.1 < P < 0.6$), and the heart rate B is as low as it can be without medication. In the standing position, the sympathetic activity S is increased, the parasympathetic activity P is reduced, and the heart rate increases. The particular balance of sympathetic and parasympathetic activity in lying and standing varies among individuals and depends upon age, fitness, health, genetics and other factors.

There are many factors that affect the variability of the RR-intervals. Two important influences take place on two different time scales: variations with periods less than around 6 seconds, and periods longer than 10 seconds.¹

- Short time scale changes, from one beat to the next, are caused primarily by changes in breathing. The dynamics are relatively simple. When one inhales, the heart rate B increases; conversely, the rate decreases when one exhales. The heart rate is thus driven at the breathing frequency. For breathing frequencies greater than 10 breaths/min, the “driving force” is related to the parasympathetic activity P .
- Changes longer than 1 or 2 breathing cycles are caused by many factors, and the dynamics can be complicated. The average heart rate wanders, and sometimes slow oscillations are produced. Oscillations with a period of around 15 to 25 s often occur. It is believed that these oscillations are related to variations in blood pressure, although the exact mechanisms are not completely understood.¹ The effect is strongest in the standing position and seems to depend on both the sympathetic and parasympathetic activity. We will refer to these oscillations as low frequency oscillations. In the literature, they are often called Mayer waves.¹⁴

The dynamics of both time regimes are of interest to students. By controlling the rate and amplitude of breathing, the response of a driven system can be investigated. Students can measure the resulting amplitude and phase of the heart rate variations for different breathing frequencies. For the longer time variations, students can see the usefulness of a Poincaré plot to separate breathing from the long-term dynamics. We demonstrate features of the two time scales and the differences in lying versus standing in Fig. 3. The subject changes posture from lying to standing. In the lying part of

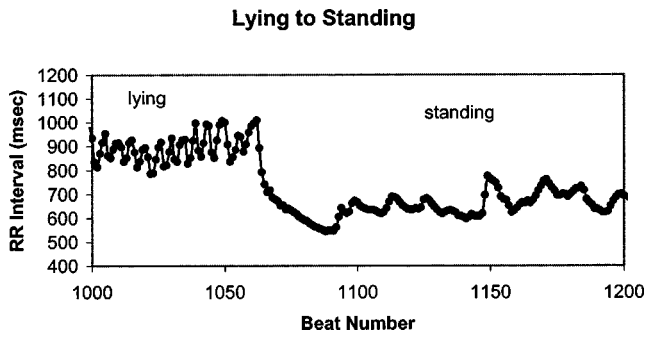


Fig. 3. Plot of the RR -interval versus beat number as a subject changes posture from lying to standing. For beat numbers 1000–1050 the subject is lying, and for beat numbers 1100 to 1200 the subject is standing.

Fig. 3, the short-term variations in the heart rate caused by breathing due to the strong parasympathetic activity are clearly seen. In the standing part of Fig. 3, the short-term oscillations due to breathing are essentially gone, but low frequency oscillations with a period of around 10 heartbeats are present. We note that heart rate control is a subject of current research, and the interpretation of the data may change as further experiments are done. Interested students should consult sources on human physiology for a more detailed treatment.^{1,24}

In the laboratory experiments described below, students collect data through the parallel port of a personal computer as explained in the Appendix. The RR times in milliseconds are displayed on the screen in real time. There is a switch option in the software which causes a beep for every heartbeat. This option allows the students to synchronize breathing and heart rate. After all the RR times are recorded, students can view the data or save the data as text to import it into a spreadsheet program. Software has been written to assist the students in their data analysis.

IV. SIMPLE STATISTICAL CALCULATIONS

The RR -interval data are well suited for instructing students in simple statistical calculations that come with spreadsheet programs and data analysis software, for example, averages and standard deviations. To save time, we have included programs in the data acquisition software to perform averages, standard deviations, discrete Fourier transform, and fast Fourier transforms (FFT). The students view the data graphically to determine the range of beat numbers that are appropriate for the calculations. The data were of particular interest to biology students who participated in data analysis workshops in which averages, standard deviation, Fourier analysis, and analysis of variance (ANOVA) were taught using the RR -interval data.

Data for statistical calculations are best taken in the lying or reclined position. In the lying position the heart rate does not wander as much as while standing. The data vary about a fairly constant average value, with the beat-to-beat variation due primarily to breathing. The coefficient of variation (COV) is defined as the ratio of the standard deviation divided by the average. Values of the COV usually are between 2% and 10%. In general, the COV decreases with age, and a large COV often is associated with fitness and overall good health.¹⁵

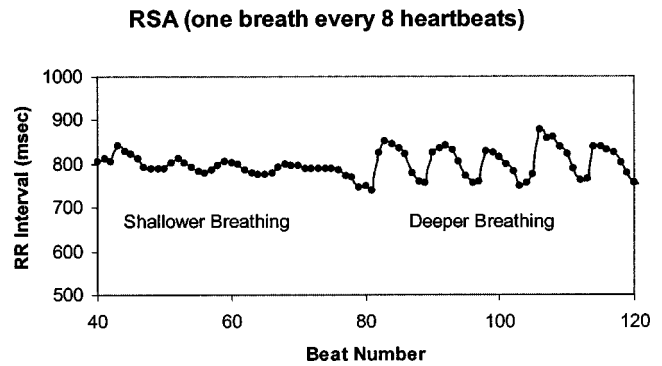


Fig. 4. Plot of the RR -interval versus beat number for a subject breathing eight heartbeats for every breath. The first five breaths are shallow breathing (beat numbers 40–80), and the next five breaths are deep breathing (beat numbers 80–120).

V. RESPONSE OF A DRIVEN SYSTEM

The damped driven pendulum and RLC circuit are systems that often are studied in undergraduate physics laboratories to examine the response of a driven system. Although heart rate control is more complicated than these two physical systems, many of the terms and basic properties are similar. The heart at rest has a steady state heart rate, a low frequency natural oscillation (Mayer waves), and can be driven by a periodic mechanism (breathing). Students can perform similar experiments on the heart as they have on the pendulum and RLC circuit by driving the heart with different breathing frequencies and driving forces and measuring the response. Periodic breathing results in a periodic heart rate response after transients have settled out. If one breathes in and out smoothly, the RR -interval times oscillate in a smooth manner with a near sinusoidal shape. This phenomenon is called respiratory sinus arrhythmia (RSA), and the oscillations are quantified by the respiratory sinus arrhythmia amplitude.

The RSA amplitude is roughly proportional to the volume of air inhaled, the tidal volume.¹⁶ If the tidal volume is not measured, the students can qualitatively verify that the RSA amplitude increases with increased tidal volume. In Fig. 4 we show data taken while the subject was standing and breathing at 8 beats/breath. The first five breaths are shallow breathing, and the next five are deeper breathing. It is clear that a larger RSA amplitude is a result of deeper breathing, or driving force.

The frequency response of the heart rate can be examined by having the subject breathe at different frequencies and measuring the resulting RSA amplitudes and relative phases.^{17,18} The heart at rest behaves quite differently in the lying compared to the standing position.^{19,20} It is most interesting to perform the experiment in the standing position, where the RSA amplitude has a much stronger frequency dependence. The subject should try to breathe comfortably at each frequency. Because the RSA amplitude depends on the tidal volume, we should normalize the RSA amplitude for the tidal volume at each frequency. We find that the average adult has a tidal volume of about 1000 ml at slow breathing rates (4 breaths/min) and a tidal volume of around 500 ml at 14 breaths/min. We could use these values and linearly interpolate to find the intermediate breathing frequencies. However such accuracy is not necessary for an introductory physics experiment. Because the increase in amplitude at the

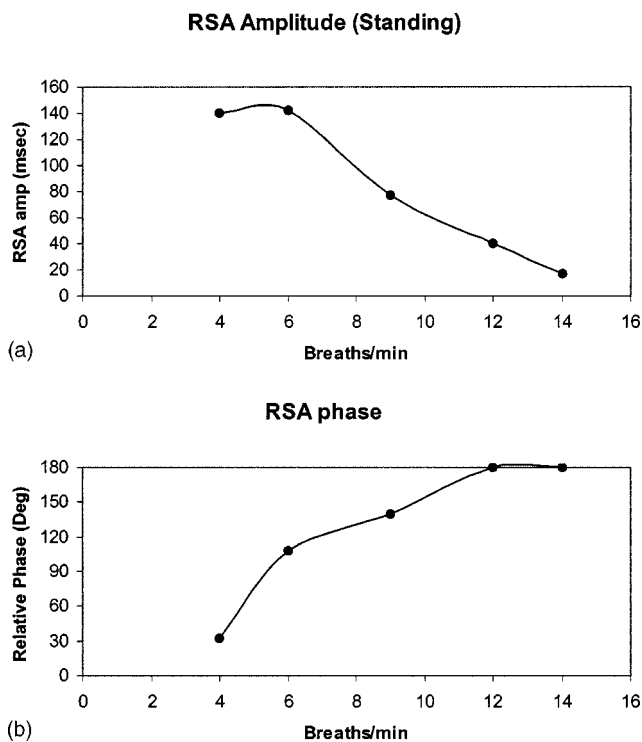


Fig. 5. The frequency response of the heart rate as a function of the breathing rate for a subject in the standing position: (a) the respiratory sinus arrhythmia amplitude and (b) the phase angle with respect to the breathing frequency.

lower frequencies is significantly more than a factor of 2 [see Fig. 5(a)], it is not due to only increased tidal volume.

The RSA amplitude is measured by displaying the RR -interval data graphically on the computer display. For breathing frequencies slower than 10 breaths/min, the oscillations are clearly visible. The students simply subtract the shortest RR -interval time from the longest RR time in each cycle and divide by 2. An average over a few cycles gives a fairly accurate RSA amplitude. For breathing frequencies greater than 10 breaths/min, the oscillations are sometimes small in the standing position, but usually a rough RSA amplitude can be obtained. Another option is to perform a discrete Fourier transform and use the amplitude of the peak at the breathing frequency for the RSA amplitude.

The phase angle between the breathing and heart rate can be estimated by noting the beat number at maximum lung volume (or any other point in the breathing cycle). The students can obtain the phase angle from the location of these beat numbers within the RSA cycle. In Fig. 5 we plot the phase angle and amplitude for a standing subject. We have taken positive phase to mean that breathing oscillations lead RSA oscillations.

The frequency response resembles that of a damped driven oscillator, with characteristics of a resonance phenomena.¹⁷ In Fig. 5 the amplitude has a maximum and the phase passes through 90 degrees at a breathing rate of around five breaths/min or a frequency of around 0.08 cycles/s. If the students have time to observe low frequency oscillations (Mayer waves), the frequency of this natural oscillation also will be close to 0.08 cycles/s. Although the data suggest a resonance phenomena is occurring, further investigation is necessary for a definitive interpretation. The phase angle being measured is between breathing and RSA oscillations. There is

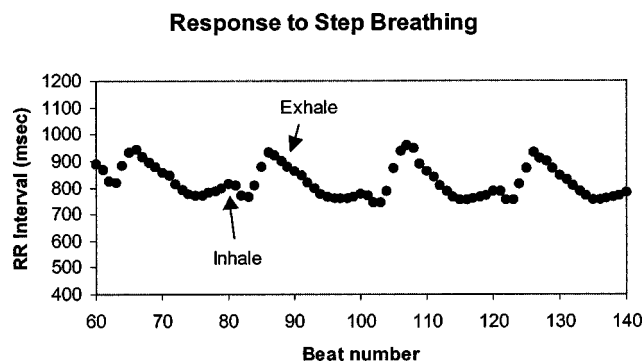


Fig. 6. Heart rate response to step function breathing. The subject inhales quickly, holds his breath for 10 heartbeats, exhales quickly, and holds his breath for 10 heartbeats.

another phase angle between the blood pressure and RSA oscillations, which might be more relevant for low frequency resonance. Although the amplitude rises quickly from high to low breathing frequencies, it does not drop as quickly at low frequencies. Some physiologists think a resonance phenomena is occurring,¹⁷ while others believe that the amplitude increase is partly caused by the increased response time when breathing slowly.^{21,22}

We also can measure the response of the heart to a step input; that is, have the subject breath in such a manner that the tidal volume is a step function of time.²¹ This is accomplished by breathing in quickly, holding one's breath for a certain number of heartbeats, breathing out quickly and then holding one's breath for the same number of heartbeats. We plot in Fig. 6 the response for a standing subject and breath holding for 10 heartbeats. It is interesting to observe that the steady state response is periodic, and that breathing in has the biggest beat-to-beat effect.

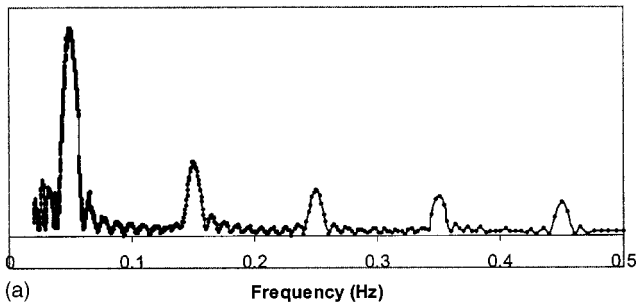
For more advanced students, the step-function input demonstrates that the response can be nonlinear. In Fig. 6 it can be seen that the response to a quick inhale is not equal to the negative of the response of a quick exhale. The nonlinearity also can be demonstrated by comparing the Fourier spectrum of the input and output. A step function only has odd spectral components and is shown in Fig. 7(a). The response function shown in Fig. 7(b) has a large amplitude at twice the input frequency, a frequency not present initially. In both Figs. 7(a) and 7(b), a discrete Fourier transform was performed over five breathing periods. If one breathes smoothly, however, the response is fairly linear. An example is given in Sec. VII.

VI. POINCARÉ PLOTS

A Poincaré plot is a plot in which one or more variables are projected out of the dynamics. We can project out a periodic variable by plotting the other variables every time the former variable obtains a particular value. The classic example is the damped pendulum driven sinusoidally. The angle of the pendulum, θ , angular velocity, ω , and the phase of the driving force are used to describe the motion. A phase space plot of θ versus ω for a particular phase of the driving force produces a Poincaré plot which demonstrates the period doubling route to chaos and strange attractors.²³

For the RR -interval data, a similar approach can be used to project out much of the effect that breathing has on the system. To accomplish this, the subject needs to breathe synchronously with the heart rate. The subject takes a complete

Fourier Spectrum of Square Wave (5 periods)



Square Wave Response (5 periods)

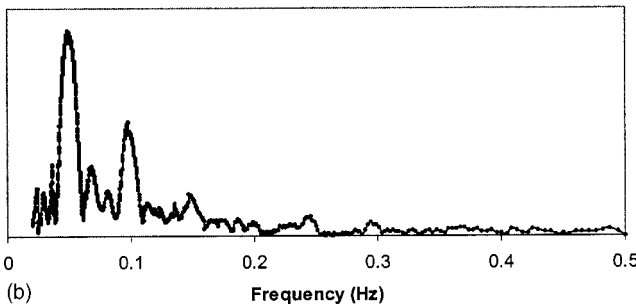


Fig. 7. Fourier spectrum of heart rate for step function breathing: (a) the spectrum of the breathing (step function) and (b) the spectrum of the heart rate response.

breath every n heartbeats, and should take each breath the same way. We then measure one or more variables for every n th heartbeat. One variable to consider is every n th RR -interval time. These times are at the same phase of the driving force (breathing). In Fig. 8 we show a plot for $n=4$ for a standing subject. In the figure, oscillations occurring every three points can be seen, particularly for (beat numbers)/4 between 130 and 150. These low frequency oscillations, with a period of 12 heartbeats, are presumed to be due to variations in the blood pressure (Mayer waves). To produce a two-dimensional plot, we can plot the blood pressure versus the RR -interval time at every n th heartbeat. Although these are not phase-space variables, a plot of system parameters that depend on each other at a constant phase of an external driving force is analogous to a Poincaré plot for mechanical systems.

Constant Phase Plot ($n=4$)

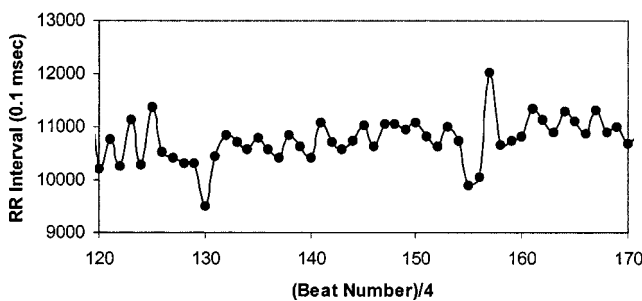
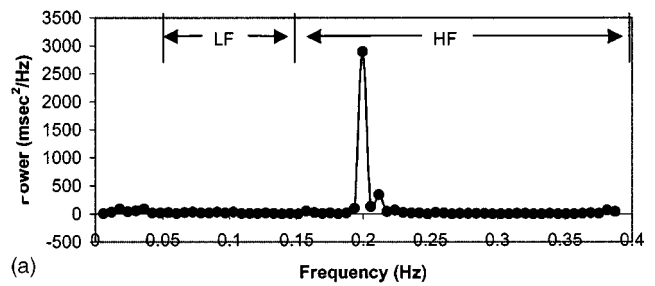


Fig. 8. Plot of the RR -interval for a subject breathing one breath every four heartbeats while standing. The RR -interval is plotted for every fourth heartbeat.

Fourier Spectrum (Lying)



Fourier Spectrum (Standing)

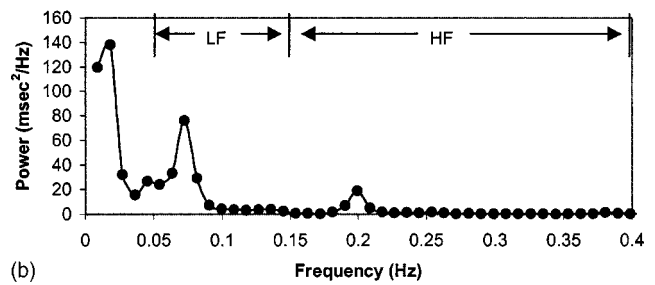


Fig. 9. Fourier spectrum of the RR -interval times for (a) a subject lying and (b) standing. The subject is breathing at 12 breaths/min in both cases. LF and HF refer to the low and high frequency bands used by exercise scientists. The FFT is calculated using 256 data points.

VII. SPECTRAL ANALYSIS

Spectral analysis is a common tool in physics and can be applied to RR -interval data. In practice it is used to separate out the high frequency beat-to-beat variations due to breathing from the low frequency variations due to interactions with the rest of the body.²⁴ The RR -interval times serve as an interesting data set for instruction in Fourier transform techniques.

To observe the low frequency oscillations, it is best to take data with the subject in the standing position breathing at a fixed rate faster than 10 breaths/min so that the higher frequency peak due to breathing does not lie in the low frequency range.²⁵ A common practice in exercise science is to plot the power density spectrum of the heart rate variability. The power spectral density is proportional to the absolute square of the FFT amplitude. We plot the spectrum in Figs. 9(a) (lying) and 9(b) (standing) for which the subject is breathing with a frequency of 12 breaths/min, or 0.2 Hz. Note the narrow peak at 0.2 Hz in both spectra. The broad low frequency peak centered at 0.07 Hz is significantly larger in the standing position.

The power density spectrum plotted in Figs. 9(a) and 9(b) is calculated using a simple FFT with 256 points. The low frequency peak is not always clean and narrow, and it is believed that the amplitude of this peak is related to sympathetic activity.²⁶ Because the low frequency peak often is difficult to observe, different methods involving filtering and autoregression have been developed to assist the analysis. Although we usually limit our analysis to an FFT of the raw

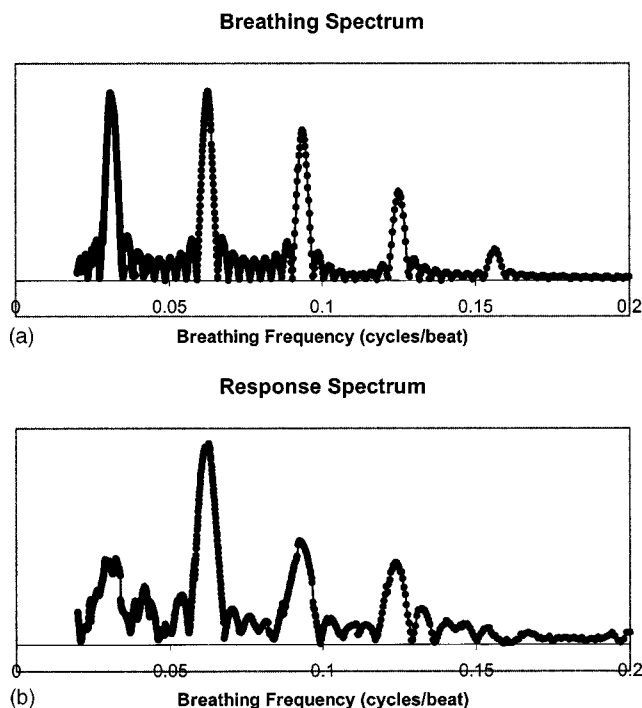


Fig. 10. (a) Fourier spectrum of breathing; (b) RR -interval for a subject breathing a deep breath lasting 16 heartbeats followed by two shallower breaths of eight heartbeats each.

data, these advanced time series analysis methods might be of interest to physics or engineering students.

An interesting application of spectral analysis is the output from breathing with a particular spectral profile. A simple breathing pattern for which the oxygen intake is constant is to breathe one slow deep breath followed by two breaths that are twice as fast and half as deep. In Fig. 10, we show data for a subject breathing the following repeating pattern: first one deep breath lasting 16 heartbeats, then two breaths, each lasting eight heartbeats each with half the depth as the first deep breath. The spectrum corresponding to a sinusoidal function of amplitude A , period T , followed by two sinusoidal functions each of amplitude $A/2$, period $T/2$, is shown in Fig. 10(a). The spectrum of the RR -interval time series for a subject breathing in this way is shown in Fig. 10(b). In both Figs. 10(a) and 10(b), a discrete Fourier transform was performed over five cycles. For each peak in the breathing spectrum, there is a corresponding peak in the heart rate spectrum. The relative response amplitudes correspond to that of Fig. 5(a), indicating a fairly linear response.

VIII. SCALING AND NONLINEAR ANALYSES

Most of the research done by physicists in heart rate variability has been done in the area of nonlinear dynamics and chaos. Physicists have contributed to the development of mathematical methods using correlations, the correlation dimension, fractal dimension, detrended fluctuation analysis,⁵ wavelets,⁶ entropy,⁸ for example, to obtain a better understanding of the underlying complex dynamics of heart rate control. Sometimes large data sets are needed for these calculations, and are best collected while the subject sleeps through the night. For a student exercise, 4000 data points are sufficient to observe interesting results and can be col-

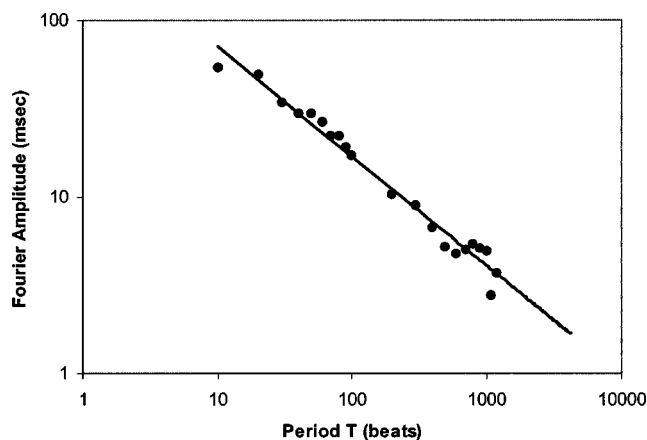


Fig. 11. A discrete Fourier transform of the RR -interval times for a subject lying and breathing normally. 4000 data points were used in the transform.

lected in a little more than 1 hour. Data collection can be done before class (for example, during a lecture) and analyzed later.

The general approach is to extract a parameter V from N heartbeats that is a measure of variability. One then examines the properties of this parameter for large N . In particular, a power law relationship often exists:

$$V \propto N^\beta. \quad (2)$$

It is beyond the scope of this article to discuss all the methods in-current use, and the interested reader is directed to the research articles.⁴⁻⁸ Here we discuss two simple applications, which can be used with 1 hour worth of data.

It has been observed²⁷ that heart rate variability data exhibit $1/f$ noise scaling. The students can demonstrate this scaling by taking a Fourier transform of the RR -interval data. Such a transform is best accomplished from a stationary time series. We first take the difference of successive RR -interval times, $\delta_i = t_{i+1} - t_i$, and then Fourier transform δ_i . In Fig. 11 we plot the discrete Fourier transform of δ_i for 4000 heart beats while a subject was sitting. Figure 11 is a log-log plot of the average Fourier amplitude versus the period $T = 1/f$. The interesting observation is the power law relation between the amplitude and period (frequency). It is believed that data from healthy hearts have a power law relation between the amplitude and the frequency. When more data is taken, the linearity of the log-log plot of Fig. 11 holds up to periods as long as 24 hours.⁴ Unhealthy heart rate control produces a kink in the log-log plot.⁴ The meaning of the power exponent, β , using a Fourier or wavelet basis⁶ is a topic of current research.

The second application is analogous to experiments done to demonstrate the statistics of nuclear counting. A standard method for showing the Poisson statistics of nuclear counting is to record data many times for a specific counting time. The mean number of counts, N , and the standard deviation, σ , are calculated from the data. The students then determine if σ is equal to the square root of N within the limits of the experiment. We can also repeat the experiment with different values for N by changing the counting time or the source-detector geometry. A graph of $\log N$ versus $\log \sigma$ should produce a straight line with slope $1/2$, demonstrating that the variability scales as $N^{1/2}$ for radiation counting.

Standard Deviation vs. N

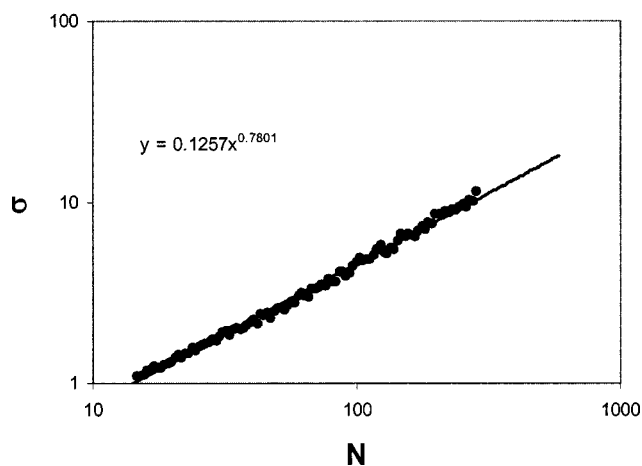


Fig. 12. A log-log graph of the standard deviation σ versus N , the average number of heartbeats.

A similar analysis can be performed with the RR -interval times. From a series of RR -interval times, we can calculate the average number of heartbeats, N , and its standard deviation, σ , for a particular counting time T_c . For example, take T_c equal to 1 minute. One hour of data gives 60 numbers corresponding to the number of heartbeats for each of the 60 1 minute intervals. From these 60 numbers, we can calculate the average and standard deviation. We then repeat the analysis using the same data, with a different counting time, and consequently a different N and σ . In Fig. 12 we plot $\log N$ versus $\log \sigma$ for 1 hour of RR -interval data. We have chosen our shortest time T_c to be 20 s, because this duration is just above the period for low frequency oscillations. For $T_c = 20$ s, 1 hour of data gives 180 counting periods and thus good statistics. We have chosen the longest time T_c to be 250 s, which gives N 250. For $T_c = 250$ s, 1 hour of data gives 14 counting intervals and the statistics become marginal. As seen in the example of Fig. 12, a remarkable scaling relation results with a slope 0.75. We find that usually power law scaling occurs, with slopes varying between 0.6 and 0.9. A slope of $1/2$ results from random processes, and a slope of 1 occurs if the variability is proportional to N . The heartbeat data lie somewhere in between these two values. The significance of the value of the scaling exponent (slope) to health and/or fitness is not known.

A common technique in the analysis of nonlinear systems is to plot a return map from a time series. The classic example is that of a dripping faucet^{28,29} in which a plot of t_i versus t_{i+1} uncovers period doubling and a strange attractor when the faucet is dripping chaotically. The same approach has been applied to RR -interval data. Usually the difference $\delta_i = t_{i+1} - t_i$ is plotted versus δ_{i+n} where n is some delay. Because the heart is a complex system with many factors affecting the RR -interval times, a return map for a healthy subject generally produces a blob of points. Even if the subject breathes one breath every n heartbeats, a plot of δ_i versus δ_{i+n} usually does not reveal any simple underlying dynamics. For subjects with heart problems, on the other hand, a return map can yield plots of distinctively different shapes.³⁰ Research is ongoing on how to make the technique of return maps more useful in the analysis of heart rate

Cool-down (Lying Position)

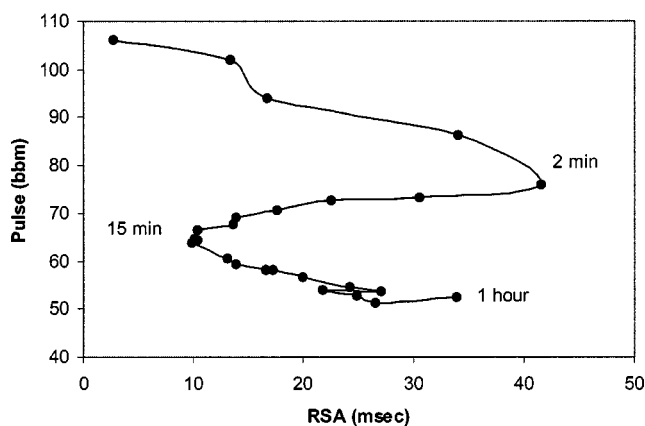


Fig. 13. A plot of the heart rate in beats/min versus the respiratory sinus arrhythmia amplitude as the subject cools down in the lying position after exercise. The subject is breathing at 12 breaths/min. Time was started ($t = 0$ min) a few minutes after the exercise was completed.

variability.³¹ In the student lab, comparing the return map from RR -interval data to that of simpler systems would be an instructive and interesting exercise.

IX. HEART-RATE VARIABILITY AFTER EXERCISE

There are numerous examples in physics and biology for which a system decays (or grows) exponentially. Immediately after exercise the heart rate drops and after a while reaches steady state. It is tempting to imagine that this decay is exponential, but there is no obvious reason to believe that the rate of change of the RR -interval times is proportional to the difference between the RR time and its value in the steady state. Exponential decay might be a good approximation for certain time intervals, but it is found that the decay is not exactly exponential.³²

Instead of examining the time change of only one variable, it is better to consider how two system parameters vary as the body changes from one state to another. This approach is used in thermodynamics, in which the relation of macroscopic parameters gives information about the process the system is undergoing. For example, if a gas undergoes a quasistatic process, a P - V plot can be used to determine if the process is isobaric, isometric, isothermal, adiabatic, or is more complicated. For the heart, the average heart rate (or RR -interval time) and its variability are good parameters to plot in order to identify the heart rate control process that is taking place.

As an example, in Fig. 13 we plot the average heart rate as a function of the respiratory sinus arrhythmia amplitude (RSA) as the subject cools after exercise. The RSA is approximately proportional to the parasympathetic activity. Thus, for processes dominated by parasympathetic change, the heart rate will decrease (or increase) and the RSA will increase (or decrease) in concert. In Fig. 13, the first 2 minutes of the cool down and the last 45 minutes have this feature. The last stage, from 15 minutes to 1 hour, takes place very slowly, and can be classified as a quasistatic process. To first order in P , the RSA amplitude equals kP , where k is a constant. Equation (1) becomes $B = mB_0(1 - (RSA)/k)$. Because the process during the final 45 minutes is approximately a straight line on the graph, this stage of the cooling

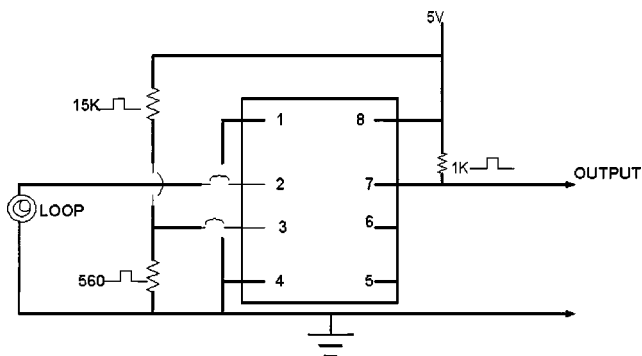


Fig. 14. Connections to the comparator chip, Motorola LM334, used to measure the RR -interval times. The loop of wire is wrapped around the heart rate monitor belt. The output is to the HC11 or the parallel port of a personal computer.

is isosympathetic (that is, m is constant). We can do a linear fit to this stage to obtain the physiological parameters mB_0 and P from the intercept and slope. Life science students may find it interesting that the same methods of analysis used in physical systems can be applied to biological systems.

X. SUMMARY

We have described several methods that can be applied to heart rate data. In general, we find that students are quite interested in heart rate dynamics, because it pertains to their health and fitness. Bringing it into the physics program can increase enthusiasm and interest, and allow the students to literally put their heart into physics.

ACKNOWLEDGMENTS

We would like to thank technician Mark Harnetiaux for his assistance with the construction of the hardware, and Professor Yi Cheng from the Cal Poly Pomona Electrical Engineering Department for all his instruction in the programming of the HC11. One of us (P.S.) would like to thank the Institute for Sports and Preventative Medicine at the University of Saarland for their kind hospitality during his sabbatical stay.

APPENDIX

The RR -interval times can be measured quite easily using a heart rate monitor belt. The heart-rate belt is worn around the chest and emits a signal whenever the R peak of the EKG signal is detected. The time from the start of one transmitted signal to the start of the next signal is the time between the successive heartbeats. The Polar heart rate monitor belt that we used transmits a 5 kHz signal which lasts 2 ms (35 cycles). The signal can be detected by placing a coil of wire around the center of the belt as shown in Fig. 2. Using a coil with 80 turns produces an electrical signal that has a peak-to-peak voltage of 0.8 volts. To obtain a clean digital signal (TTL), we connect the coil to the input of a comparator chip. The circuit is shown in Fig. 14. For the Motorola LM334 comparator chip, the voltage on pin 2 is compared to the reference voltage on pin 3. Dividing 5 volts by a 15 000 Ω and 556 Ω resistors in series produces a reference voltage of

0.1 volts for pin 3. Because the signal is very clean, the start of the first cycle of the transmitted wave triggers a 5 volt output on pin 7.

The time between successive heartbeats can be measured by sampling the output voltage on pin 7 of the chip. When the voltage jumps to 5 volts, a timer is read. After a pause of greater than 7 ms, the transmitted signal has finished and the voltage is back to zero. When the voltage jumps to 5 V again, the timer is read. This process is repeated, and the differences in the times are the RR -intervals. One can use the timer on a microprocessor or the system clock on a personal computer as a timer.

We use the parallel port to interface to a personal computer. The chip ground is connected to pin 24 on the parallel port, and the chip output from pin 7 is connected to pin 10 on the parallel port. Use of the PC timer is described in Ref. 33. The RR -interval times are stored in an array and then saved on disk after the measurements have ended.

When using the HC11, we use the input capture interrupt to detect the signal. We use interrupt service routines to detect the signal, read the timer, and update the timer overflow. The RR -interval times are stored in an array and transferred to a personal computer via the serial port after the measurements are made.

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³Between 1993 and 2002 there were 14 articles on heart rate variability published in *Phys. Rev. Lett.* and 9 articles in *Phys. Rev. E*. References 4–8 listed might be of interest to undergraduate students.

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¹²We used a monitor belt from Polar, Inc.

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