Attenuation of Radiation in Matter

In this experiment we will examine how radiation decreases in intensity as it passes through a substance. Since radiation interacts with matter, its intensity will decrease as it travels through a material. The attenuation properties of radiation will effect how much shielding is necessary and how much dosage one recieves. Since radiation deposits its energy as it travels though matter, the amount of dosage one receives depends on how the radiation is attenuated. Radiation that attenuates fast, i.e. loses its energy in a short distance, will give a higher dosage (near the surface) than radiation that attenuates slowly, i.e. loses its energy over a long distance in the material.

First we consider how the intensity of gamma particles decreases as they pass through a material. We will see that the attenuation properties of gamma particles can be described fairly simply. The attenuation of β and α particles is more complicated.

Attenuation of gamma particles

As gamma particles pass through matter, they interact with the material and get absorbed. Consider the following situation shown in the figure below:

Figure of gamma particles incident on a slab.

Gamma particles are incident from the left on a slab. The slab has a thickness of x. After passing through the slab of material, the gamma particles emerge on the right. Let the intensity of the gammas incident from the left be denoted as I_0, the initial intensity. Let the intensity of the gammas that emerge on the right, after passing though the slab, be denoted as $I(x)$, the final intensity. The unit for intensity, I_0 and $I(x)$, is (number of gamma particles)/area/time. For units of intensity we could also use energy/area/time. Either type of unit could be used in our applications, but since our detectors measure counts, we will use (number of gammas)/time. As the slab gets thicker, x gets larger, and $I(x)$ becomes smaller. As x increases, more radiation is absorbed in the material and less passes through and emerges on the right side.

How does I(x) depend on x? To a very good approximation the number of gammas that pass through the slab decrease exponentially with thickness:

$$I(x) = I_0 e^{-\alpha x} \tag{1}$$

This equation is referred to as Lambert's law, and is applicable for linear attenuation. The geometry for linear attenuation is that the material is a rectangular slab, and the gamma's are incident perpendicular to the slab. The attenuation is in a "line" and is uniform in the plane of the slab. The units of α are (1/distance) since αx is unitless. In practice x often has units of cm, so α will have units of cm^{-1}. The parameter α is called the linear attenuation coefficient.

The linear attenuation coefficient, α, will depend on the type of material and on the energy of the gamma radiation, E_γ. When gamma particles travel through matter, any interaction that occurs will be with the electrons in the substance. The higher the density of electrons, the more the gamma particles will attenuate. That is, if you want to shield yourself from gamma radiation, you need to put lots of electrons between yourself and the source. Since the electron density is approximately proportional to the density of the material, the attenuation coefficient α depends strongly on the density of the substance. For this reason, it is convenient to define a mass attenuation coefficient by dividing α by the density ρ of the material. Starting with the equation for linear attenuation:

$$I(x) = I_0 e^{-\alpha x} = I_0 e^{\frac{\alpha}{\rho}(\rho x)} \tag{2}$$

$$I(x) = I_0 e^{-\mu z} \tag{3}$$

where $z \equiv \rho x$, and the mass attenuation coefficient μ is defined as α/ρ. The linear attenuation coefficient is just the mass attenuation coefficient times the density of the material:

$$\alpha = \mu\rho \tag{4}$$

The units of μ are (area/mass), and are commonly given in (cm^2/g). Since μz is unitless, z is in units of (mass/area) or (g/cm^2). The "thickness" z can be thought of as the number of grams of "thickness" per square centimeter area of the slab. That is, for one square centimeter of area, z is the number of grams of the substance that the gamma's travel through. The more mass there is, the more electrons there are, and the more attenuation. Materials with a high density will have a large z (lots of electrons) for a small thickness, and are good for shielding gammas.

In tables of attenuation coefficients, it is the mass attenuation coefficient that is always listed. This is because the mass attenuation coefficient, μ, has the density dependence somewhat factored out. The mass attenuation coefficient mainly depends on energy. For a fixed energy, μ is roughly the same for a large class of materials. When performing an attenuation calculation, one looks up the mass attenuation coefficient in the tables, multiplies by the density of the substance to obtain α, then uses the linear attenuation equation.

Why does the attenuation follow an exponential formula? Such a simple relationship must derive from a simple reason. The derivation of Lambert's law can obtained by considering a very thin slice of the slab of thickness Δx:

Figure of thin slice of slab

Let $I(x)$ be the intensity of the radiation incident on the thin slab from the left, and let $I(x + \Delta x)$ be the intensity of the radiation after passing through the thin slab. From quantum mechanics, the interaction of gamma particles (photons) with electrons is probabilistic. Thus, any particular gamma particle has a certain probability of being scattered with an electron in the thin slab. Since the number of electrons the gamma will encounter is proportional to Δx, then for small enough Δx the probability that a gamma will interact with an electron in the thin slab is proportional to its thickness:

$$Probability\ for\ an\ interaction = \alpha(\Delta x) \tag{5}$$

As with radioactive decay, probability enters in the description of nature. In radioactive decay, the parameter λ was equal to the probability to decay per unit time. Here, the parameter α equals the probability for the gamma to interact per unit length. Now if one has $I(x)$ particles at a distance x, and $I(x + \Delta x)$ particles at a distance $x + \Delta x$ into the slab, the number of particles lost per area is just the difference $I(x) - I(x + \Delta x)$. However, the number lost per area equals the number incident times the probability for any one gamma to interact with an electron in the slab:

$$I(x) - I(x + \Delta x) = (probability\ for\ one\ gamma\ to\ interact)I(x) \tag{6}$$

$$I(x) - I(x + \Delta x) = \alpha(\Delta x)I(x) \tag{7}$$

Dividing both sides by Δx and multiplying by -1 gives

$$\frac{I(x + \Delta x) - I(x)}{\Delta x} = -\alpha I(x) \tag{8}$$

Taking the limit as Δx goes to zero gives a simple differential equation for I:

$$\frac{dI}{dx} = -\alpha I \tag{9}$$

The general solution to this differential equation is Lambert's law:

$$I(x) = I_0 e^{-\alpha x} \tag{10}$$

where I_0 is the initial intensity at x=0. The derivation gives us two results: the reason Lambert's law is valid, and the meaning of the linear attenuation coefficient. The validity of Lambert's law follows from the fact that the interaction of photons with electrons is probabilistic, and a single interaction removes the gamma from the beam. Since the parameter α in our derivation is the same as the linear attenuation coefficient, we have a physical interpretation for α. The linear attenuation coefficient is the probability per unit length that a gamma particle will undergo an interaction in the material. If α is large(small), the probability of an interaction is high(low) and the beam attenuates in a short(long) distance.

We note that Lambert's law applies to gamma particles of a single energy, mono-energetic gammas. In the derivation, we assumed that once the gamma particle interacted with an electron, it was removed. The gamma is removed when the process photo-absorption, but if Compton scattering occurs the gamma has not been absorbed. In Compton scattering, the gamma particle is scattered out of the beam with a lower energy. Thus, $I(x)$ in Lambert's law refers to the intensity of gammas with a particular energy.

The mass attenuation coefficient μ does not have a particularly strong dependence on the type of material, but μ will vary dramatically with energy. For low energy gamma particles (or X-rays) μ will be much larger than for high energy gammas. This is because photo-absorption is much more probable at lower energies. Below we show a graph of the mass attenuation coefficient as a function of energy. Shown are three different curves for three different processes: photo-absorption, Compton scattering, and pair production. Note the different energy dependences for each process.

Figure of mass attenuation coefficient as a function of energy.

Since photo-absorption is the process for producing a photopeak in our scintillation detectors, μ for photo-absorption is related to the efficiency of our detectors. Note the particularly strong energy dependence of the mass attenuation coefficient for photo-absorption.

Examples of Gamma Shielding Calculations:

Example 1: 5×10^6 gamma's, each with energy 600KeV, are incident on a lead shielding of thickness 2cm. How many pass through?

One first needs to find the mass attenuation coefficient for lead for a gamma with energy 600KeV. From the tables, we find that $\mu = 0.125$ cm^2/g. Since the density of lead is 11.35 g/cm^3, the linear attenuation coefficient is: $\alpha = \mu\rho = 11.35(0.125) = 1.42$ cm^{-1}. Using the attenuation equation we have:

$$I = 5 \times 10^6 \, e^{-1.42(2)} \approx 292,000 \tag{11}$$

That is, 292,000 gamma's pass through the lead.

Example 2: How thick should a lead shield be in order to reduce the intensity of 600KeV gamma radiation by a factor of 1000?

As before, one needs to find the value of the mass attenuation coefficient for lead for 600KeV gammas. From the tables we find that $\mu = 0.125$ cm^2/g. The density of lead is 11.35 g/cm^3, so $\alpha = \mu\rho = 0.125(11.35) = 1.42$ cm^{-1}. Since $I = I_0/1000$, we have

$$\frac{1}{1000} = e^{-1.42x} \tag{12}$$

x can be solved by taking the natural log (or log) of both sides:

$$x = \frac{ln(0.001)}{-1.42} = 4.86 \; cm \tag{13}$$

Example 3: How thick should a lead shield be in order to reduce the intensity of 30 KeV X-ray radiation by a factor of 1000?

As before, one needs to find the value of the mass attenuation coefficient for lead for 30 KeV X-rays. From the tables we find that $\mu = 29.7$ cm^2/g. The density of lead is 11.35 g/cm^3, so $\alpha = \mu\rho = 29.7(11.35) \approx 337$ cm^{-1}. Since $I = I_0/1000$, we have

$$\frac{1}{1000} = e^{-337x} \tag{14}$$

x can be solved by taking the natural log (or log) of both sides:

$$x = \frac{ln(0.001)}{-337} = 0.02 \; cm \tag{15}$$

It is much easier to shield against 30 KeV X-rays than 600 KeV gamma radiation.

Example 4. Suppose now that you want to use aluminum to shield the X-rays of example 3. What thickness of aluminum is needed to reduce the intensity of 30 KeV X-rays by a factor os 1000?

The mass attenuation coefficient for aluminum for X-rays of energy 30 KeV is $\mu = 1.12$ cm^2/g. The density of aluminum is 2.7 g/cm^3, which gives a value of $\alpha = \rho\mu \approx 3.02$ cm^{-1}. Since $I = I_0/1000$,

$$\frac{1}{1000} = e^{-3.02x} \tag{16}$$

As before, x can be solved by taking the natural log (or log) of both sides:

$$x = \frac{ln(0.001)}{-3.02} \approx 2.29\ cm \tag{17}$$

You can see why lead is a better material for shielding X-rays than aluminum.

Absorption of Alpha Particles

Alpha particles are Helium nuclei, and are hence relatively massive particles. They do not penetrate as far into matter as do beta's or gamma's. Their penetrability depends on the energy of the alpha particle. For alpha's with an energy less than 8 MeV, the range in air in centimeters is approximately equal to the energy of the alpha in MeV. That is $R \approx E$ cm/MeV.

In matter, alphas hardly penetrate at all. For example, in water, 8 MeV alpha's only penetrate 40×10^{-6} meters. If the energy of the alpha is less than 7.5 MeV, it will not penetrate the skin (for humans).

When dealing with alpha emitters, one must be sure that they do not get ingested. As long as alpha emitters are kept outside the body, they can be easily shielded.

Absorption of Beta Particles

Beta particles are electrons, and travel further in materials than alphas, but not as far as gamma particles. In materials, the attenuation of betas is approximately exponential with distance as with gammas. However, as beta particles slow down, they lose energy quicker.

In air, the range for beta particles is approximately 12 feet for every MeV of energy. In water, a 1 MeV beta has a range of around 0.2 inches. A beta needs 70

KeV of energy to penetrate the skin (for humans). Externally, the main danger for beta emitters is if they get too close to the skin. Radiation damage can occur close to the skin, within the first 1 or 2 mm of the surface. Thus, it is always good practice to use tongs when handling beta emitters. If ingested, beta emitters can be very dangerous.

The energy of the beta particles emitted in beta decay is continuous. The average energy is around 1/3 of the maximum energy. Betas that are emitted with high energy are more dangerous than low energy emitters. Some common beta emitters that are used in biology are:

Isotope	Half-life	Maximum Energy (KeV)
^{32}P	14 days	1710 KeV (Watch out!!)
^{3}H	12.26 years	18.6 KeV
^{14}C	5730 years	156 KeV
^{35}S	88 days	167 KeV