Exponentials and Logarithms

The purpose of this appendix is to review some properties of exponentials and logarithms. Exponential and logarithm functions arise often in radiation detection since the activity of a radioisotope and the attenuation of the intensity decrease via an exponential function. First we summarize the properties of exponentials, then logarithms.

Properties of Exponentials

There are two main properties of operations with exponentials that pertain to our calculations:

$$\begin{aligned} a^x a^y &= a^{x+y} \\ (a^x)^y &= a^{xy} \end{aligned}$$

where a, x and y are real numbers. From these properties, we can derive the following:

$$a^0 a^x = a^{0+x} = a^x \tag{1}$$

this gives

$$a^0 = 1 \tag{2}$$

One can interpret negative exponents using:

$$a^{-1} a^1 = a^{1-1} = a^0 = 1 \tag{3}$$

Thus we see that

$$a^{-1} = \frac{1}{a} \tag{4}$$

Likewise, for fractional exponents

$$a^{1/2} a^{1/2} = a^{1/2+1/2} = a^1 = a \tag{5}$$

Thus, $a^{1/2}$ is equal to the square root of a. An important base for taking exponentials is the natural logarithm base e:

$$e = \lim_{n\to\infty} (1 + \frac{1}{n})^n \approx 2.71828 \tag{6}$$

You need to become familiar with how to use your calculator to calculate expoentials. Check out the 4 examples below:

$$
\begin{aligned}
8e^{-2} &\approx 1.08 \\
10(\frac{1}{2})^{7/2} &\approx 0.884 \\
e^{-1.5} &\approx 0.223 \\
10(\frac{1}{2})^{2} &= 2.5
\end{aligned}
$$

Properties of Logarithms

The logarithm is the inverse of the exponential function. If $z = y^x$, then $log_y z = x$. Two useful properties of logarythms (which can be derived from the properties of exponentials) are

$$
\begin{aligned}
log_y(ab) &= log_y(a) + log_y(b) \\
log_y(a^b) &= b\ log_y(a)
\end{aligned}
$$

An example of when you might use logs in this class is the following equation for which you need to solve for t:

$$6 = 17(\frac{1}{2})^{t/10} \tag{7}$$

t might be the time it takes for the activity to decrease from 17 to 6 for an isotope with a half-live of 10. t is found by carrying out the following steps. First divide both sides by 17:

$$\frac{6}{17} = (\frac{1}{2})^{t/10} \tag{8}$$

Next take the log of both sides:

$$log(\frac{6}{17}) = \frac{t}{10} log(\frac{1}{2}) \tag{9}$$

Finally, solve for t:

$$t = 10\ \frac{log(6/17)}{log(1/2)} \approx 15.0 \tag{10}$$

For the equation above, it doesn't matter which base you use for the logarithm. You can use base 10, which is probably the log function on your calculator, or base e, which is the ln function.

Here is another example for which you need to solve for x:

$$.02 = .1e^{-5x} \tag{11}$$

In this case, x could be the shielding thickness you need to reduce the dosage from .1 Rem/hr to .02 Rem/hr. In this case one first divides both sides of the equation by .1

$$.2 = e^{-5x} \tag{12}$$

Then take the natural log of both sides:

$$ln(.2) = -5x \tag{13}$$

Now one can solve for x:

$$x = \frac{ln(.2)}{-5} = .32 \tag{14}$$

Be sure you know how do use the expontial and logarithm functions on your calculator.